Optimal Fiscal Policy in a Small Open Economy with Limited Commitment*

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May 2009

Abstract

In this paper we analyze how the tax-smoothing result obtained in models of optimal fiscal policy is altered in a context of international risk sharing with limited commitment. We consider the problem of a benevolent government that has to choose optimally distortionary taxes on labor income and transfers from the rest of the world. The contract between the government and the rest of the world is designed so that at any point in time, neither agent has incentives to exit the contract and there is no net transfer of wealth between them. Our analytical results suggest that the presence of limited commitment alters substantially the dynamics of the fiscal variables with respect to the full commitment case. In particular, the volatility of the tax rate is higher than the volatility of the government expenditure shock since the former responds strongly to the incentives to default of both agents. Moreover, optimal taxes are procyclical. Our findings are in line with the evidence of fiscal policy in developing countries.

*We would like to thank Albert Marcet for constant encouragement and guidance. We also benefited from useful discussions with Alexis Anagnostopoulos, Eva Cárceles-Poveda, Begoña Dominguez, Patrick Kehoe, Michael Reiter, Jaume Ventura and seminar participants at the UPF Macro Break, XII Spring Meeting of Young Economists, EEA - ESEM 2007 Congress, ASSET 2007 Conference, 32° Simposio de Análisis Económico and La Pietra-Mondragone Workshop 2008 for useful comments and suggestions.

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1 Introduction

A fundamental question in macroeconomics is how a policymaker has to set distortionary taxes in order to finance an exogenous public expenditure shock. The answer to this question depends on both the degree of openness of the economy and on the commitment technology of the government to fulfill its obligations towards foreign investors.

Consider a setup in which a small open economy, which we call the *home country*, can trade assets with the rest of the world. The government of the home country has to collect revenues optimally in order to finance an exogenous stream of public expenditure, while the rest of the world is subject to no shock. In this case, a benevolent government of the home country would set taxes roughly constant over the business cycle. When a bad shock hits the economy, the government can borrow from abroad and pay back the debt later on, when the economy faces instead a good shock. In this way, the possibility to do risk-sharing with the rest of the world implies that the deadweight losses associated to distortionary taxation are minimized. In the extreme case in which the rest of the world is risk neutral, the optimal tax rate is perfectly flat and all fluctuations in public expenditure can be absorbed by international capital flows. It follows that, at least from a theoretical point of view, tax volatility in small open economies should be lower than the tax volatility in large or closed economies, thanks to the insurance role played by international borrowing and lending.

Nevertheless, this conclusion does not seem to be validated in the data. Table 1 shows some statistics for government expenditure and average tax rate series in Argentina and in the USA.[1]. Although the variability of the government expenditure series is roughly the same in the two countries, tax rates in Argentina are much more volatile than in the USA: the standard deviation of the series for Argentina is almost 60% higher than the one for the US economy. As can be seen from Table 2, this empirical evidence applies to other countries as well, for the same sample period.

In this paper we introduce sovereign risk into a standard optimal fiscal policy open economy model as the one described before by relaxing the assumption of full commitment from the home country and the rest of the world towards their contracted obligations. We show that this

[1] The series for USA are from the Bureau of Economic Analysis of the US Department of Commerce. In the case of Argentina, the data we use is from the IMF, INDEC and Ministerio de Economia. We use quarterly data of current government expenditure net of interest payments plus gross government investment as a measure of government expenditure, and total tax revenues plus contributions to social security as a measure of total tax revenues. The average tax rate is calculated as the ratio between tax revenues over GDP. Due to reliability/availability of data for Argentina, we use data for the period $1993-2005$.

Table 1: Fiscal variables for the USA and Argentina

	USA		ARG	
	Govt. expend.	Tax rate	Govt. expend.	Tax rate
Mean	0.1755	0.1850	0.1704	0.1828
St. deviation	0.0092	0.0130	0.0098	0.0214
Coef. of variation	0.0525	0.0704	0.0573	0.117

Table 2:

Country	Tax rate coefficient of variation
Bulgaria	0.104
Guatemala	0.136
Nicaragua	0.139
Venezuela	0.13

framework provides a theoretical justification for the tax rate volatility observed in small open economies that have commitment problems to repay their external obligations.

In the model the home country is populated by risk adverse households. The fiscal authority has to finance an exogenous public expenditure shock either through distortionary labor income taxes or by issuance of internal and/or international debt. The rest of the of world is inhabited by risk-neutral agents that receive a constant endowment and have to decide how much to consume and how much to borrow/lend in the international capital market. We assume that neither the government in the home country nor the rest of the world can commit to pay back the debt contracted among themselves.

A contract, signed by the two countries, regulates international capital flows. The terms of the contract depend on the commitment technology available to the two parts to honor their external obligations. When both countries can fully commit to stay in the contract in all states of nature, the only condition to be met is that *ex-ante* there is no exchange of net wealth among them. Instead, when the countries may at some point decide to leave the contract, further conditions need to be imposed. In particular, since default takes place if the benefit a country obtains from staying in the contract is smaller than its outside option, the contract must specify an adjustment in the allocation necessary to rule out default in equilibrium.

We show that the presence of sovereign default risk, i.e., the possibility that a country may exit the contract with the other country, limits the amount of risk-sharing among countries. Consequently, the classical tax-smoothing result is broken since now the optimal tax rate depends on the incentives to default of both countries. In particular, when the home country wants to exit the contract it has to be compensated so that the benefits of staying in it equal the value of its outside option. Therefore, consumption and leisure have to increase, and the tax rate decreases. On the other hand, when the rest of the world has incentives to default, the tax rate in the home country increases to pay back the external debt and induce the rest of the world not to leave the contract.

In our model, the home country has incentives to exit the contract when the realization of the public expenditure shock is low. There are two reasons behind this feature. First, the benefits from staying in the contract decrease, since in this case the home country has to repay its foreign debt. The second reason relies on our definition of the outside option for the home country. We assume that, if the home country defaults, its government is forced to run a balanced-budget thereafter. The outside option is the expected life-time utility of the households under this fiscal policy plan. When the shock is low, the tax rate is low as well, so the outside option increases. Therefore, an important corollary of the analysis is that the optimal fiscal policy is pro-cyclical: tax rates decrease when the country has incentives to leave the contract, and this happens when public consumption is low. This conclusion is in line with recent evidence for developing countries (see e.g. Ilzetzki and Vegh (2008) and Cuadra and Sapriza (2007)).

Some possible alternative explanations for the high volatility of tax rates observed in developing economies rely on the quality of their institutions and the sources of tax collection. It is argued that developing countries are more prone to switches in political and economic regimes that, almost by definition, translate into unstable tax systems. Moreover, in booms these countries' often tax heavily those economic sectors that are responsible for the higher economic activity[2]. As a consequence, when economic conditions deteriorate, necessarily tax revenues go down dramatically. We are aware that these considerations are relevant sources of tax variability and that our study does not incorporate them in the analysis. However, we do not intend to provide an exhaustive description of such sources. In this sense, by focusing on sovereign risk and incomplete international capital markets as causes for the high tax rate volatility of developing economies we are carrying out a partial analysis of the phenomenon.

In the recent years there have been some attempts to add default to dynamic macroeco-

[2]As an example, in the recent years Argentina has been experiencing rapid export-led growth, mainly due to exports of commodities such as soya. In this period, the government's main source of tax revenues has come from taxation of these exports.

nomic models. A number of papers (Arellano (2008), Aguiar and Gopinath (2006), Hamann (2004)) have introduced sovereign default in otherwise standard business cycle models in order to quantitatively match some empirical regularities of small open developing economies. More specifically, they adapt the framework of Eaton and Gersovitz (1981) to a dynamic stochastic general equilibrium model. These models are usually able to explain with relative success the evolution of the interest rate, current account, output, consumption and the real exchange rate. Nevertheless, since they all consider endowment economies, they fail to capture the effects of default risk over the taxation scheme. Our contribution is to extend the analysis to be able to characterize the shape of fiscal policy and the links between the risk of default and taxes in a limited commitment framework.

Many papers have introduced the idea of limited commitment to study many important issues. Among others, Kehoe and Perri (2002) introduce credit arrangement between countries to reconcile international business cycle models with complete markets and the data, Krueger and Perri (2006) look at consumption inequality, Chien and Lee (2008) look at capital taxation in the long-run, Marcet and Marimon (1992) study the evolution of consumption, investment and output, and Kocherlakota (1996) analyzes the properties of efficient allocations in a model with symmetric information and two-sided lack of commitment. To our knowledge, none of them has focused on the impact of the possibility of default on the volatility of optimal taxation.

The closest papers to ours are probably those by Cuadra and Sapriza (2007), Pouzo (2008) and Scholl (2009). The first paper focuses on matching some stylized facts in developing countries, namely the positive correlation between risk premia and the level of external debt, higher risk premia during recessions and the procyclicality of fiscal policy in developing economies. The second paper studies the optimal taxation problem in a closed economy under incomplete markets allowing for default on internal debt. Finally, the third paper analyzes the problem of a donor that has to decide how much aid to give to a government that has an incentive to use these external resources to increase its own personal consumption without decreasing the distortive tax income it levies on private agents.

We differentiate from these papers along various dimensions. In the first place, we consider the full commitment solution instead of the time-consistent one. We do this to isolate the effect of endogenously incomplete markets on the optimal fiscal plan, while giving the government all the usual tools to distribute the burden of taxation across periods and states of the world. In particular, in our framework there is a complete set of state-contingent bonds the government can issue internally. This has important implications for consumption smoothing as it allows the government to distribute the burden of taxation across states. Finally, in contrast with the assumption in Scholl (2009), we focus on the scenario in which the government of the small open economy is benevolent, i.e., its objective is to maximize the expected life-time utility of its citizens.

The rest of the paper proceeds as follows. Section 2 describes the model. Section 3 shows how the optimal fiscal plan is affected by the possibility of default in the case study of a perfectly anticipated one-time fiscal shock. In section 4 we solve the model for the general case of correlated government expenditure shock. Section 5 is devoted to show that our economy can be reinterpret as one in which the government can issue debt subject to debt limits, both on internal and external debt. Section 6 offers some empirical evidence on the relationship between tax volatility and default risk. Section 7 concludes.

2 The Model

We assume that the economy is constituted by two countries: the home country (HC) and the rest of the world (RW). The HC is populated by risk-averse agents, which enjoy consumption and leisure, and by a benevolent government that has to finance an exogenous public expenditure shock either by levying distortionary taxes, by issuing state-contingent internal bonds, or by receiving transfers from the RW. The RW is populated by risk-neutral agents that receive a fixed endowment each period. These resources can be either consumed or lent to the HC. Being an endowment economy without shocks, there is no government activity in this country.

2.1 The contract

The government of the HC can do risk-sharing with the RW by contracting transfers[3]. Let T_t be the amount of transfers received by the HC at time t. There are three conditions that have to be met by $\{T_t\}_{t=0}^{\infty}$.

First, the expected present discounted value of transfers exchanged with the RW must equal zero:

$$E_0 \sum_{t=0}^{\infty} \beta^t T_t = 0 \tag{1}$$

where β is the discount factor of households in the RW and the HC. This condition rules out the possibility that the government of the HC uses resources from the RW for reasons other than the risk-sharing one. In other words, we do not allow for net redistribution of wealth between

[3] In section 5 we show that these transfers can be reinterpreted as bonds traded in the international capital market.

countries at time 0. We call this condition the *fairness condition*, since it implies that *ex-ante* the contract is fair from an actuarial point of view[4].

If we assumed that the two parts in the contract have full commitment to pay back the debt contracted with each other, equation (1) would be the only condition regulating international flows. The allocations compatible with this situation will be our benchmark for comparison purposes. However, when the government in the HC does not have a commitment technology, it may decide to leave the contract if it finds it profitable to do so. Denote by V_t^a the value of the government's outside option, i.e., the expected life-time utility of households in the HC if the government leaves the contract, and by V_t the continuation value associated to staying in the contract in any given period t. Then, in order to rule out default in equilibrium, the following condition has to be satisfied

$$V_t \equiv E_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j}, l_{t+j}) \geq V_t^a \ \forall t \tag{2}$$

This condition constitutes a *participation constraint for the HC*. We assume that, if the government chooses to leave the contract at any given period, it remains in autarky from then on. Moreover, when the government defaults on its external obligations, it also default on its outstanding domestic debt. Consequently, the government is forced to run a balanced budget thereafter[5]. Alternative assumptions to identify the costs of default could be made, for example that the government cannot use external funds, but it still has access to the domestic bonds market to smooth the distortions caused by the expenditure shock. We have chosen the current specification for two reasons. First, this allows us to keep the problem tractable, both from an analytical and a numerical point of view. Second, this specification is consistent with the interpretation that the government is subject to debt limits, as shown in section 5.

Similar to the case of the HC, the RW also lacks a commitment technology and can potentially exit the contract at any point in time. Therefore, we need to impose a *participation constraint for the RM*:

$$E_t \sum_{j=0}^{\infty} \beta^j T_{t+j} \leq \underline{B} \ \forall t \tag{3}$$

[4]This condition implies that the contract is actuarially fair only if the RW has full commitment. This is due to the fact that, if the RW has limited commitment, the risk-free interest rate will not always be $1/\beta$ (see Section 5 for further details). This condition is useful because it allows us to pin down the allocations. However, one can impose other similar conditions that will yield different allocations.

[5]It follows that the only state variable influencing the outside option is the government expenditure shock. Therefore $V_t^a = V_t^a(g_t)$.

This condition is analogous to (17) and states that, at each point in time and for any contingency, the expected discounted value of future transfers the HC is going to receive cannot exceed an exogenous threshold value $\underline{B}$. This restriction, together with the fairness condition, poses an upper limit on how much indebted the RW can get.

As long as conditions (1), (17) and (18) are satisfied, the government of the HC can choose any given sequence $\{T_t\}_{t=0}^{\infty}$ to partially absorb its expenditure shocks.

2.2 Households in the HC

Households in the HC derive utility from consumption and leisure, and each period are endowed with one unit of disposable time. The production function is linear in labor and one unit of labor produces one unit of the consumption good. Therefore, wages $w_t = 1 \ \forall t$.

The representative agent in the HC maximizes her expected lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$

subject to the period-by-period budget constraint

$$b_{t-1}(g_t) + (1-\tau_t)(1-l_t) = c_t + \sum_{g^{t+1}|g^t} b_t(g_{t+1}) p_t^b(g_{t+1}) \tag{4}$$

where c_t is private consumption, l_t is leisure, $b_t(g_{t+1})$ denotes the amount of bonds issued at time t contingent on the government shock in period $t+1$, τ_t is the flat tax rate on labor earnings and $p_t^b(g_{t+1})$ is the price of a bond contingent on the government shock realization in the next period.

The optimality condition with respect to the state-contingent bond is:

$$p_t^b(g_{t+1}) = \beta \frac{u_{c,t+1}(g_{t+1})}{u_{c,t}} \pi(g^{t+1}|g^t) \tag{5}$$

.

where $\pi(g^{t+1}|g^t)$ is the conditional probability of the government expenditure shock. Combining the optimality conditions with respect to consumption and leisure we obtain the intratemporal condition

$$1 - \tau_t = \frac{u_{l,t}}{u_{c,t}} \tag{6}$$

2.3 Government of the HC

The government finances its exogenous stream of public consumption $\{g_t\}_{t=0}^{\infty}$ by levying a distortionary tax on labor income, by trading one-period state-contingent bonds with domestic consumers and by contracting transfers with the RW. The government's budget constraint is

$$g_t = \tau_t(1 - l_t) + \sum_{g^{t+1}|g^t} b_t(g_t + 1)p_t^b(g_t + 1) - b_{t-1}(g_t) + T_t \tag{7}$$

2.4 Equilibrium

We proceed to define a *competitive equilibrium with transfers* in this economy.

Definition 1. *A competitive equilibrium with transfers is given by allocations $\{c, l\}$, a price system $\{p^b\}$, government policies $\{g, \tau^l, b\}$ and transfers T such that*[6]*:*

1. *Given prices and government policies, allocations satisfy the household's optimality conditions* (4), (5) *and* (6).

2. *Given the allocations and prices, government policies satisfy the sequence of government budget constraints* (7).

3. *Given the allocations, prices and government policies, transfers satisfy conditions* (1), (17) *and* (18).

4. *Allocations satisfy the sequence of feasibility constraints:*

$$c_t + g_t = 1 - l_t + T_t \tag{8}$$

2.5 Optimal policy

The government of the HC behaves as a benevolent Ramsey Planner and chooses tax rates, bonds and transfers $\{c_t, b_t, T_t\}_{t=0}^{\infty}$ in order to maximize the representative household's life-time expected utility, subject to the constraints imposed by the definition of competitive equilibrium.

Before studying the consequences of introducing default in terms of the optimal fiscal plan, it is instructive to analyze the benchmark scenario in which both the government in the HC and the RW have a full commitment technology.

[6]We follow the notation of ? and use symbols without subscripts to denote the one-sided infinite sequence for the corresponding variable, e.g., $c \equiv \{c_t\}_{t=0}^{\infty}$.

2.5.1 Full commitment

If both the HC and the RW can commit to honor their external obligations in all states of nature, then conditions (17) and (18) need not be specified in the contract. Then, the problem of the Ramsey planner is

$$\max_{\{c_t,l_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$

s.t.

$$b_{-1} u_{c,0} = E_0 \sum_{t=0}^{\infty} \beta^t (u_{c,t} c_t - u_{l,t}(1 - l_t)) \tag{9}$$

$$c_t + g_t = 1 - l_t + T_t \tag{10}$$

$$E_0 \sum_{t=0}^{\infty} \beta^t T_t = 0 \tag{11}$$

The optimality conditions for $t \geq 1$ are:

$$u_{c,t} + \Delta(u_{cc,t} c_t + u_{c,t} + u_{cl,t}(1 - l_t)) = \lambda \tag{12}$$

$$u_{l,t} + \Delta(u_{cl,t} c_t + u_{l,t} - u_{ll,t}(1 - l_t)) = \lambda \tag{13}$$

where λ is the multiplier associated with constraint (11), and Δ is the multiplier associated with the implementability condition (9). The next proposition characterizes the equilibrium.

Proposition 1. *Under full commitment, consumption, labor and taxes are constant* $\forall t \geq 1$*. Moreover, if* $b_{-1} = 0$*,* $b_t(g_{t+1}) = 0$ $\forall t$*,* $\forall g_{t+1}$ *and the government perfectly absorbs the public expenditure shocks through transfers* T_t*.*

Proof. Using optimality conditions (12) and (13) we have two equations to determine two unknowns, c_t and l_t, given the lagrange multipliers λ and Δ. Since these two equations are independent of the current shock g_t, the allocations are constant $\forall t \geq 1$. From the intratemporal optimality condition of households (6) it can be seen that the tax rate τ_t^l is also constant $\forall t \geq 1$[7]. Finally, the intertemporal budget constraint of households at time 0 (equation (9)) can be written as

[7]Notice that at $t = 0$ the optimality conditions of the Ramsey planner differ from those at $t \geq 1$. This is the standard source of time-inconsistency in this type of problems.

$$\frac{1}{1-\beta}(u_c c - u_l(1-l)) = 0$$

Notice that, for any given time $t+1$, domestic bond holdings $b_t(g_{t+1})$ are obtained from the intertemporal budget constraint of households in that period, i.e.,

$$b_t(g_{t+1})u_{c,t+1} = E_{t+1}\sum_{j=0}^{\infty}\beta^j(u_{c,t+1+j}c_{t+1+j} - u_{l,t+1+j}(1-l_{t+1+j}))$$

However, since the allocations are constant over time, it is the case that

$$b_t(g_{t+1})u_c = E_{t+1}\sum_{j=0}^{\infty}\beta^j(u_c c - u_l(1-l)) = \frac{1}{1-\beta}(u_c c - u_l(1-l)) = 0$$

Therefore, $b_t(g_{t+1}) = 0 \; \forall g_{t+1}$ and, from the feasibility constraint (10) it follows that all fluctuations in g_t must be absorbed by T_t.

□

Proposition 1 illustrates the effect of full risk-sharing on the optimal fiscal policy plan: being consumption and leisure constant along the business cycle, the optimal tax rate is constant as well. The government in the HC uses transfers from the RW to absorb completely the exogenous shock. When g_t is higher than average, the government uses transfers to finance its expenditure; conversely, when g_t is below average, the government uses the proceeds from taxation to pay back transfers received in the past[8]. The RW, which is a risk neutral agent, provides full insurance to the domestic economy.

2.5.2 Limited Commitment

We consider the case in which neither the government in the HC nor the RW can commit to repay external debt. The problem of the Ramsey planner is identical to the one in the previous section, but now conditions (17) and (18) have to be explicitly taken into account:

$$\max_{\{c_t,l_t,T_t\}_{t=0}^{\infty}} E_0\sum_{t=0}^{\infty}\beta^t u(c_t,l_t)$$

subject to

$$c_t + g_t = (1-l_t) + T_t \tag{14}$$

[8] In Appendix A.1 we study the case in which the utility function is logarithmic in its two arguments. In such a case, it is easy to see that transfers behave exactly as described here.

$$E_0 \sum_{t=0}^{\infty} \beta^t (u_{c,t} c_t - u_{l,t}(1 - l_t)) = u_{c^1,0}(b_{-1}) \tag{15}$$

$$E_0 \sum_{t=0}^{\infty} \beta^t T_t = 0 \tag{16}$$

$$E_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j}, l_{t+j}) \geq V^a(g_t) \forall t \tag{17}$$

$$E_t \sum_{j=0}^{\infty} \beta^j T_{t+j} \leq \underline{B} \ \forall t \tag{18}$$

Since the participation constraint at time t (17) includes future endogenous variables that influence the current allocation, standard dynamic programming results do not apply directly. To overcome this problem we apply the approach described in Marcet and Marimon (2009) and write the Lagrangean as:

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t [(1 + \gamma_t^1) u(c_t, l_t) - \psi_t (c_t + g_t - (1 - l_t) - T_t)$$
$$- \mu_t^1 (V_t^a) + \mu_t^2 (\underline{B}) - \Delta (u_{c,t} c_t - u_{l,t}(1 - l_t)) - T_t (\lambda + \gamma_t^2)] + \Delta (u_{c,0}(b_{-1}))$$

where

$$\gamma_t^1 = \gamma_{t-1}^1 + \mu_t^1$$

$$\gamma_t^2 = \gamma_{t-1}^2 + \mu_t^2$$

for $\gamma_{-1}^1 = 0$ and $\gamma_{-1}^2 = 0$. Δ is the Lagrange multiplier associated to equation (15), ψ_t is the Lagrange multiplier associated to equation (14), λ is the Lagrange multiplier associated to equation (16), μ_t^1 is the Lagrange multiplier associated to equation (17) and μ_t^2 is the Lagrange multiplier associated to equation (18). γ_t^1 and γ_t^2 are the sum of past Lagrange multipliers μ^1 and μ^2 respectively, and summarize all the past periods in which either constraint has been binding. Intuitively, γ^1 and γ^2 can be thought of as the collection of past compensations promised to each country so that it would not have incentives to leave the contract.

In can be shown that, for $t \geq 1$[9], the solution to the problem stated above is given by time-invariant policy functions that depend on the *augmented* state space $\mathcal{G} \times \Gamma^1 \times \Gamma^2$, where

[9]Once again, for $t = 0$ the FOCs of the problem are different. Applying Marcet and Marimon (2009), the problem only becomes recursive from $t \geq 1$ onwards.

$G = \{g_1, g_2, \ldots, g_n\}$ is the set of all possible realizations of the public expenditure shock g_t and Γ^1 and Γ^2 are the sets of all possible realizations of the costate variables γ^1 and γ^2, respectively. Therefore,

$$\begin{bmatrix} c_t \\ l_t \\ T_t \\ \mu_t^1 \\ \mu_t^2 \end{bmatrix} = H(g_t, \gamma_{t-1}^1, \gamma_{t-1}^2) \qquad \forall t \geq 1$$

More specifically, the government's optimality conditions for $t \geq 1$ are:

$$u_{c,t}(1+\gamma_t^1) - \psi_t - \Delta(u_{cc,t}c_t + u_{c,t} - u_{cl,t}(1-l_t)) = 0 \tag{19}$$

$$u_{l,t}(1+\gamma_t^1) - \psi_t - \Delta(u_{cl,t}c_t + u_{l,t} - u_{ll,t}(1-l_t)) = 0 \tag{20}$$

$$\psi_t = \lambda + \gamma_t^2 \tag{21}$$

Other optimality conditions are:

$$c_t + g_t = (1 - l_t) + T_t \tag{22}$$

$$E_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j}, l_{t+j}) \geq V^a(g_t) \forall t \tag{23}$$

$$\mu_t^1 (E_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j}, l_{t+j}) - V^a(g_t)) = 0 \tag{24}$$

$$\mu_t^2 (E_t \sum_{j=0}^{\infty} \beta^j T_{t+j} - \underline{B}) = 0 \tag{25}$$

$$\gamma_t^1 = \mu_t^1 + \gamma_{t-1}^1 \tag{26}$$

$$\gamma_t^2 = \mu_t^2 + \gamma_{t-1}^2 \tag{27}$$

$$\mu_t^1 \geq 0 \tag{28}$$

$$\mu_t^2 \geq 0 \tag{29}$$

Two observations are worth mentioning. First, from (19), (20) and (21) it is immediate to see that now the presence of γ_{t-1}^1 and γ_{t-1}^2 makes the allocations state-dependent. Moreover, being γ_{t-1}^1 and γ_{t-1}^2 functions of all the past shocks hitting the economy, the allocations are actually history-dependent. Second, the presence of these Lagrange multipliers makes the cost of distortionary taxation state-dependent. While in the full-commitment case this cost is constant over time and across states, in the limited commitment case it changes depending on the incentives to default that the HC and the RW have[10]. We will discuss this is further detail in section 5.

The next proposition characterizes the equilibrium for a logarithmic utility function.

Proposition 2. *Consider a utility function logarithmic in consumption and leisure and separable in the two arguments:*

$$u(c_t, l_t) = \alpha * log(c_t) + \delta * log(l_t) \tag{30}$$

with $\alpha > 0$ and $\delta > 0$. Define $t < t'$:

1. *If the participation constraint* (17) *binds such that $\gamma_t^1 < \gamma_{t'}^1$, then $c_t < c_{t'}$, $l_t < l_{t'}$ and $\tau_t > \tau_{t'}$.*

2. *If the participation constraint* (18) *binds such that $\gamma_t^2 < \gamma_{t'}^2$, then $c_t > c_{t'}$, $l_t > l_{t'}$ and $\tau_t < \tau_{t'}$.*

Proof. See Appendix A.2. □

Proposition 2 states the way the allocations and tax rates adjust in order to make the contract incentive-compatible for the HC and the RW. The optimal tax rate decreases whenever constraint (17) is binding and increases when constraint (18) binds instead[11]. Since the government in the HC has incentives to leave the contract when the government expenditure shock is low, because the the value of the ouside option in that case is high, the model implies a procyclical fiscal policy: the tax rate decreases following a low realization of the public expenditure process and increases when a the realization instead is high. This conclusion is in line with some recent empirical evidence for developing countries (see Ilzetzki and Vegh (2008) and Cuadra and Sapriza (2007)).

[10] It can be shown that, in the full commitment case, this cost is given by Δ, while in the limited commitment one is determined by $\frac{\Delta}{1+\gamma_t^1}$.

[11] Notice that it cannot be the case that the two participation constraints bind at the same time.

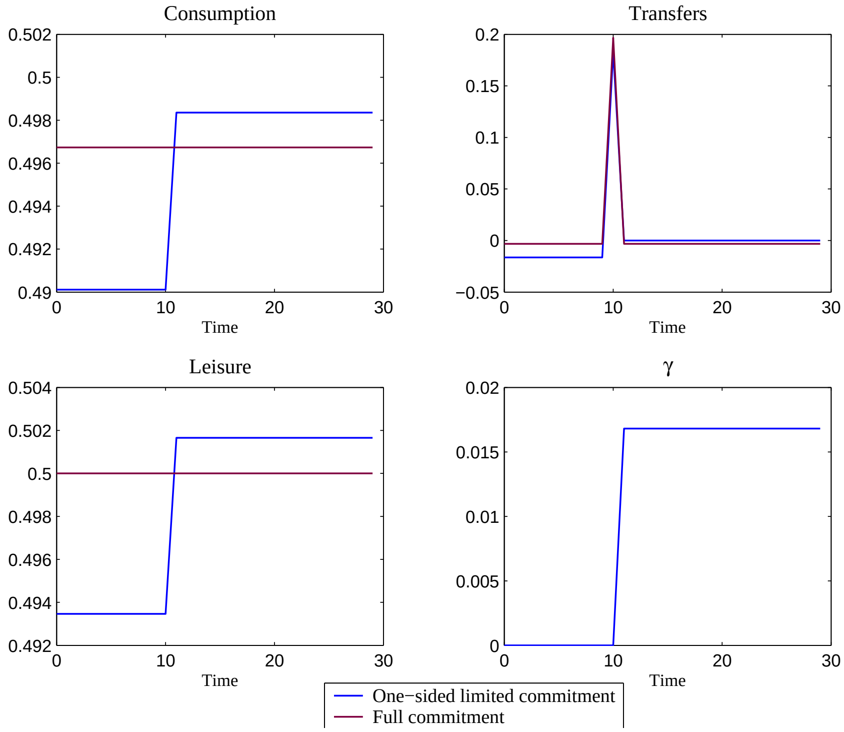

Figure 1: Example: $g_t = 0$ for $t \neq T$, and $g_T > 0$

3 An example of labor tax-smoothing

In order to understand better the impact of limited commitment on the ability of the government to smooth taxes, in this section we analyze the case study of a perfectly anticipated government expenditure shock. Suppose that government expenditure is known to be constant and equal to 0 in all periods except in T, when $g_T > 0$. In order to simplify the analysis, throughout this section we assume that only the HC can default, while the RW has a commitment technology. Moreover, we assume that $b_{-1} = 0$ and that households have a logarithmic utility function as (30).

Since equilibrium allocations depend on γ_t^1, understanding the dynamics of the incentives to default is crucial. The next proposition states that, given the assumptions previously made, the participation constraint (17) binds only at $t = T + 1$[12].:

Proposition 3. *Suppose that government expenditure is known to be constant and equal to* 0 *in all periods except in* T*, when* $g_T > 0$*. Assume further that* $b_{-1} = 0$*. Then, the participation*

[12] The reader may wonder why the participation constraint binds just after the shock. The reason is that agents know the bad shock will happen in T, so this decreases the outside option value in every period before the shock effectively takes place. Once the shock is over, the autarky value goes up.

Table 3: Parameter values

Preferences	$\alpha = \delta = 1$	
Intertemporal discount factor	$\beta = 0.98$	
Government expenditure	T = 10	$g_T = 0.2$

constraint (17) *binds at exactly* $T + 1$.

Proof. See Appendix A.3. □

From the results of Proposition 2 we can characterize the allocations for $t < T + 1 \leq t'$. Given that $\gamma_t^1 < \gamma_{t'}^1$, it follows that $c_t < c_{t'}$, $l_t < l_{t'}$ and $\tau_t > \tau_{t'}$[13]. The limited commitment by the government exerts a permanent effect on the tax rate and alters its entire dynamics, since the tax rate level after the shock is permanently lower than before the shock.

The intuition for this result is as follows. Since at $T + 1$ the continuation value of staying in the contract has to increase in order to prevent default, utility of households in the HC has to increase. By the intratemporal optimality condition, a positive tax rate implies that the marginal utility of consumption is higher than the marginal utility of leisure. Therefore, increasing consumption is relatively more efficient than increasing labor and, as a consequence, the tax rate decreases.

3.1 The example in numbers

In this section we solve numerically the example depicted above. Table 3 contains the parameters values used in the simulation.

Figures 1 and 2 show the evolution of the allocations c_t, l_t, the tax rate τ_t, international capital flows T_t, domestic bonds b_t and the costate variable γ_t^1. We compare the allocations with limited commitment to the ones under full commitment by the government towards the RW. There are two forces determining the dynamics of the economy. On one side the government has to finance the higher and expected expenditure outflow at T in the most efficient way; on the other, the participation constraint has to be satisfied. For $t \leq T$ the higher and expected shock at T keeps the continuation value of autarky low, and for this reason leaving the contract is not optimal. Therefore, before T the government accumulates assets towards the RW, and uses them to finance part of the high expenditure outflow in T. The remaining part is covered both

[13] In Appendix A.5 we show that $\Delta < 0$ in this case.

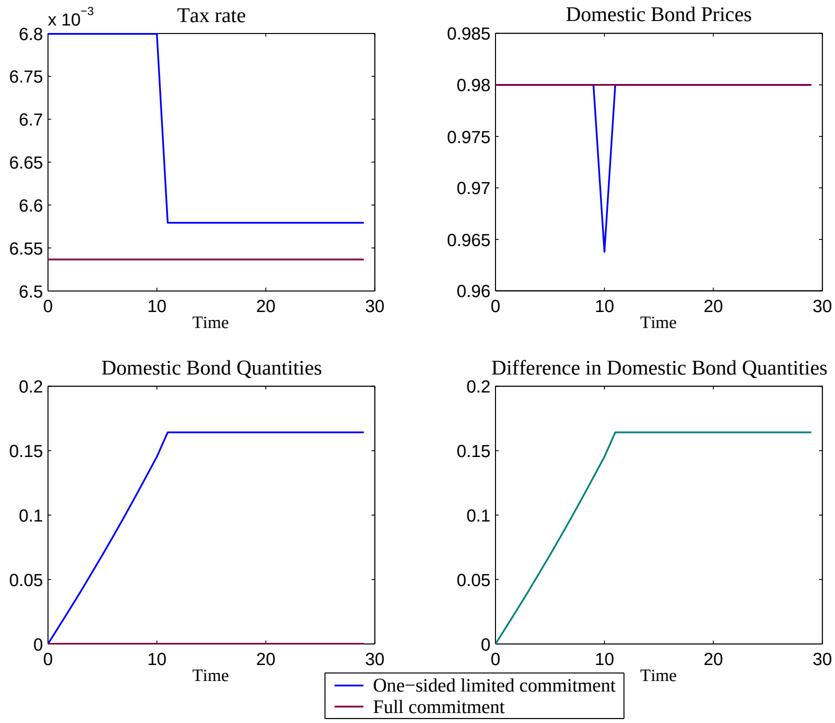

Figure 2: Example: $g_t = 0$ for $t \neq T$, and $g_T > 0$

through tax revenues and through transfers from the RW. After the high shock has taken place, the outside option value increases. In order to prevent default, the government lowers the tax rate to allow domestic households to enjoy a higher level of consumption and leisure. Moreover, from $T+1$ onwards, the transfer with the RW adjusts to guarantee that the expected present discounted value of net international flows is zero.

Notice the difference with the full commitment scenario, where the allocations are constant and transfers absorb completely the shock. The high inflow in period T is repaid forever by the government through small outflows after the shock. Taxes remain constant *even in period* T and do not react to the shock at all. The limited commitment technology constraints the amount of insurance offered by international capital markets, and perfect risk-sharing among countries is no longer possible. Consequently, the negative expenditure shock has to be absorbed through external debt and higher tax revenues in the initial periods.

4 Numerical results

In section 2 we have characterized the equilibrium allocations arising from the Ramsey planner's maximization problem under two different scenarios. First, we studied the case in which the

Table 4: Parameter values

Preferences	$\alpha = \delta = 1$
Intertemporal discount factor	$\beta = 0.98$
Government expenditure process	$g_t = g * + \rho^g g_{t-1} + \epsilon_t$
$g*$	$0.1820 * 0.33$
ρ^g	0.9107
σ_g^2	$0,1320 * 0.0607$
$\underline{B}$	0.031
$b_{-1} = b_{-1}^G$	0

two parts in the contract have full commitment. In this case, we have seen that the risk-neutral households of the RW fully insure the HC and, consequently, consumption, labor and tax rates are perfectly constant in the RW. When both parts have limited commitment, however, it is no longer possible to do perfect risk-sharing and the allocations are no longer constant.

In this section we proceed to solve the model numerically assuming that the government spending follows an $AR(1)$ process. We calibrate the parameters of this process to the argentinean economy. We use quarterly series of current government expenditure net of interest payments plus gross government investment as our measure of government expenditure for the period 1993-I to 2005-IV.

Given that we need to calibrate the process for government expenditure, we estimate an AR(1) process in levels for the argentinean data. We find that, for the broader measure of real government expenditure, $\hat{\rho} = 0,9107$ for a specification as

$$g_t = \alpha + \rho g_{t-1} + \epsilon_t$$

We also need to obtain a value for the variance of the shock associated to g_t. Since the variance of g_t and, similarly, of ϵ_t are influenced by the units in which government expenditure is measured, in order to make meaningful international comparison across countries we need to find a statistic that is not influenced by neither the currency in which expenditure is denominated nor the size of the government itself. We therefore use the coefficient of variation (CV), defined as

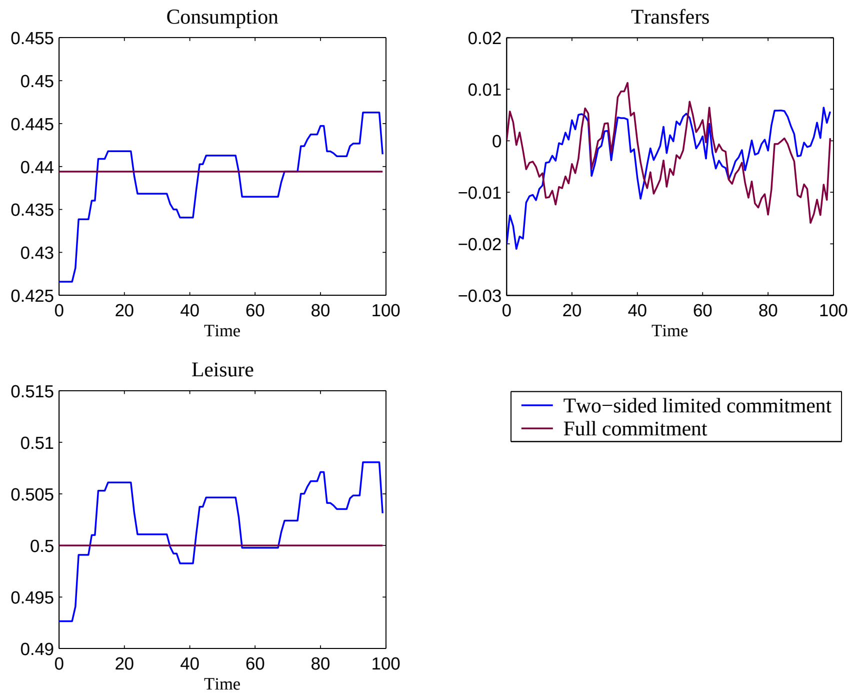

Figure 3: Two-sided limited commitment - Allocations

$$CV = \frac{\text{Std. Dev}}{\text{Mean}}$$

In the data for Argentina, $CV = 0,1320$. We estimate the mean of g_t as the value of g_t in steady state, given the mean of $\frac{g_t}{GDP_t}$ in the data. This value is $\overline{\frac{g}{GDP}} = 0,182$. Since our problem does not have a well defined steady state, we consider, as others in the literature, that $1 - l_t = \frac{1}{3}$ in steady state. Then $\overline{\frac{g}{GDP}} = \overline{\frac{g}{1-l}} = 0,182$. Therefore $\overline{g} = 0,33 * 0,182 = 0,0607$. Finally, the variance of $g_t = (0,1320 * 0.0607)^2 = 0,0000641$. We obtain the variance of ϵ_t in the following way:

$$\sigma_\epsilon^2 = \sigma_g^2(1 - \rho^2)$$

Figures 3, 4 and 5 show the allocations, co-state variables and fiscal variables respectively for the calibrated government expenditure shock, for the case in which both the government and the international institution have limited commitment (blue line). For comparison purposes, we show the same variables under full commitment (red line). Compared to the case in which international flows allow the government to smooth completely the distortion in the consumption-leisure choice, under partial commitment the current realization of the shock influences the equilibrium. Tables 5 and 6 summarize some statistics for the allocations and the fiscal variables for the cases

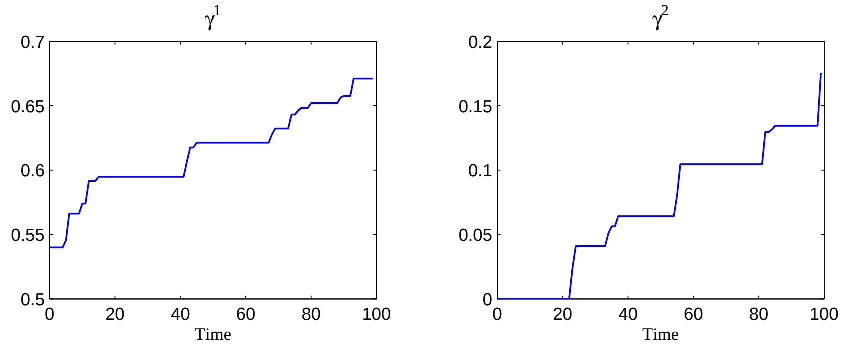

Figure 4: Two-sided limited commitment - Co-state variables

of full and limited commitment.

Table 5: Statistics of allocations for the first 50 periods

	Partial Comm.			Full Comm.		
	Mean	St.Dev.	Autocorr.	Mean	St.Dev.	Autocorr.
consumption	.43	.004	.92	.44	0	1
leisure	.5	.003	0.6	.92	0	1
labor tax rate	.13	.001	.92	.12	0	1
international flows	-.005	.006	.86	-.01	.003	.7

Table 6: Correlation with past promises

	Partial Comm.		Full Comm.	
	$Corr(x, \gamma^1)$	$Corr(x, g)$	$Corr(x, \gamma^1)$	$Corr(x, g)$
consumption	.5	-48	-	0
labor tax rate	-.97	.36	-	0

Three observations are worth making. First, while the average values of the allocations are roughly the same in the two frameworks, their variance is much higher under partial commitment. Second, the allocations under full commitment are uncorrelated with the government expenditure shock; however, under partial commitment this correlation is negative. The reason is that an increase in public consumption induces the standard crowding out effect to operate and, additionally, the participation constraint of the RW to be binding. This second channel reinforces the decrease in private consumption due to the first effect. Third, there is a positive correlation between consumption in the HC and the Lagrange multiplier associated to the participation

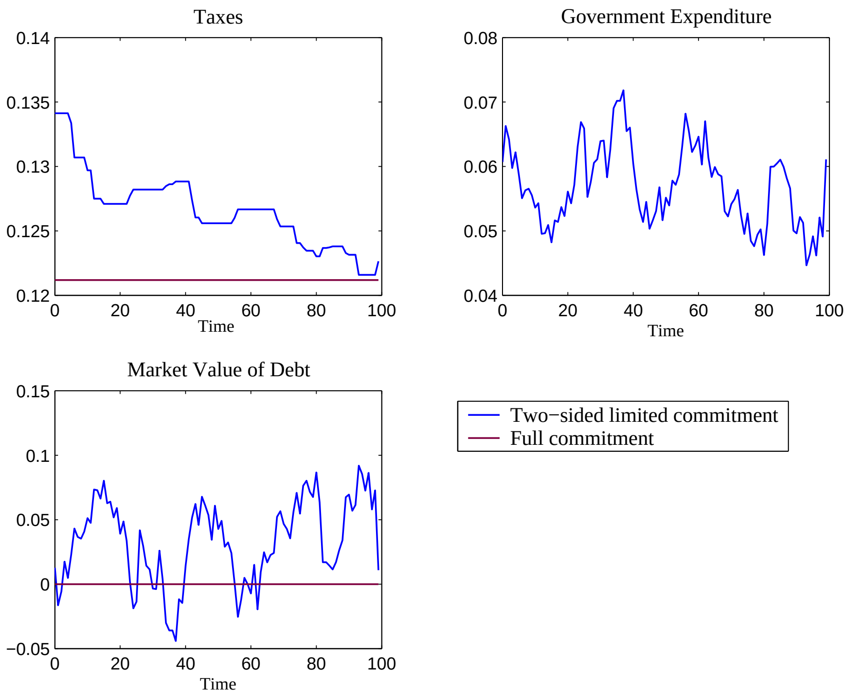

Figure 5: Two-sided limited commitment - Fiscal variables

constraint of the HC. The same is true for the correlation between international flows and the incentives to default. In periods in which it is optimal for the government of the HC to leave the contract, it receives a positive amount of transfers from abroad so as to adjust the continuation value of staying in the contract upwards to equate it with the continuation value associated to the outside option. Thus, default does not take place in equilibrium.

5 Borrowing constraints

In this section we show that it is possible to reinterpret the problem depicted in the previous sections as one in which the the HC and the RW trade one-period state-contingent bonds in the international financial market, but their trading is limited by borrowing constraints. To do so, we follow the strategy proposed by Alvarez and Jermann (2000) and Abraham and Cárceles-Poveda (2009)[14]. We show that, if we impose limits on international borrowing only, the allocations obtained in section 2 and the ones obtained in the setup of this section do not coincide. An

[14]In the Appendix we show that the government's problem coincides with the one of an international institution in charge of distributing resources among the HC and the RW, taking into account the aggregate resource constraint, the implementability condition of the HC, and the fact that countries have limited commitment. Therefore, the problem laid out in section 2 can be thought of one in which a central planner determines the constrained efficient allocations.

additional constraint on the value of domestic debt that the government of the HC can issue is required.

In what follows, we will denote with a superscript 1 variables corresponding to the HC, and with superscript 2 variables corresponding to the RW. $Z_t^1(g_{t+1})$ is an international one-period bond bought at t by the government of the HC contingent on next period's realization of the government expenditure shock. Symmetrically, call $Z_t^2(g_{t+1})$ an international one-period bond bought at t by households of the RW contingent on next period's realization of the government expenditure shock. Denote the price of these bonds by $q_t(g_{t+1})$, and assume that there are lower bounds, denoted by $A_t^1(g_{t+1})$ and $A_t^2(g_{t+1})$, on the amount of bonds that the government of the HC and the households in the RW can hold, respectively.

In the current setup, the problem of the households in the HC is exactly identical to the one described in section 2.2, so we do not reproduce it here. The problem of the government in the HC is slightly different from the one in previous sections. In order to finance its public expenditure, in addition to distortionary taxes on labor income and domestic bonds, now the government has available one-period state-contingent bonds traded with the RW. Therefore, the budget constraint of the government is:

$$g_t + \sum_{g^{t+1}|g^t} Z_t^1(g_{t+1})q_t(g_{t+1}) - Z_{t-1}^1(g_t) = \tau_t(1-l_t) + \sum_{g^{t+1}|g^t} b_t(g_{t+1})p_t^b(g_{t+1}) - b_{t-1}(g_t) \qquad (31)$$

The government faces a constraint on the amount of debt that can issue in the international financial market:

$$Z_t^1(g_{t+1}) \geq A_t^1(g_{t+1}) \qquad (32)$$

The problem of households in the RW that trade bonds with the government in the HC now is

$$\max_{\{c_t^2, Z_t^2\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t c_t^2 \qquad (33)$$

$$y + Z_{t-1}^2(g_t) = c_t^2 + \sum_{g_{t+1}} q_t(g_{t+1})Z_t^2(g_{t+1}) \qquad (34)$$

$$Z_t^2(g_{t+1}) \geq A_t^2(g_{t+1}) \qquad (35)$$

Notice that the RW is also constrained in the amount of debt it can trade with the HC. The optimality conditions of this problem are equation (34) and

$$q_t(g_{t+1}) = \beta(1+\gamma_t^2)\pi(g^{t+1}|g^t) \tag{36}$$

$$\gamma_t^2(Z_t^2(g_{t+1}) - A_t^2(g_{t+1})) = 0 \tag{37}$$

$$\gamma_t^2 \geq 0 \tag{38}$$

where γ_t^2 is the Lagrange multiplier associated to the borrowing constraint (35).

Definition 2. *A competitive equilibrium with borrowing constraints is given by allocations* $\{c^1, c^2, l\}$*, a price system* $\{p^b, q\}$*, government policies* $\{g, \tau, b\}$ *and international bonds* $\{Z^1, Z^2\}$ *such that:*

1. *Given prices and government policies, allocations c and l satisfy the HC household's optimality condition* (4)*,* (5) *and* (6)*.*
2. *Given the allocations and prices, government policies and bonds* Z^1 *satisfy the sequence of government budget constraints* (31) *and borrowing constraints* (32)*.*
3. *Prices q and bonds* Z^2 *satisfy the RW optimality conditions* (36) *and* (35)*.*
4. *Allocations satisfy the sequence of feasibility constraints:*

$$c_t^1 + g_t + \sum_{g^{t+1}|g^t} Z_t^1(g_{t+1})q_t(g_{t+1}) = 1 - l_t + Z_{t-1}^1(g_t) \tag{39}$$

$$c_t^2 + \sum_{g^{t+1}|g^t} Z_t^2(g_{t+1})q_t(g_{t+1}) = y + Z_{t-1}^2(g_t) \tag{40}$$

5. *International financial markets clear:*

$$Z_t^1(g_{t+1}) + Z_t^2(g_{t+1}) = 0$$

We need to specify borrowing constraints that prevent default by prohibiting agents from accumulating more contingent debt than they are willing to pay back, but at the same time allow as much risk-sharing as possible. Define first

$$V_t^1(Z_{t-1}^1(g_t), g_t) = u(c_t^1, l_t) + \beta E_t V_{t+1}^1(Z_t^1(g_{t+1}), g_{t+1})$$

$$V_t^2(Z_{t-1}^2(g_t), g_t) = c_t^2 + \beta E_t V_{t+1}^2(Z_t^2(g_{t+1}), g_{t+1})$$

Then we define the notion of borrowing constraints that are not too tight:

DEFINITION 3. *An equilibrium has borrowing constraints that are not too tight if*

$$V_{t+1}^1(A_t^1(g_{t+1}), g_{t+1}) = V_{t+1}^a \ \forall t \geq 0, g_{t+1}$$

and

$$V_{t+1}^2(A_t^2(g_{t+1}), g_{t+1}) = \underline{B} \ \forall t \geq 0, g_{t+1}$$

where V_{t+1}^a *and* $\underline{B}$ *are defined as in section 2.1.*

We continue to assume that the government of the HC behaves as a benevolent Ramsey Planner and chooses tax rates, domestic and international one-period state contingent bonds $\{\tau_t, b_t, Z_t^1\}_{t=0}^{\infty}$ in order to maximize the representative household's life-time expected utility, subject to the constraints imposed by the definition of competitive equilibrium with borrowing constraints. We can write the problem of the government in the HC as

$$\max_{\{c_t^1, l_t, Z_t^1\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^1, l_t) \tag{41}$$

s.t.

$$E_0 \sum_{t=0}^{\infty} \beta^t (u_{c^1,t} c_t^1 - u_{l,t}(1 - l_t)) = b_{-1} u_{c^1,0} \tag{42}$$

$$1 - l_t + Z_{t-1}^1(g_t) = c_t^1 + g_t + \sum_{g_{t+1}} q_t(g_{t+1}) Z_t^1(g_{t+1}) \tag{43}$$

$$Z_t^1(g_{t+1}) \geq A_t^1(g_{t+1}) \tag{44}$$

Proposition 4. *When the only borrowing constraints imposed on the competitive equilibrium are* (32) *and* (35)*, the allocations satisfying* (19)-(21) *do not solve the government's problem* (41)-(44)*.*

Proof. The proof is immediate. Taking the first order conditions of the problem (41)-(44)

$$u_{c_t^1} - \Delta(u_{c_t^1} c_t^1 + u_{cc_t^1} c_t^1 - u_{c^1 l,t}(1 - l_t)) = \lambda_{1,t} \tag{45}$$

$$u_{l_t} - \Delta(u_{c^1 l,t} c_t^1 + u_{l,t} l_t - u_{ll,t}(1 - l_t)) = \lambda_{1,t} \tag{46}$$

Clearly, the allocations satisfying equations (45)-(46) cannot coincide with the solution of the system of equations (19)-(20), because in the latter case the weight attached to the term $u_{c_t^1}c_t^1 + u_{cc_t^1}c_t^1 + u_{c^1l,t}(1-l_t)$ is constant and equal to Δ, while in the former it is given by $\frac{\Delta}{1+\gamma_t^1}$ and varies over time. □

This proposition states that the economy with transfers among countries cannot be reinterpreted as an economy in which there are international bond markets and limits to international debt issuance only.

From the proof of the proposition, it is again evident what has already been pointed out in section 2.5.2. When there is full commitment, the cost of distortionary taxation is given by the Lagrange multiplier associated to the implementability constraint, Δ. This cost is constant due to the presence of complete bond markets. However, when we relax the assumption of full commitment and consider instead the case in which the government of the HC has limited commitment, the cost of distortionary taxation becomes state-dependent and is given by $\frac{\Delta}{1+\gamma_t^1}$. The reason for this is that now the government faces endogenously incomplete international bond markets[15]. Since allocations and tax rates vary permanently every time the participation constraint of the HC binds, so does the burden of taxation.

The previous discussion leads us to impose borrowing constraints on the value of domestic debt in addition to the constraints on international debt[16]. The next proposition states that, in this case, it is possible to establish a mapping between the economy with transfers and the one with borrowing constraints on domestic as well as international debt:

Proposition 5. *Suppose that, in addition to constraints* (32) *and* (35)*, we impose a lower bound on the value of state contingent domestic debt (expressed in terms of marginal utility)*

$$b_{t-1}^1(g_t)u_{c^1,t} = E_t \sum_{j=0}^{\infty} \beta^j (u_{c^1,t+j}c_{t+j}^1 - u_{l,t+j}(1-l_{t+j})) \leq B_{t-1}(g_t) \tag{47}$$

Then the allocations solving the system (19)-(21) *also solve the government's problem* (41)-(44) *and* (47)*.*

Proof. See Appendix A.6 □

This result provides a rationale for our specification of the outside option of the government in the HC. In section 2 we assumed that if the government defaulted, it would lose access to the international and domestic bond markets and would remain in financial autarky thereafter.

[15] A similar result is obtained in the incomplete markets literature (see **?**).

[16] Sleet (2004) also defines a borrowing constraint in terms of the value of debt.

It seems natural then to impose constraints on the amount of debt that it can issue in both markets.

6 Stylized Facts

In this section we want to check if in the data tax rate volatility is affected by the availability of external sources to finance domestic shocks. We use the Emerging Markets Bond Index (EMBI) to measure the degree of insurance against internal shocks governments in emerging markets can get offshore. EMBI tracks total returns for traded external debt instruments, and gives a measure of the riskiness of the sovereign bonds issued by a country. We compute the annual average of EMBI for 6 emerging economies (Argentina, Mexico, Nigeria, Venezuela, Panama, Peru) from 1995 until 2001 and we look at the relationship with the average tax rate volatility referring to the same period.[17][18] The idea is that the higher the (mean) EMBI specific to a country, the higher the perceived investment risk that investors from abroad associate to that country, and the lower the amount of international flows the country can use to hedge against government revenues shocks.

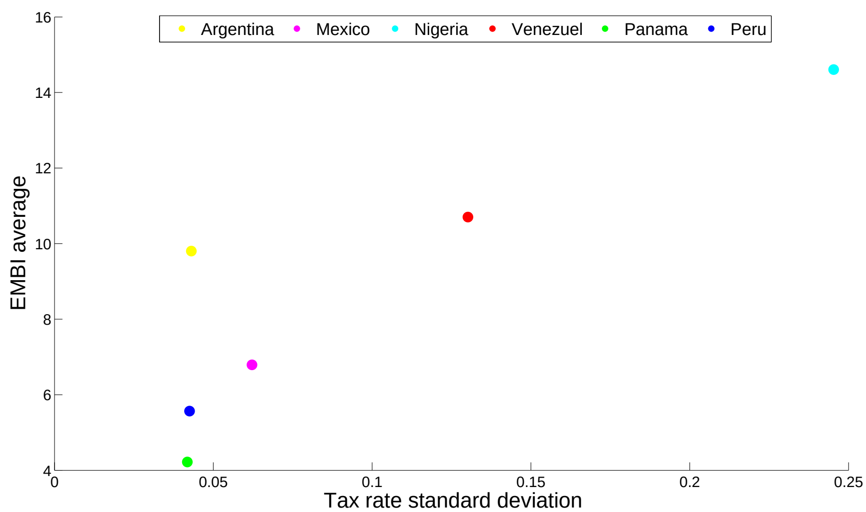

Figure 6:

Figure 6 plots this relationship. The horizontal axis measures tax rate standard deviation, and the vertical axis measures the EMBI. The graph shows that the higher the EMBI the more

[17] We restrict our analysis to this period and to these countries for a question of availability of data.

[18] We consider a broad measure of tax rate as we define it as the ratio between total government revenues over GDP.

the government has to vary taxes to satisfy its budget constraint: in a sense the government can rely less on international debt to minimize the taxation distortion.

Although it supports the main result of this paper, the evidence we suggest is very preliminary. Data on fiscal variables for emerging economies are difficult to obtain and, when available, the series are either very short or they are not always reliable. For this reason we can not perform any time series analysis. Apart from this problem, what really matters for tax rate variability in the model is the availability of international lending/borrowing and it is not obvious how to measure this variable. As we use EMBI, which refers to emerging markets, we cannot run any cross-sectional estimation as there are too few observations. Nevertheless using some other indicator for limited commitment across countries would allow us to address the empirical estimation of the model in a more formal way. We leave this issue as future research.

7 Conclusions

A key issue in macroeconomics is the study of the optimal determination of the tax rate schedule when the government has to finance (stochastic) public expenditure and only has available distortionary tools[19]. Under this restriction, a benevolent planner seeks to minimize the intertemporal and intratemporal distortions caused by taxes. Since consumption should be smooth, a general result is that taxes should also be smooth across time and states.

When considering a small open economy that can borrow from international risk-neutral lenders and both parts can fully commit to repay the debt, this result is amplified because there is perfect risk-sharing. Consumption and leisure are perfectly flat, thus the tax rate is also flat. The domestic public expenditure shock is absorbed completely by external debt and there is no role for internal debt. In light of this result, one would expect small open economies to have less volatile tax rate schedules than large economies. However, the available data seems to contradict this intuition.

When we relax the assumption of full commitment from both the small open economy and from international lenders towards their international obligations, perfect risk-sharing is no longer possible. The presence of limited commitment hinders the ability of the government to fully insure against the public expenditure shock through use of international capital markets. Consequently, the government has to resort to taxes and internal debt in order to absorb part of the shock, and it is no longer possible to have constant allocations and tax rates.

[19]When the government can levy non-distortionary taxes, such as lump-sum taxes, the Ricardian Equivalence holds and the first best can be achieved.

Our simulation results show that the volatility of the tax rate increases substantially when there is limited commitment. Moreover, fiscal policy is procyclical: when the government expenditure shock is low, the country has incentives to leave the contract with the international institution. Therefore, taxes should decrease in order to allow consumption and leisure to be higher and, in this way, increase household utility. On the contrary, when the government expenditure is high, taxes need to be high as well to repay the external debt.

These two features of our model are in line with what we observe in the data for developing countries, which are the ones more likely to lack a commitment technology towards international obligations. In section 6 we confirm this by looking at some stylized facts regarding fiscal variables, public debt and sovereign risk. We find that there seems to be a positive relation between the volatility of the average tax rate and that of a measure of the country risk premium.

The results presented in the paper suggest that the volatility and cyclicality of tax rates observed in developing countries is not necessarily an outcome of reckless policy-making, as one could think a priori. We have shown that, in order to establish the optimal fiscal policy plan in small developing countries, it is important to take into account the degree of commitment that the economy has towards its external obligations, as this element is crucial in determining the extent of risk-sharing that can be achieved.

A question that remains unanswered is where the allocations converge in the long run. It could be the case that the participation constraints continue to bind in some states of nature, even in the long run. In that case γ^1 and γ^2 diverge to infinity. The other possibility is that the economy arrives to a point in which neither the HC nor the RW has further incentives to leave the contract, and from that moment onwards c, l, τ, γ^1 and γ^2 remain constant. In that case, there is partial risk-sharing only in the short-run. This is an issue that we plan to address in the future.

Finally, the theoretical results outlined in the paper suggest a new mechanism to check in the data for developing countries. Our claim is that governments that are suspected to have commitment problems in repaying their debt necessarily have to set more volatile taxation schemes than more reliable ones. We have presented some very preliminar evidence that this might be supported by the data, but evidently a deeper analysis is called for.

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A Appendix

A.1 Optimal policy under full commitment: logarithmic utility

In this section we show a particular case of Proposition 1 when the utility function of households is logarithmic both in consumption and leisure, which corresponds to the utility function used for the numerical exercises in the paper.

Consider a utility function of the form:

$$u(c_t, l_t) = \alpha * log(c_t) + \delta * log(l_t)$$

with $\alpha > 0$ and $\delta > 0$. Assume that initial wealth $b_{-1} = 0$. Then the allocations and government policies can be easily computed from the optimality conditions (9) to (13). From the intertemporal budget constraint of households (9) it can be derived that:

$$l = \frac{\delta}{\alpha + \delta} \tag{48}$$

Plugging in this expression in (12), $c = \frac{\alpha}{\lambda}$. Combining this expression for consumption, together with (48), (10) and (11) we arrive to the following expression:

$$\frac{1}{1-\beta}\left(\frac{\alpha}{\lambda} - 1 + \frac{\delta}{\alpha+\delta}\right) + E_0 \sum_{t=0}^{\infty} \beta^t g_t = 0$$

Define the last term of the previous expression as

$$E_0 \sum_{t=0}^{\infty} \beta^t g_t \equiv \frac{1}{1-\beta}\tilde{g}$$

where $\tilde{g}$ is known at $t = 0$. Then

$$\lambda = \frac{\alpha + \delta}{1 - \frac{\alpha+\delta}{\alpha}\tilde{g}}$$

Substituting in the expression for c, we obtain

$$c = \frac{\alpha - (\alpha + \delta)\tilde{g}}{\alpha + \delta} \tag{49}$$

From the feasibility constraint (10), transfers are given by the difference between the actual realization of public expenditure g_t and its expected discounted value $\tilde{g}$:

$$T_t = g_t - \tilde{g} \tag{50}$$

Finally, from the intratemporal optimality condition of households (6) we can obtain an expression for the tax rate:

$$\tau = \frac{\delta(\alpha + \delta)}{\alpha}\tilde{g} \tag{51}$$

A.2 Proof of Proposition 2

In order to prove Proposition 2, we first need to establish some intermediate results. We begin with a discussion about the sign of Δ, the Lagrange multiplier associated to the intertemporal budget constraint in the Ramsey planner's problem.

A.2.1 The Ramsey problem with limited commitment

For ease of exposition, we will assume that only the HC has limited commitment. Since the problem of the Ramsey planner is identical to the one in section 2.5.2, but without imposing constraint 18, we do not reproduce it here.

The optimality conditions for $t \geq 1$ are:

$$u_{c,t}(1 + \gamma_t^1) - \psi_t - \Delta(u_{cc,t}c_t + u_{c,t} - u_{cl,t}(1 - l_t)) = 0 \tag{52}$$

$$u_{l,t}(1 + \gamma_t^1) - \psi_t - \Delta(u_{cl,t}c_t + u_{l,t} - u_{ll,t}(1 - l_t)) = 0 \tag{53}$$

$$\psi_t - \lambda = 0 \tag{54}$$

$$c_t + g_t = (1 - l_t) + T_t \tag{55}$$

$$E_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j}, l_{t+j}) \geq V^a(g_t) \forall t \tag{56}$$

$$\mu_t^1(E_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j}, l_{t+j}) - V^a(g_t)) = 0 \tag{57}$$

$$\gamma_t^1 = \mu_t^1 + \gamma_{t-1}^1 \tag{58}$$

$$\mu_t^1 \geq 0 \tag{59}$$

Multiplying equations (52) and (53) by c_t and $-(1-l_t)$ respectively, and summing:

$$\begin{aligned}&(1+\gamma_t^1-\Delta)(u_{c,t}c_t-u_{l,t}(1-l_t))-\psi_t(c_t-(1-l_t))\\&-\Delta\underbrace{(u_{cc,t}c_t^2-2u_{cl,t}(1-l_t)c_t+u_{ll,t}(1-l_t)^2)}_{A_t}=0\end{aligned}\tag{60}$$

Notice that given that the utility function is strictly concave, expression A is strictly negative. By a similar procedure we can write down an equivalent expression at $t=0$:

$$\begin{aligned}&(1+\gamma_0^1-\Delta)(u_{c,0}(c_0-b_{-1})-u_{l,0}(1-l_0))-\psi_0(c_0-(1-l_0)-b_{-1})\\&-\Delta\underbrace{(u_{cc,0}(c_0-b_{-1})^2-2u_{cl,0}(1-l_0)(c_0-b_{-1})+u_{ll,0}(1-l_0)^2)}_{A_0}=0\end{aligned}\tag{61}$$

Multiplying (60) by $\beta^t\pi(s^t)$[20], summing over t and s^t and adding expression (61):

$$\begin{aligned}&E_0\sum_{t=0}^{\infty}\beta^t(1+\gamma_t^1-\Delta)(u_{c,t}c_t-u_{l,t}(1-l_t))-(1+\gamma_0-\Delta)u_{c,0}b_{-1}\\&-\Delta Q-E_0\sum_{t=0}^{\infty}\beta^t\psi_t(c_t-(1-l_t))+\psi_0 b_{-1}=0\end{aligned}$$

where Q is the expected value of the sum of negative quadratic terms A_t. Using the implementability constraint (15) and the resource constraint (14) we obtain equation (62)

$$\begin{aligned}&E_0\sum_{t=0}^{\infty}\beta^t(\gamma_t^1-\gamma_0^1)(u_{c,t}((1-l_t)+T_t-g_t)-u_{l,t}(1-l_t))\\&-\Delta Q+E_0\sum_{t=0}^{\infty}\beta^t\psi_t(g_t-T_t)+\psi_0 b_{-1}=0\end{aligned}\tag{62}$$

For later purposes, using the intratemporal optimality condition of households (6) we can reexpress this equation as[21].

$$E_0\sum_{t=0}^{\infty}\beta^t(\gamma_t^1-\gamma_0^1)u_{c,t}(\tau_t(1-l_t)-g_t+T_t)-\Delta Q+E_0\sum_{t=0}^{\infty}\beta^t\lambda g_t+\lambda b_{-1}=0\tag{63}$$

Notice that, in the case of full commitment, expression (62) simplifies to

[20] $\pi(s^t)$ is the probability of history s^t taking place given that the event s_0 has been observed.

[21] Notice that if the participation constraint was never binding, then $\gamma_t^1=\gamma_0^1=0$ and we would recover an identical condition to the one obtained in the Lucas and Stokey (1983) model.

$$-\Delta Q + \lambda \left(E_0 \sum_{t=0}^{\infty} \beta^t g_t + b_{-1} \right) = 0 \tag{64}$$

Since $\lambda = \psi_t > 0 \, \forall t$, it is straightforward to see that when the present value of all government expenditures exceeds the value of any initial government wealth, the Lagrange multiplier $\Delta < 0$.

In the presence of limited commitment, however, there is an extra term involving the costate variable γ_t^1 which prevents us from applying the same reasoning. Nevertheless, we will show that this is the case for the specific example of section 3, and we will assume this result extends to the general setup. In the numerical exercise we perform in section 4 we confirm that this assumption holds.

We show now under which conditions $\Delta = 0$. Setting $\Delta = 0$, from equations (52) and (53) we know that

$$u_{c,t}(1+\gamma_t) = u_{l,t}(1+\gamma_t) \tag{65}$$

$$u_{c,t} = u_{l,t} \tag{66}$$

This last expression and equation (6) in the text imply that $\tau_t = 0 \, \forall t$. Inserting these results into equation (63):

$$E_0 \sum_{t=0}^{\infty} \beta^t (\gamma_t - \gamma_0)(u_{c,t}(T_t - g_t) + E_0 \sum_{t=0}^{\infty} \beta^t \lambda (g_t - T_t) + \lambda b_{-1} = 0$$

Using (65)

$$\Rightarrow E_0 \sum_{t=0}^{\infty} \beta^t (-\gamma_t + \gamma_0 + 1 + \gamma_t) u_{c,t}(g_t - T_t) + u_{c,0}(1+\gamma_0) b_{-1} = 0$$

$$\Rightarrow E_0 \sum_{t=0}^{\infty} \beta^t \frac{u_{c,t}}{u_{c,0}} (g_t - T_t) = -b_{-1} \tag{67}$$

We can rewrite (67) as

$$\sum_{t=0}^{\infty} \sum_{s^t} p_t^0 (g_t - T_t) = -b_{-1} = b_{-1}^g \tag{68}$$

where p_t^0 is the price of a hypothetical bond issued in period 0 with maturity in period t contingent on the realization of s_t. Equation (68) states that when the government's initial claims b_{-1}^g against the private sector equal the present-value of all future government expenditures net of transfers, the Lagrange multiplier Δ is zero. Since the government does not need to resort to any distortionary taxation, the household's present-value budget does not exert any additional constraining effect on welfare maximization beyond what is already present through the economy's technology.

Finally, we will follow Ljungqvist and Sargent (2000) and assume that if the government's initial claims against the private sector were to exceed the present value of future government expenditures, the government would return its excess financial wealth as lump-sum transfers and Δ would remain to be zero.

A.2.2 Proof of Proposition 2

We begin by proving the first part of the Proposition. Given a logarithmic utility function as (30), optimality conditions (19) to (21) become

$$\frac{\alpha}{c_t}(1+\gamma_t^1) - (\lambda + \gamma_t^2) - \Delta\left(-\frac{\alpha}{c_t^2}c_t + \frac{\alpha}{c_t}\right) = 0$$

$$\Longrightarrow c_t = \frac{\alpha(1+\gamma_t^1)}{\lambda + \gamma_t^2} \tag{69}$$

$$\frac{\delta}{l_t}(1+\gamma_t^1) - (\lambda + \gamma_t^2) - \Delta\left(\frac{\delta}{l_t} + \frac{\delta}{l_t^2}(1-l_t)\right) = 0$$

$$\Longrightarrow l_t = \frac{\delta(1+\gamma_t^1) \pm \sqrt{\delta^2(1+\gamma_t^1)^2 - 4\Delta\delta(\lambda+\gamma_t^2)}}{2(\lambda+\gamma_t^2)} \tag{70}$$

Notice from equation (70) that if $\Delta < 0$ then we need to take the square root with positive sign in order to have $l_t > 0$. To show that consumption and leisure increase with γ_t^1, we take the derivatives of c_t and l_t with respect to γ_t^1

$$\frac{\partial c_t}{\partial \gamma_t^1} = \frac{\alpha}{\lambda + \gamma_t^2} > 0$$

$$\frac{\partial l_t}{\partial \gamma_t^1} = \frac{\delta + \left(\delta^2(1+\gamma_t^1)^2 - 4\Delta\delta(\lambda+\gamma_t^2)\right)^{-\frac{1}{2}}\delta^2(1+\gamma_t^1)}{2(\lambda+\gamma_t^2)} > 0$$

We can write the intratemporal optimality condition of households (6) as

$$\tau_t = \frac{u_{c,t} - u_{l,t}}{u_{c,t}} = 1 - \frac{\delta c_t}{\alpha l_t} \tag{71}$$

Given $t < t'$, assume $\gamma_t^1 < \gamma_{t'}^1$ while $\gamma_t^2 = \gamma_{t'}^2$. Now we compare the tax rates at t and t', and show that τ_t decreases with γ_t^1 by contradiction. Then, using (71)

$$\tau_{t'} - \tau_t = \frac{\delta}{\alpha}\left(\frac{c_t}{l_t} - \frac{c_{t'}}{l_{t'}}\right) > 0$$

It follows that it must be the case that $c_t l_{t'} - c_{t'} l_t > 0$. After some algebra this condition translates into

$$\left(\frac{1+\gamma_t^1}{1+\gamma_{t'}^1}\right)^2 > \frac{\delta^2(1+\gamma_t^1)^2 - 4\Delta\delta(\lambda+\gamma_t^2)}{\delta^2(1+\gamma_{t'}^1)^2 - 4\Delta\delta(\lambda+\gamma_t^2)}$$

$$(1+\gamma_t^1)^2 > (1+\gamma_{t'}^1)^2$$

which is clearly a contradiction. Thus, τ_t increases with γ_t^1.

Now we proceed to prove the second part of the Proposition. We can immediately check that c_t decreases with γ_t^2 by taking partial derivates:

$$\frac{\partial c_t}{\partial \gamma_t^2} = -\frac{\alpha(1+\gamma_t^1)}{(\lambda+\gamma_t^2)^2} < 0$$

Suppose l_t is an increasing function of γ_t^2. Then the partial derivative of l_t w.r.t γ_t^2 must be positive

$$\frac{\partial l_t}{\partial \gamma_t^2} = \frac{-2\Delta\delta(\lambda+\gamma_t^2)A^{-\frac{1}{2}} - \delta(1+\gamma_t^1) - A^{\frac{1}{2}}}{4(\lambda+\gamma_t^2)^2} > 0$$

where $A = \delta^2(1+\gamma_t^1)^2 - 4\Delta\delta(\lambda+\gamma_t^2)$. This last expression implies that

$$-2\Delta\delta(\lambda+\gamma_t^2) > \delta(1+\gamma_t^1)A^{\frac{1}{2}} + A$$

Then,

$$2\Delta\delta(\lambda+\gamma_t^2) - \delta^2(1+\gamma_t^1)^2 > A^{\frac{1}{2}}\delta(1+\gamma_t^1)$$

Since the left hand side of the previous expression is negative, while the right hand side is positive, this statement is clearly a contradiction. Then it must be the case that l_t is a decreasing function of γ_t^2.

Finally, suppose that $t' > t$, $\gamma_{t'}^2 > \gamma_t^2$ but $\gamma_{t'}^1 = \gamma_t^1$. Assume that τ_t is a decreasing function of γ_t^2. Then, using (71), it must be the case that

$$u_{c,t'} u_{l,t} < u_{c,t} u_{l,t'}$$

This implies that

$$\frac{\delta(1+\gamma_t^1)+\sqrt{\delta^2(1+\gamma_t^1)^2-4\delta\Delta(\lambda+\gamma_{t'}^2)}}{2(\lambda+\gamma_{t'}^2)}\frac{\alpha(1+\gamma_t^1)}{\lambda+\gamma_t^2}< \frac{\delta(1+\gamma_t^1)+\sqrt{\delta^2(1+\gamma_t^1)^2-4\delta\Delta(\lambda+\gamma_t^2)}}{2(\lambda+\gamma_t^2)}\frac{\alpha(1+\gamma_t^1)}{\lambda+\gamma_{t'}^2} \tag{72}$$

Simplifying and remembering that $\Delta < 0$, the previous inequality is a contradiction. Therefore, τ_t increases with γ_t^2. This completes the proof.

A.3 Proof of Proposition 3

Notice first that at $t = 0$ and for $\gamma_0^1 = 0$, the continuation value of staying in the contract has to be (weakly) greater than the value of the outside option (financial autarky):

$$\sum_{t=0}^{\infty}\beta^t u(c_t, l_t) \geq \sum_{t=0}^{\infty}\beta^t u(c_{t,A}, l_{t,A}) \tag{73}$$

The reason for this statement is that, for the government, subscribing the contract with the rest of the world represents the possibility to do risk-sharing and, consequently, to smooth consumption of domestic households. Since utility is concave, a smoother consumption path translates into a higher life-time utility value. Obviously, this result hinges on the fact that the initial debt of the government is zero and that equation (1) must hold[22].

Now we show that equation (17) holds with strict inequality for $1 \leq t \leq T$. It is important to bear in mind that the allocations could change in time only due to a different γ_t^1. Since $\gamma_{t-1}^1 \leq \gamma_t^1\ \forall t$, then $u(c_{t-1}) \leq u(c_t)$. Assume that $\mu_1^1 > 0$. This implies that, if μ_1^1 was equal to zero, the PC would be violated, that is,

$$\begin{aligned} u(c_0, l_0) + \sum_{t=2}^{T-1}\beta^{t-1}u(c_t, l_t) + \beta^{T-1}u(c_T, l_T) + \sum_{t'=T+1}^{\infty}\beta^{t'-1}u(c_{t'}, l_{t'}) \\ < \sum_{t=0}^{T-2}\beta^t u(c_A, l_A) + \beta^{T-1}u(c_{A'}, l_{A'}) + \sum_{t=T}^{\infty}\beta^t u(c_A, l_A) \end{aligned} \tag{74}$$

Equation (73) can be rewritten as

[22] If, for example, the initial level of government debt b_{-1} was very high, then the government could find it optimal to default on this debt and run a balanced budget thereafter. On the other hand, if condition (1) was not imposed, then the contract could mean a redistribution of resources from the HC to the RW that could potentially lead the HC to have incentives not to accept the contract.

$$\begin{aligned}&\sum_{t=0}^{T}\beta^{t}u(c_t,l_t)+\sum_{t'=T+1}^{\infty}\beta^{t'}u(c_{t'},l_{t'})\\&\qquad>\sum_{t=0}^{T-1}\beta^{t}u(c_A,l_A)+\beta^{T}u(c_{A'},l_{A'})+\sum_{t=T+1}^{\infty}\beta^{t}u(c_A,l_A)\end{aligned}\tag{75}$$

Subtracting (75) from (74):

$$\begin{aligned}&\beta[u(c_2,l_2)-u(c_1,l_1)]+\beta^2[u(c_3,l_3)-u(c_2,l_2)]+\ldots+\beta^{T-1}[u(c_T,l_T)-u(c_{T-1},l_{T-1})]+\\&\beta^{T}[u(c_{T+1},l_{T+1})-u(c_T,l_T)]+\beta^{T+1}[u(c_{T+2},l_{T+2})-u(c_{T+1},l_{T+1})]+\ldots\\&<\beta^{T-1}[u(c_{A'},l_{A'})-u(c_A,l_A)]+\beta^{T}[u(c_A,l_A)-u(c_{A'},l_{A'})]\end{aligned}\tag{76}$$

Reordering terms we arrive at:

$$\begin{aligned}&\beta\underbrace{[u(c_2,l_2)-u(c_1,l_1)]}_{\geq 0}+\beta^2\underbrace{[u(c_3,l_3)-u(c_2,l_2)]}_{\geq 0}+\ldots+\beta^{T-1}\underbrace{[u(c_T,l_T)-u(c_{T-1},l_{T-1})]}_{\geq 0}+\\&+\beta^{T}\underbrace{[u(c_{T+1},l_{T+1})-u(c_{T-1},l_{T-1})]}_{\geq 0}+\beta^{T+1}\underbrace{[u(c_{T+2},l_{T+2})-u(c_{T+1},l_{T+1})]}_{\geq 0}+\ldots\\&<\underbrace{[u(c_{A'},l_{A'})-u(c_A,l_A)]}_{<0}\underbrace{(\beta^{T-1}-\beta^{T})}_{>0}\end{aligned}\tag{77}$$

Expression (77) is clearly a contradiction, since the left hand side of the inequality is greater or equal to 0, but the right hand side is strictly smaller than 0. We conclude then that it cannot be that $\mu_1^1>0$. Therefore, equation(17) is not binding in period $t=1$. The same reasoning can be extended to periods $t=2,3,\ldots,T$. Therefore, $\gamma_t^1=\gamma_0^1=0$ for $t=1,2,...,T$ and the allocations $\{c_t\}_{t=0}^{T},\{l_t\}_{t=0}^{T}$ are constant.

Notice that, from $T+1$ onwards, $g_t=0$ so the allocations do not change. Therefore, $\gamma_t^1=\gamma_{T+1}^1$ for $t=T+2,T+3,\ldots,\infty$.

Finally, we show that $\mu_{T+1}^1>0$[23]. We prove this by contradiction. Assume that $\mu_{T+1}=0$. From the previous discussion, this implies that $\gamma_t^1=0\ \forall t$. Then the allocations are identical to the case of limited commitment, and from the results of section A.1, we know that $T_t<0$ for $t\neq T$ and $T_T>0$. Thus, from the feasibility constraint (14) we can see that $c_{T+1}<c_A$ and $l_{T+1}<l_A$. But this implies that utility $u(c_{T+1},l_{T+1})<u(c_A,l_A)$, so

[23]Notice that, given that our shock in this example is not a Markov process, neither γ_t nor the allocations c_t and l_t are time-invariant functions of the state variables g_t,γ_{t-1} but, on the contrary, the depend on t.

$$\frac{1}{1-\beta}u(c_{T+1}, l_{T+1}) < \frac{1}{1-\beta}u(c_A, l_A)$$
$$\sum_{j=0}^{\infty}\beta^j u(c_{T+1+j}, l_{T+1+j}) < \sum_{j=0}^{\infty}\beta^j u(c_A, l_A)$$

which clearly contradicts with the fact that $\mu^1_{T+1} = 0$. Therefore, it must be the case that $\mu^1_{T+1} > 0$. This completes the proof.

A.4 Proof that $\Delta < 0$ in Section 3

Since in the example of Section 3 we have a full analytical characterization of the equilibrium, it is possible to determine the sign of Δ.

Given our assumption about the government expenditure shock and the result of Proposition 3, equation (63) can be written as

$$\sum_{t=T+1}^{\infty}\beta^t(\gamma_{T+1}-\gamma_0)u_{\bar{c}}(\bar{\tau}(1-\bar{l})+\bar{T}) - \Delta Q + \beta^T\lambda g_T = 0 \tag{78}$$

where $\bar{c}, \bar{l}, \bar{\tau}$ and $\bar{T}$ are the constant allocations and fiscal variables from $t = T+1$ onwards. In order to determine the sign of the first term of the previous expression, we recall the period by period budget constraint of the government for $t \geq T+1$:

$$(\beta - 1)\bar{b}^G = \bar{\tau}(1-\bar{l}) + \bar{T}$$

The sign of the first term of equation (63) depends on whether government bonds are positive or negative after the big shock has taken place. From equation (15)

$$\sum_{j=0}^{T}\beta^t(u_{\tilde{c}}\tilde{c} - u_{\tilde{l}}(1-\tilde{l})) + \sum_{j=T+1}^{\infty}\beta^t(u_{\bar{c}}\bar{c} - u_{\bar{l}}(1-\bar{l})) = 0$$
$$\Rightarrow \frac{1-\beta^{T+1}}{1-\beta}\left(\alpha - \frac{\delta}{\tilde{l}}(1-\tilde{l})\right) + \frac{\beta^{T+1}}{1-\beta}\left(\alpha - \frac{\delta}{\bar{l}}(1-\bar{l})\right) = 0 \tag{79}$$

where $\tilde{c}$ and $\tilde{l}$ are the constant allocations from $t = 0$ to $t = T$. We know that the participation constraint binds in period $T+1$ and consequently $\bar{l} > \tilde{l}$. But this implies that

$$\alpha - \frac{\delta}{\tilde{l}}(1-\tilde{l}) < 0$$
$$\alpha - \frac{\delta}{\bar{l}}(1-\bar{l}) > 0 \tag{80}$$

because the two terms of (79) have to add up to zero. Now we recover b_t for $t \geq T+1$ from the intertemporal budget constraint (15) of households at time $T+1$:

$$
\begin{aligned}
u_{\bar{c}}\bar{b} =& \sum_{j=0}^{\infty} \beta^j (u_{\bar{c}}\bar{c} - u_{\bar{l}}(1-\bar{l})) = \frac{1}{1-\beta}(u_{\bar{c}}\bar{c} - u_{\bar{l}}(1-\bar{l})) \\
=& \frac{1}{1-\beta}\left(\alpha - \frac{\delta}{\bar{l}}(1-\bar{l})\right) > 0
\end{aligned}
$$

If $\bar{b} > 0$, $\bar{b}^G < 0$ so the first term in equation (78) is positive. But then from this equation it is immediate to see that $\Delta < 0$.

A.5 The International Institution Problem and the Government Problem: Equivalence of Results

Suppose that there exists an international financial institution that distributes resources among the HC and the RW, taking into account the aggregate feasibility constraint, the implementability condition (15), and participation constraints (17) and (18). The Lagrangian associated to the international institution is

$$
\begin{aligned}
&\max_{\{c_t^1, c_t^2, T_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t (\alpha u(c_t^1, l_t^1) + (1-\alpha) u(c_t^2) + \\
&+ \tilde{\mu}_{1,t}(E_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j}^1, l_{t+j}^1) - V_t^{1,a}) + \tilde{\mu}_{2,t}(\underline{B} - E_t \sum_{j=0}^{\infty} \beta^j T_{t+j}) + \\
&- \tilde{\Delta}(u_{c^1,t} c_t^1 - u_{l_{1_t}}(1 - l_t^1)) + \tilde{\psi}_t (c_t^1 + c_t^2 + g - (1 - l_t^1 + y)))
\end{aligned} \tag{81}
$$

where α is the Pareto weight that the international institution assigns to the HC. Since by assumption households in the RW are risk-neutral, $u(c_t^2) = c_t^2$. The feasibility constraint in the RW implies that $c_t^2 = y - T_t$. Substituting this into 81 and applying **?** we can recast problem 81 as

$$
\begin{aligned}
&\max_{\{c_t^1, c_t^2, T_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t ((\alpha + \tilde{\gamma}_{1,t}) u(c_t^1, l_t^1) + (1-\alpha)(y - T_t) + \\
&- \tilde{\mu}_{1,t} V_t^{1,a} + \tilde{\mu}_{2,t}\underline{B} - \tilde{\gamma}_{2,t} T_t) - \tilde{\Delta}(u_{c^1,t} c_t^1 - u_{l_{1_t}}(1 - l_t^1)) + \tilde{\psi}_t (c_t^1 + g - (1 - l_t^1 + T_t))
\end{aligned} \tag{82}
$$

Dividing each term by α does not change the solution, since α is a constant. Let $\frac{\tilde{\gamma}_t^i}{\alpha} \equiv \gamma_t^i$, for $i = 1, 2$, $\frac{\tilde{\Delta}}{\alpha} \equiv \Delta$ and $\frac{\tilde{\psi}_t}{\alpha} \equiv \psi_t$.

The first-order conditions are

$$u_{c,t}(1+\tilde{\gamma}_t^1) - \psi_t - \Delta(u_{cc,t}c_t + u_{c,t} - u_{cl,t}(1-l_t)) = 0 \tag{83}$$

$$u_{l,t}(1+\gamma_t^1) - \psi_t - \Delta(u_{cl,t}c_t + u_{l,t} - u_{ll,t}(1-l_t)) = 0 \tag{84}$$

$$\psi_t = \frac{1-\alpha}{\alpha} + \gamma_t^2 \tag{85}$$

where $c_t \equiv c_t^1$. Posing $\lambda \equiv \frac{1-\alpha}{\alpha}$ makes the system of equations (83)-(85) coincide with (19)-(21).

A.6 Proof of Proposition 5

First notice that, when we introduce (47) as one of the constraints of the Ramsey Planner, the problem becomes non-recursive because future endogenous variables appear in a constraint valid in period t. Once more, we need to apply the recursive contract's approach of Marcet and Marimon (2009) to write a recursive problem which solution coincides with the one of the original problem. The Lagrangean of this new problem can be written as

$$\begin{aligned}\mathcal{L} =& E_0 \sum_{t=0}^{\infty} \beta^t \{u(c_t^1, l_t^1) - (\Delta + \Lambda_{1,t})(u_{c^1,t}c_t^1 - u_{l^1,t}(1-l_t^1)) + \lambda_{1,t}B_{t-1}(g_t) - \lambda_{2,t}(A_t^1(g_{t+1}) \\ &- T_t^1(g_{t+1})) + \lambda_{3,t}(1 - l_t^1 - \sum_{g_{t+1}} q_t T_t^1(g_{t+1}) + T_{t-1}^1(g_t) - c_t^1 - g_t) - \Delta b_{-1} u_{c^1,0}\}\end{aligned}$$

where $\Lambda_{1,t} = \Lambda_{1,t-1} + \lambda_{1,t}$ is a costate variable representing the sum of all past Lagrange multipliers λ_1 attached to constraint (47), and its initial condition is $\Lambda_{1,-1} = 0$. This costate variable keeps track of all past periods in which the constraint on the value of the domestic debt has been binding.

The optimality conditions are

$$u_{c^1,t} - (\Delta + \Lambda_{1,t})(u_{cc^1,t}c_t^1 + u_{c^1,t} - u_{c^1l,t}(1-l_t)) = \lambda_{3,t} \tag{86}$$

$$u_{l^1,t} - (\Delta + \Lambda_{1,t})(u_{c^1l,t}c_t^1 + u_{l^1,t} - u_{ll^1,t}(1-l_t^1)) = \lambda_{3,t} \tag{87}$$

$$q_t(g_{t+1}) = \frac{\beta\lambda_{3,t+1}(g_{t+1})\pi((g_{t+1})) + \lambda_{2,t}}{\lambda_{3,t}} \tag{88}$$

The problem of households in the RW and the optimality conditions associated to it are given by (33)-(35) and (36)-(38), respectively.

Following Alvarez and Jermann (2000), we prove the proposition by construction. We consider three possible scenarios:

1. Neither the HC's nor the RW's participation constraints are binding in $t+1$, $\forall g' = g_{t+1}$
2. The RW's participation constraint is binding in $t+1$ for $g' = g_{t+1}$
3. The HC's participation constraint is binding in $t+1$ for $g' = g_{t+1}$

A.6.1 Neither the HC's nor the RW's participation constraints are binding in $t+1$, $\forall g' = g_{t+1}$

Assume, for simplicity, that the HC has never been borrowing constrained and, consequently, $\Lambda_{1,t-1} = 0$. Set debt limits $A_t^1(g')$, $A_t^2(g')$ and $B_t(g')$ to be very large in absolute value such that constraints (32), (35) and (47) do not bind for $g' = g_{t+1}$. Then multipliers $\lambda_{1,t} = \lambda_{2,t} = \gamma_t^2 = 0$. Given the allocations, equations (36) and (88) define prices. Notice that, from these two equations and the optimality conditions (86) and (87), it has to be the case that $c_t^1 = c_{t+1}^1$ and $l_t = l_{t+1}$. The allocations of the problem of section 2 satisfy these conditions. Finally, from the optimality conditions of the two problems, (19)-(21) and (86)-(88), it is easy to see that it has to be the case that $\lambda_{3,t} = \lambda$. The initial level of international bonds $Z_{-1}^1 = -Z_{-1}^2$ is chosen so that this equality holds.

A.6.2 The RW's participation constraint is binding in $t+1$ for $g' = g_{t+1}$

Given the allocations, the price of the bond is determined by the HC. From equation (36), it is clear that the borrowing constraint of the RW has to be binding, therefore, we set the debt limit $A_t^2(g')$ to be equal to the holding of the corresponding bond. We will explain later how such holdings are determined. The rest of the debt limits, $A_t^1(g')$ and $B_t(g')$, are again set to be very large in absolute value so that constraints (32) and (47) do not bind for $g' = g_{t+1}$. From (19)-(21) and (86)-(88), we know that $\lambda_{3,t+1} > \lambda_{3,t}$ because $\gamma_{t+1}^2 > 0$, which is exactly what the pricing equation is reflecting.

A.6.3 The HC's participation constraint is binding in $t+1$ for $g' = g_{t+1}$

When the participation constraint of the HC binds in $t+1$ for $g' = g_{t+1}$, the price of the bond is determined by the RW from equation (36), given the allocations. From (19)-(21) and (86)-(88), it is clear that $\lambda_{3,t+1} < \lambda_{3,t}$ because $\gamma_{t+1}^1 > 0$. Therefore, from equation (88) we conclude that $\lambda_{2,t} > 0$. Consequently, we set the debt limit $A_t^1(g')$ to be equal to the holding of the corresponding international bond. Moreover, we set the limit $B_t(g')$ to be equal to the holding of the domestic bond, which is already known from the allocations of section 2. The debt limit $A_t^2(g')$ is set to be very large in absolute value so that constraint (35) does not bind.

Notice that equations (19) and (20) imply that:

$$u_{c,t}(1+\gamma_t^1) - \Delta(u_{cc,t}c_t + u_{c,t} - u_{cl,t}(1-l_t)) = u_{l,t}(1+\gamma_t^1) - \Delta(u_{cl,t}c_t + u_{l,t} - u_{ll,t}(1-l_t)) \quad (89)$$

On the other hand, equations (86) and (87) imply:

$$u_{c^1,t} - (\Delta+\Lambda_{1,t})(u_{cc^1,t}c_t^1 + u_{c^1,t} - u_{c^1l,t}(1-l_t)) = u_{l^1,t} - (\Delta+\Lambda_{1,t})(u_{c^1l,t}c_t^1 + u_{l^1,t} - u_{ll^1,t}(1-l_t^1)) \quad (90)$$

If we did not introduce a limit of the value of domestic debt, then the allocations that solve (89) would never solve (90) as well, because the possibility of default in the first case changes the marginal rate of substitution between consumption and labor, but the binding limits on international debt would not do so in the second case. Therefore, we need to introduce a limit on the value of domestic debt as well.

Finally, we need to determine the holdings of the corresponding international bonds. From the budget constraints of the HC and the RW, (43) and (34) respectively, and given prices and allocations, iterate forward to obtain the holding of the international bond for each possible realization of the public expenditure shock g_t. This ensures that c_t^1, l_t and c_t^2 are budget feasible. It is easy to see that, of constructed in this way, $Z_t^1(g') + Z_t^2(g') = 0 \; \forall g'$.