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Nº 278 UNEMPLOYMENT, PARTICIPATION AND WORKER FLOWS OVER
THE LIFE CYCLE

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Unemployment, Participation and Worker Flows Over the Life Cycle *

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Abstract

We estimate and report life cycle transition probabilities between employment, unemployment and inactivity for male workers using Current Population Survey monthly files. We assess the relative importance of each probability in explaining the life cycle profiles of participation and unemployment rates using a novel decomposition method. A key robust finding is that most differences in participation and unemployment over the life cycle can be attributed to the probability of leaving employment and the probability of transiting from inactivity to unemployment, while transitions from unemployment to employment (the job finding probability) play secondary roles. We then show that a simple life cycle extension of a three-state labor search model with leisure shocks can qualitatively replicate the empirical unemployment and participation life cycle profiles, without introducing age or worker heterogeneity in market abilities. We conclude that models that seek to explain life cycle work patterns should not ignore transitions to and from inactivity.

Keywords: Life cycle, Unemployment, Participation, Worker flows, Search.

JEL Codes: **D91, E24, J64**

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1 Introduction

Labor market experiences are significantly different over the life cycle: unemployment rates are higher for younger individuals while participation rates fall dramatically for workers after certain ages. There is also evidence of heterogeneous labor supply volatilities by age groups, as noted by [Blanchard and Diamond \(1990\)](#), [Rios-Rull \(1996\)](#) and [Jaimovich and Siu \(2009\)](#). However, little is known about the worker flows involved in this process: Is high unemployment among the young the result of high job separation or low job finding probabilities? Are transitions in and out of the labor force important?

In this paper we explore the dynamics of transition probabilities between employment, unemployment and inactivity over the life cycle, using Current Population Survey (CPS) monthly data for male workers.¹ We construct measures of worker flows between labor force states and aggregate according to worker ages. From this procedure we can estimate age-dependent job finding, separation, and labor force exit and entry probabilities.

We propose a novel way to account for the relative importance of each of these transition probabilities in shaping the life cycle profiles of unemployment and participation rates. We simulate the labor status of a cohort of individuals by using age dependent Markov chains estimated from the CPS monthly data. In this fashion, we can compare the empirical participation and unemployment profiles with those obtained by our simulations when shutting down lifetime heterogeneity of a subset of transition probabilities. We perform our analysis also by controlling for cohort and time effects and for compositional changes in the population's education and family characteristics.

We find that most differences in participation and unemployment rates over the life cycle can be attributed to the probability of leaving employment and the probability of transiting from inactivity to unemployment. However, transitions from unemployment to employment play only minor roles. The decrease in the probability of leaving employment at the beginning of the life cycle is important to explain the changes in unemployment and participation stocks among the young. Moreover, the decrease in the probability of transiting from inactivity to unemployment at the end of the life cycle is relevant to account for the increase of inactivity in old workers. In light of these facts, we argue that models trying to explain unemployment over the life cycle should not ignore ins and outs

¹In terms of methodology, we follow a rich literature interested in the cyclical behavior of worker flows. See for example [Abowd and Zellner \(1985\)](#), [Darby, Haltiwanger, and Plant \(1986\)](#), [Davis \(1987\)](#), [Blanchard and Diamond \(1990\)](#), [Hall \(2006\)](#), [Shimer \(2007\)](#) and [Fujita and Ramey \(2009\)](#).

of inactivity. Although labor market models that consider employment and unemployment only can achieve good fit of unemployment stocks, they are not useful tools to perform counterfactual experiments nor policy analysis.

Can this evidence be reconciled with standard labor search models? To the best of our knowledge, there is no model considering all the relevant ingredients to answer this question in the literature. Hence, we put forward a basic search and matching model of the job market to provide a basic framework that can account for the evidence. The model is built in the spirit of [Mortensen and Pissarides \(1994\)](#) and extended as in [Garibaldi and Wasmer \(2005\)](#) to consider employment, unemployment and inactivity.² As in the latter paper, the engine generating labor market transitions is a stochastic shock to the flow value of inactivity. We further extend this framework by adding a life cycle, i.e. agents enter for the first time when young as jobless and exit forever the market at a fixed age. Even assuming that all workers have the same market productivity, we are able to qualitatively replicate the profiles of life cycle participation and unemployment rates. This is a remarkable result since all workers are equally productive throughout their lifetime and the stochastic structure of the model does not depend on age. All worker flows are driven by the changing value of inactivity versus the expected value of unemployment/employment in a finite horizon, and the fact that workers start their lives as nonemployed.

However, the model has some quantitative limitations. In the calibrated economy, the decrease in unemployment observed at the beginning of the life cycle lacks persistence. The same critic applies to changes in participation: the increase in participation for the young and the decrease at the end of the life cycle occur too quickly. As a result, unemployment and participation rates in the calibrated economy are similar to the rates of an economy with no life cycle structure. Moreover, our model is unable to reproduce the changes in the transitions out of employment and the flows from inactivity to unemployment, as described by our empirical estimates. We conclude that extensions of the standard model should provide mechanisms able to reproduce those empirical regularities and leave the door open for future research.

Our results show the importance of taking into account life cycle considerations and the age composition of the workforce in order to understand aggregate unemployment and participation rates. This is crucial, for example, when comparing experiences across economies: aggregate un-

²See also [Alvarez and Veracierto \(1999\)](#), [Ravn \(2006\)](#), [Krussel, Mukoyama, Rogerson, and Şahin \(2008\)](#), [Veracierto \(2008\)](#), [Pries and Rogerson \(2009\)](#) and [Krussel, Mukoyama, Rogerson, and Şahin \(2010\)](#).

employment rates for OECD countries exhibit marked heterogeneity (see for example, table 3 in [Jaimovich and Siu \(2009\)](#)) which is driven mainly by the fate of the young and the old, but not the “prime age” workers.³ In turn, our results also show that effective employment creation or unemployment alleviation policies depend crucially on which transition probabilities affect workers the most in each phase of their life cycle and that these forces might be affected by different market arrangements differently across economies.

The model we propose resembles the most to [Chéron, Hairault, and Langot \(2008\)](#) in terms of the market structure and wage setting mechanisms, and to [Menzio, Telyukova, and Visschers \(2010\)](#) in that they also are interested in understanding job flows from employment to unemployment and viceversa. However, both papers completely ignore transitions to and from inactivity and focus on reproducing the decrease in the job finding rate observed along the life cycle. Our exercise shows that this particular flow’s contribution to unemployment and participation over the life cycle is slim. Abstracting from transitions to and from inactivity provides an incomplete picture in that they a priori discard the most important source of unemployment and participation variation over the life cycle.

Other related papers are [Low, Meghir, and Pistaferri \(2010\)](#) and [Kitao, Ljunqvist, and Sargent \(2008\)](#). In the first paper, the authors focus more on the influence of shocks to transition probabilities into worker’s welfare, while, in the second paper, it is analyzed how unemployment interacts with human capital acquisition in a life cycle context. But in both papers only two employment states (employment versus unemployment) are also considered.

The structure of the paper is as follows. In section 2 we discuss our empirical strategy, including data sources, cleaning procedure, the decomposition method we use to account for relative importance of job market flows, and robustness analysis. In section 3, we present our basic model, with discussion of calibration and results. The last section concludes.

2 Empirical Analysis

Our main data source are the basic monthly data files from the Current Population Survey (CPS).⁴ Our sample consists of male workers, observed between January of 1976 to July of 2010, inclusively. In each month (period t) we identify workers according to their labor force status: employment

³Evidence of this can be found in [OECD \(1996\)](#) for the young and in [OECD \(1998\)](#) for older workers.

⁴Available from the National Bureau of Economic Research, at http://www.nber.org/data/cps_basic.html

(e_t), unemployment (u_t) and inactivity/out of the labor force (o_t). Following Shimer (2007), we match individuals across consecutive months based on interview identification numbers, gender, race and age.⁵ This limited longitudinal aspect of the data⁶ is enough to calculate flows between these three employment states for each month.⁷ We define the set of indicator variables D_{nt}^{xz} that take the value of 1 if individual n has transitioned from labor status $x \in \{e, u, o\}$ in period $t - 1$ to labor status $z \in \{e, u, o\}$ in t . Then, we take weighted averages of these indicator variables for each month t , for each age a , and for each birth cohort c to obtain a measure of monthly, age and cohort specific transition probabilities between employment, unemployment and out of labor force states. We denote $\mathbb{I}(a, t, c)$ as an indicator variable that takes the value of 1 if the individual is observed in month t , belongs to cohort c and is a years old, and 0 otherwise. Then, we define the corresponding worker flow f_{atc}^{xz} as follows:

$$f_{atc}^{xz} = \frac{\sum_{n=1}^N D_{nt}^{xz} \cdot \omega_{nt} \mathbb{I}(a, t, c)}{\sum_{n=1}^N D_{nt}^x \cdot \omega_{nt} \mathbb{I}(a, t, c)}$$

where D_{nt}^x equals 1 if the individual was in state x in $t - 1$ and ω_{nt} is the sample weight. In order to extract life cycle profiles for each transition, we run regressions of the form⁸

$$f_{atc}^{xz} = \sum_{a=1}^A \gamma_a^{xz} D_{atc} + \beta W_{atc} + \epsilon_{atc} \quad (1)$$

where $\{D_{atc}\}$ are age dummy variables and $\{\gamma_a^{xz}\}$ are their corresponding estimated coefficients, our statistics of interest. W_{atc} is a vector of potential control variables. However, our main results show the unconditional age-specific transition probabilities because the lessons we obtain from the data remain roughly unchanged after controlling for several variables. In the Appendix we perform a series of robustness analysis of our results by computing transition probabilities conditional on variables such that time dummies, cohort dummies, the proportions of workers with several classes

⁵The unit of analysis in the CPS is a physical address, hence, the same identification number during two consecutive months might not correspond to the same person. Admittedly, the estimates we provide may be slightly biased since the relatively small sample of movers are qualitatively different from stayers. Other papers using this dataset have the same shortcoming.

⁶The matching of individuals can only be done for a maximum of 3 consecutive months, given the rotating panel aspect of the CPS.

⁷We built upon Robert Shimer's Stata codes, which are publicly available at <http://sites.google.com/site/robertshimer/research/flows>

⁸In our subsequent analysis, we do not need estimates of staying probability transitions ee, uu, oo , so we discard them from the presentation.

of educational attainment, and the proportion of workers with specific family characteristics among those characterized by age a , cohort c in month t .⁹

It has been noted that in computing these transition probabilities using CPS data, one can incur in *margin error*, due to the inability of contacting individuals across consecutive surveys. We follow the common practice of assuming that these observations are missing at random (MAR). Although Frazis, Robinson, Evans, and Duff (2006) argue that the predicted stocks of employed, unemployed and inactive workers given estimated transition probabilities under the MAR approach are inconsistent with independently measured stocks, below we show that this doesn't seem to be the case for life cycle averages.

We depict the unconditional estimated life cycle profiles (the collection of $\{\gamma_a^{xz}\}$) in Figure 1 below.¹⁰ The shaded areas around the profiles are confidence intervals constructed from point estimates plus and minus two times their estimated standard deviations.

In Figure 1 we show the transition probabilities for male individuals between the ages of 16 and 70. Employment-to-unemployment (EU), employment-to-inactivity (EO) and the unemployment-to-inactivity (UO) transition probabilities are quite stable between 30 to 60 years of age, while they show a negative slope at younger ages and an increase for older workers. The so-called job finding probability (UE) shows a sharp increase until the mid-20's and then a slight but persistent decrease until age 60. For workers between 60 and 70 this probability increases. Finally, the probability of going from inactivity to both employment and unemployment (OE and OU) shows a hump shaped pattern, peaking around age 30 and steadily decreasing from that point to age 70. Confidence bands show that the estimated age profiles are quite precise.

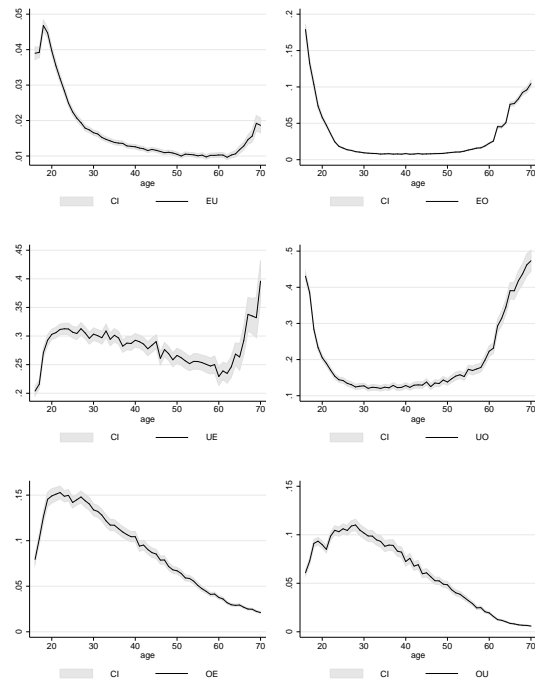
2.1 Markov Chain Analysis

In this subsection, we propose a way to account for the contribution of each transition probability into the determination of participation $P_a = 1 - o_a$ and unemployment $U_a = u_a/(e_a + u_a)$ profiles over the life cycle. Once we get our estimates for transition probabilities, we construct age-specific Markov transition matrices denoted Γ_a . Starting from initial conditions of labor force status at

⁹Since we obtain our age-specific estimates from individuals of the same age at different time periods, it makes sense to control for cohort effects. Similarly, we control for time effects because there are significant fluctuations in labor market transition probabilities at business cycle frequencies and control for family characteristics given the changing structure of household composition in the US.

¹⁰Since we are interested in the average transition probability conditional on W_{atc} , our linear regression model does not have an intercept.

Figure 1: Unconditional life cycle profile of job flows between three states: employment (e), unemployment (u) and out of the labor force (o)



some starting age S_1 , we compute the predicted labor market states after twelve months as

$$S_2 = \Gamma_1^{12} S_1 \quad \text{with } \Gamma_1 = \begin{pmatrix} \gamma_1^{EE} & \gamma_1^{EU} & \gamma_1^{EO} \\ \gamma_1^{UE} & \gamma_1^{UU} & \gamma_1^{UO} \\ \gamma_1^{OE} & \gamma_1^{OU} & \gamma_1^{OO} \end{pmatrix} \quad \text{and } S_1 = \begin{pmatrix} e_1 \\ u_1 \\ o_1 \end{pmatrix}$$

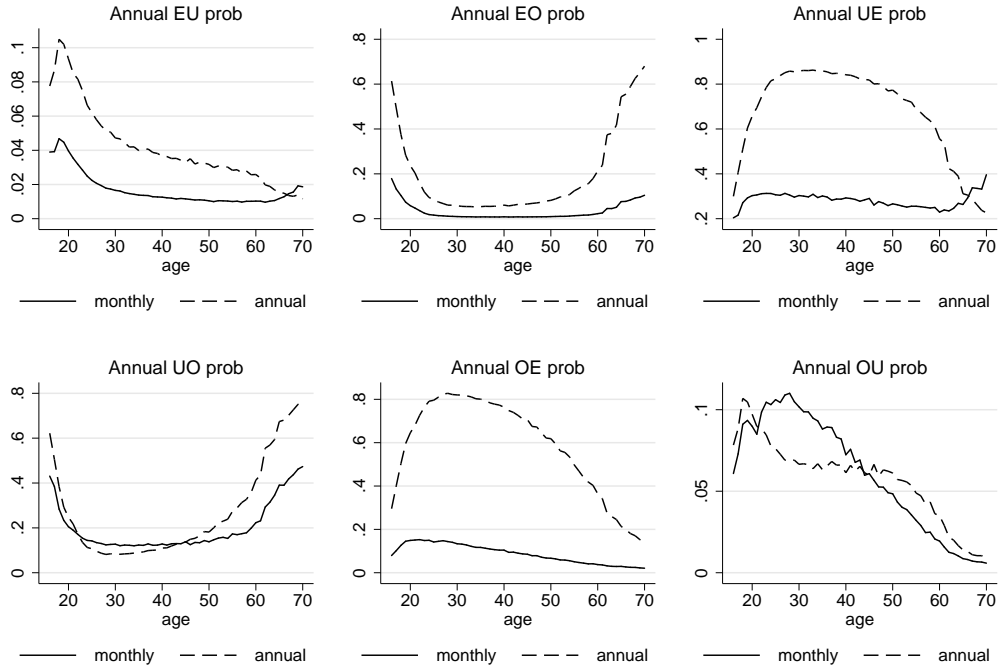
Using the same logic, we can obtain the probability of labor market states at any age a by doing the following calculation

$$S_a = \left(\prod_{i=1}^{a-1} \Gamma_i^{12} \right) S_1 \quad (2)$$

It is important for the exercise to distinguish between annual and monthly transition probabilities.

Figure 2 shows the differences between the two time horizons.

Figure 2: Monthly vs Annual transition probabilities

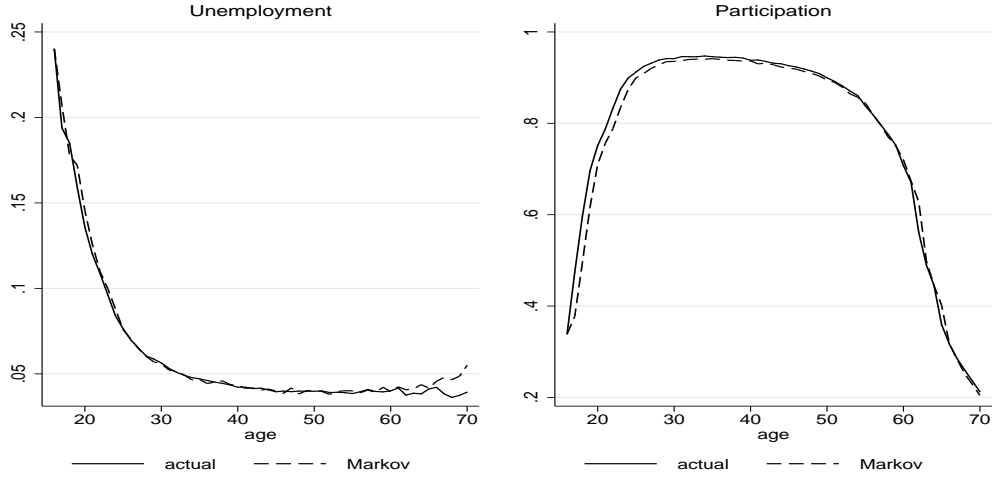


Note: Life cycle profiles computed with no controls.

Using equation (2), we can obtain complete lifetime profiles implied by the estimated transition probabilities, i.e., $U_a = u_a/(e_a + u_a)$ and $P_a = 1 - o_a$ using observed initial conditions. We compare the computed lifetime sequences of participation and unemployment to the actual lifetime

profiles obtained from the data.¹¹ The results are depicted in Figure 3. The estimated transition probabilities come remarkably close to replicate the actual profiles.

Figure 3: Unemployment and Participation according to annual probabilities



Note: Life cycle profiles computed with no controls.

With these constructed transition matrices, we perform a set of decomposition exercises. We want to understand how influential is each specific flow on its own for the determination of unemployment and participation profiles. Thus, we fix the transition probability of each flow (EU , EO , UE , UO , OE , OU) at an arbitrary age. Then, we adjust the probability of “staying” flows (EE , UU , OO) so that the monthly transition matrices are well defined.¹² The remaining five transition probabilities are left unchanged. Using this alternative set of age-specific transition matrices, we can assess the contribution of a specific flow by inspecting the loss of goodness-of-fit derived from such a change. We call this method “all but one change”(AB1C). We interpret this procedure as an approximated model-free counterfactual profile of unemployment and participation, the closest thing to an “all else constant” exercise. This has the advantage of introducing minimal changes to what an underlying structural general equilibrium model would propose for these transition probabilities.

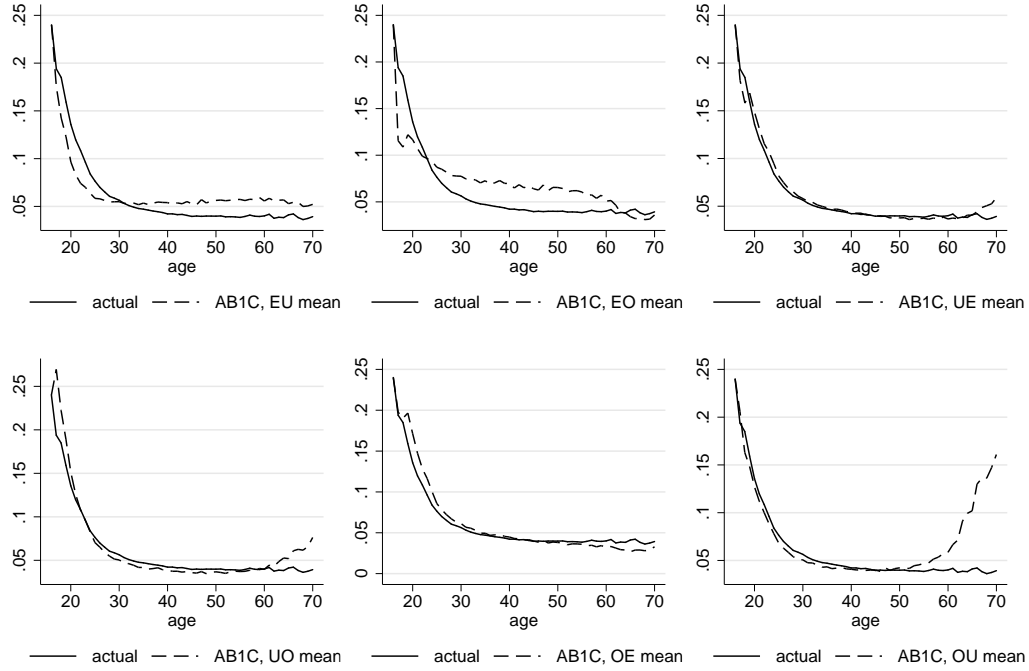
Figures 4 and 5 depict the alternative unemployment and participation profiles when the particular transition probabilities related to each subfigure are replaced by its life cycle average. For

¹¹Note that the actual profiles are computed using the whole monthly CPS sample, while our Markov simulation uses only the matched sample, which is on average 3/4 of the whole sample. Thus, the moving attrition and *margin error* in monthly CPS seems to be unimportant for life cycle averages.

¹²Suppose we fix $\gamma_a^{EU} = \bar{\gamma}^{EU}$ for all a . We adjust the transition matrices by computing $\tilde{\gamma}_a^{EE} = 1 - \bar{\gamma}^{EU} - \gamma_a^{EO}$.

example, the first subfigure in Figure 4 shows how different life cycle participation rates would be if the separation rate (EU) were the same across all ages, instead of being age-specific. Hence, whenever there is a significant difference between both lines, we argue that the particular transition probability contributes to the shape of the life-cycle profile in either participation or unemployment rates.¹³

Figure 4: Importance of flows in Markov chains (AB1C): Unemployment

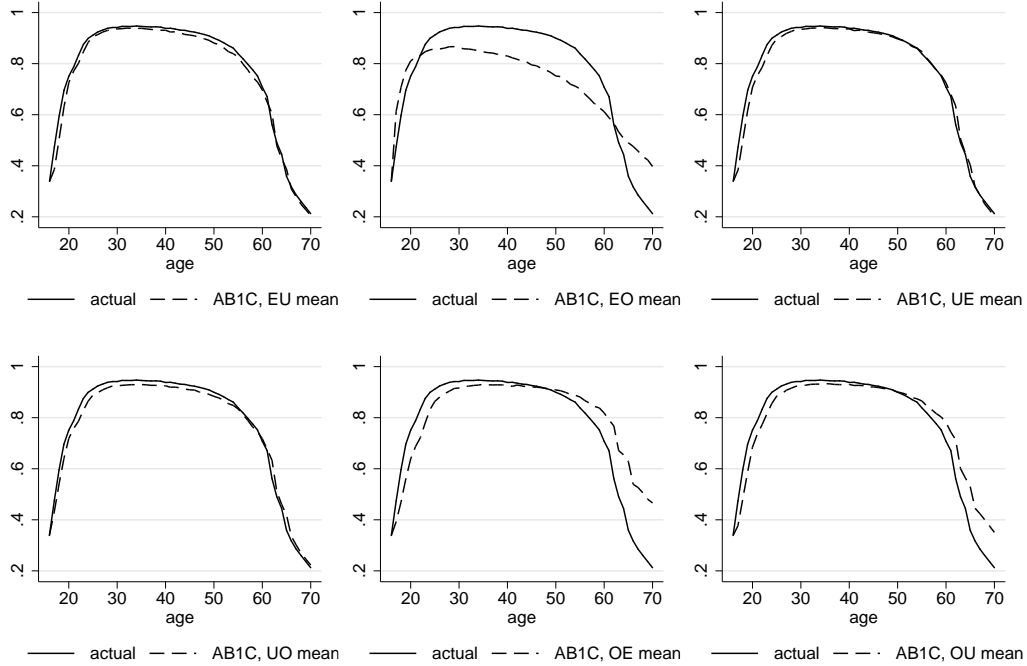


Life cycle profiles computed with no controls.

Looking at Figure 4 we observe that implied unemployment profiles are barely affected by changes in the job finding (UE) and the inactivity-to-employment transition (OE) probabilities. Changes in EU (separation probability) matter to explain higher unemployment of the young and lower unemployment of the old. A larger effect occurs with changes in “retirement” probabilities. If

¹³We also perform an alternative exercise. We assess the specific contribution of only one transition rate by constructing Markov transition matrices that keep fixed all but one transition probabilities at the average life cycle level. We label this method as the “all but one fixed” (AB1F) decomposition. The AB1F method need five out of six transitions to remain fixed, which presumably would imply severe general equilibrium effects in an underlying structural model. Accordingly, we strongly prefer the AB1C method, since it resembles more closely a counterfactual analysis, departing only marginally from estimated values. Nevertheless, results from the AB1F decomposition are quite similar to those obtained from the preferred method. They are available upon request.

Figure 5: Importance of flows in Markov chains (AB1C): Participation



Life cycle profiles computed with no controls.

the *EO* flows were at its average level, we would observe smaller young unemployment and higher old unemployment rates. Finally, the probability of transiting from inactivity to unemployment plays a role in explaining low unemployment rates for workers older than 55 years of age. This decomposition exercise shows that the decision of searching for a job once workers have been inactive is the most important factor explaining low unemployment rates for old workers.

Figure 5 shows the decomposition for the participation rate. The pictures show that neither separation (*EU*), unemployment-to-inactivity (*UO*) nor job finding probabilities (*UE*) explain very much the participation age profile. As much, if they were fixed at they average life cycle level, we would observe a somehow lower participation for workers older than 40. The most important life cycle changes that influence participation profiles come from the “retirement” probability (*EO*). The evolution of this transition probability over the cycle increases labor force participation by as much as 20% at age 60. Finally, the *OE* flow seems to be quite important to explain higher participation for the young and, especially, lower participation for the old.

Admittedly, the choice of keeping each transition probability fixed at its average life cycle value is arbitrary. We could also focus on another type of question. For example, what would be the unemployment or participation life cycle profiles if all workers had the same job finding probability of a 20 year old worker (UE transition)? We can answer this question by using the AB1C decomposition with fixed probabilities at arbitrary ages. Below we focus on young (20), prime age (40) and old (60) workers.

Table 1 summarizes the results from the decomposition exercise as well as the ones of the baseline (average probability) case. This table quantifies the qualitative examinations we perform on Figures 4-5. Here we assess the explanatory power for the AB1C method as the drop in goodness-of-fit generated by keeping constant a particular probability of transition. Hence, our measure of explanatory power in this case is $1 - R_{xz}^2$, where the last term is the R-squared from a regression between the actual life cycle profile of unemployment/participation and the one simulated via an adjusted Markov chain with the flow xz fixed at the specified age.

From Table 1 we see that the EO and OU flows are the most influential ones in determining the life cycle trajectories of unemployment. For instance, if a worker keeps his EO transition probability fixed at its 20-year level, the Markov chain analysis would explain 85% less than what it could if we allowed age dependent EO values. Once we fix the EO probability at other ages, this number decreases to 20%. Thus, the EO flow is particularly important to determine unemployment for young male workers. In contrast, the OU flow plays an important role in shaping unemployment at all ages.

The OE flow plays a small role in shaping unemployment, but is quite relevant for participation. We can interpret this latter flow as the job finding probability of workers exerting little effort to find a job. These transitions are particularly important for young and old workers. An interesting observation is the lack of relevance of the job finding probability (UE flow) at any stage in the life cycle. The latter observation contrasts with the protagonism of this flow in explaining business cycle variation of unemployment, as noted by Hall (2006) and Shimer (2007).

2.2 Relation to existing methods

Our decomposition method is similar to the one used by Pissarides (1986) and Shimer (2007). More specifically, unemployment and labor force participation approximations in the latter are the result of iterating the Markov chains used in the analysis above, an infinite number of times: labor states

Table 1: Unemployment and Participation Flow Decompositions

Fixed probs		<i>EU</i>	<i>EO</i>	<i>UE</i>	<i>UO</i>	<i>OE</i>	<i>OU</i>
γ^{xz} fixed at 20	<i>U</i>	17.0	85.2	4.0	4.1	3.1	46.3
	<i>P</i>	1.0	32.6	1.3	1.3	17.4	6.9
γ^{xz} fixed at 40	<i>U</i>	12.3	17.0	2.4	8.7	3.0	49.7
	<i>P</i>	0.8	18.6	1.3	0.7	22.2	9.1
γ^{xz} fixed at 60	<i>U</i>	12.6	18.3	2.7	7.3	5.0	35.7
	<i>P</i>	0.9	12.6	1.3	0.7	12.4	4.7
γ^{xz} fixed at avg	<i>U</i>	12.1	23.9	2.4	6.5	3.0	46.2
	<i>P</i>	0.7	12.6	1.3	0.8	18.6	6.9

NOTES: The numbers represent the loss of goodness of fit measured by $1 - R^2_{xz}$, where R^2 is the R-squared from a regression between the actual life cycle profile of unemployment/participation and the one simulated via an adjusted Markov chain with the flow xz fixed at the specified age.

obtained from twelve months of transitions (to simulate one year in the life of a worker) with rather high transitional probabilities are not very different from the Markov chain limit. In most cases, the approximation is accurate so that we can construct theoretical counterparts to the observed proportion of individuals in each of the three considered states $\{e, u, o\}$ at age a using the Markov chain limit. Therefore, the approximation at any age a can be constructed by solving the following linear system¹⁴

$$\begin{aligned}
 (\gamma_a^{EU} + \gamma_a^{eo}) \tilde{e}_a &= \gamma_a^{UE} \tilde{u}_a + \gamma_a^{oe} \tilde{o}_a \\
 (\gamma_a^{UE} + \gamma_a^{uo}) \tilde{u}_a &= \gamma_a^{EU} \tilde{e}_a + \gamma_a^{ou} \tilde{o}_a \\
 (\gamma_a^{oe} + \gamma_a^{OU}) \tilde{o}_a &= \gamma_a^{eo} \tilde{e}_a + \gamma_a^{uo} \tilde{u}_a
 \end{aligned}$$

The interpretation of these equations is straightforward. The left hand sides of these equations represent the flow of individuals transiting away from states $\{e, u, o\}$ respectively, at the end of age a . The right hand side accounts for the number of individuals transiting into those same states. These two numbers must be the same, assuming a stationary age-specific population structure and stationary transition probabilities γ_a^{xz} . Solving for the states, we get functional forms that relate

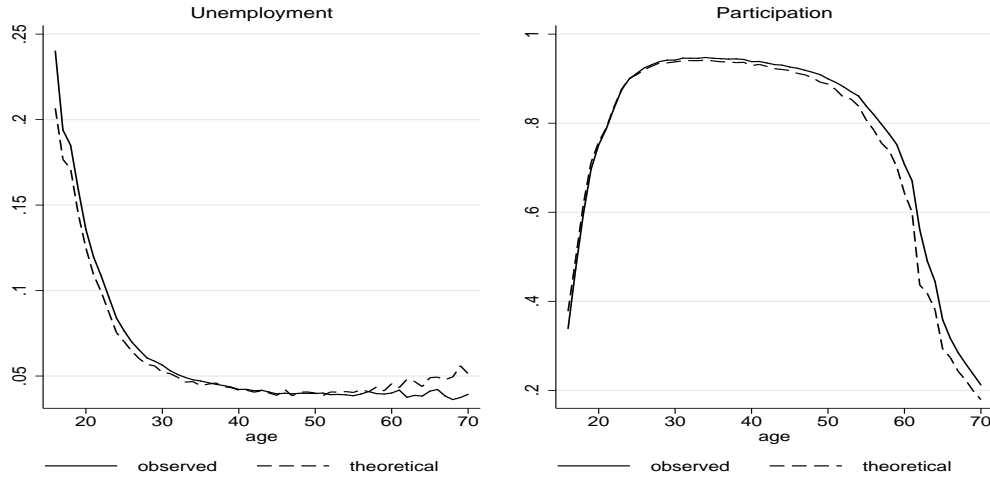
¹⁴The limiting labor states e, u, o are just the normalized eigenvector (so that its components add up to 1) associated to an eigenvalue 1

them to age specific transition rates only.

$$\begin{aligned}\tilde{e}_a &= \tilde{e}(\gamma_a^{UE}, \gamma_a^{UO}, \gamma_a^{OE}, \gamma_a^{OU}) \\ \tilde{u}_a &= \tilde{u}(\gamma_a^{EU}, \gamma_a^{EO}, \gamma_a^{OE}, \gamma_a^{OU}) \\ \tilde{o}_a &= \tilde{o}(\gamma_a^{EU}, \gamma_a^{EO}, \gamma_a^{UE}, \gamma_a^{UO})\end{aligned}$$

accordingly, we can construct these “theoretical” counterparts for participation ($\tilde{P}_a = 1 - \tilde{o}_a$) and unemployment rates ($\tilde{U}_a = \tilde{u}_a / (\tilde{e}_a + \tilde{u}_a)$) using the above equations and our estimates of $\{\gamma_a^{xz}\}$ from regression ((1)). In Figure 6 we plot the observed versus theoretical (constructed) rates.

Figure 6: Participation and Unemployment rates



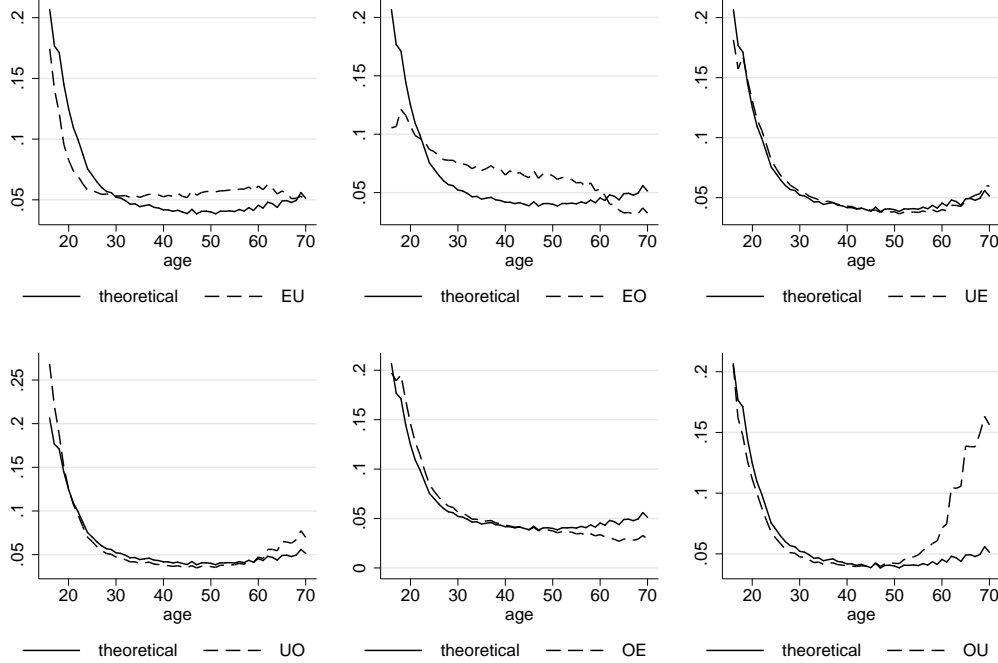
As seen from the Figure, the theoretical rates follow closely their observed counterparts. Thus, the theoretical rates pose a reasonable approximation to the observed profiles. Notice that in order to calculate stocks of unemployed, employed and inactive workers, the method above does not rely on initial conditions/distribution of workers across employment states but only age-specific transition probabilities. The goodness of fit of the theoretical rates is due to high monthly transition probabilities, which dwarfs the effect of initial conditions.

Given that theoretical participation and unemployment rates depend only on age-specific transition probabilities, we can assess their relative importance in explaining aggregate life cycle profiles. Using the same logic as in the “all but one change” (AB1C) method,¹⁵ we compute the limiting

¹⁵This is in contrast to [Shimer \(2007\)](#), who fixes all transition probabilities at their mean and changes only one,

states at each age by using our estimates γ_a^{xz} . However, we keep fixed a particular transition probability at its mean life cycle value, one at a time, and we allow the rest of them to change according to age. We present these decompositions for unemployment and participation in Figures 7 and 8 below.

Figure 7: Decomposition of Life Cycle Unemployment rates

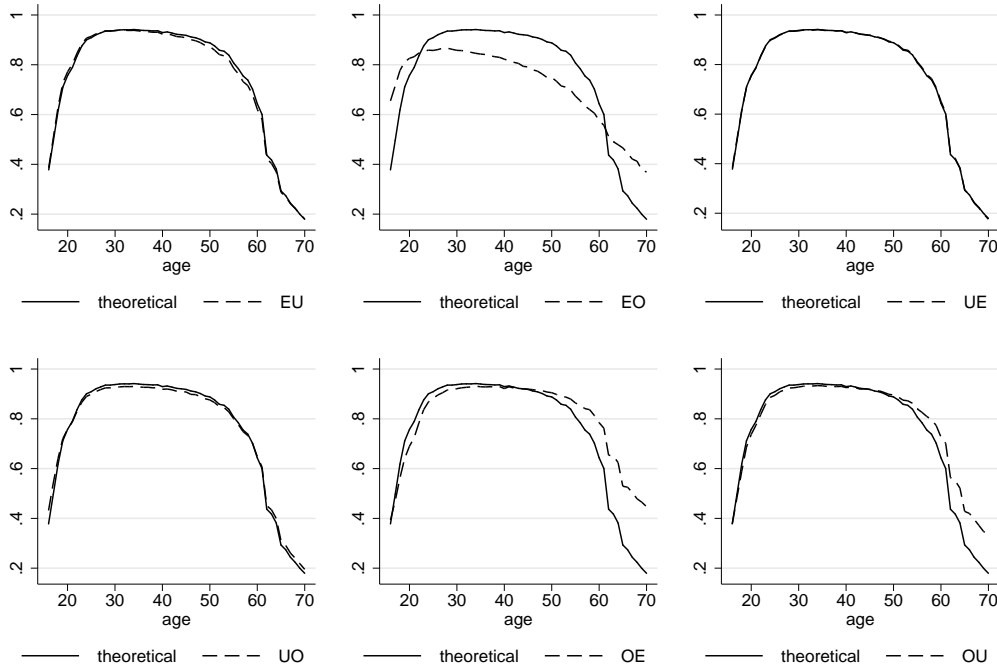


When comparing the results from this “limit” method to the ones we see in the Markov chain analysis, we get roughly identical results. In terms of participation, the most important transition probability is the one from employment to inactivity EO . If this transition probability were to be constant throughout the life cycle, the participation profile would be flatter. The EO probability is very important to determine early and late life employment status. Also, movements from inactivity into the labor force (both OE and OU probabilities) determine to a great extent unemployment after the age of 60.

As for the life cycle profile of the unemployment rate, again the EO probability plays an important role, followed by the EU as well as the OU transition probabilities. The job finding probability (UE) affects differences in life cycle participation and unemployment significantly. It

what we labeled the AB1F method above.

Figure 8: Decomposition of Life Cycle Participation rates



turns out that the transitions between being in versus out of the labor force are quite important in shaping unemployment and participation rates life cycle profiles.

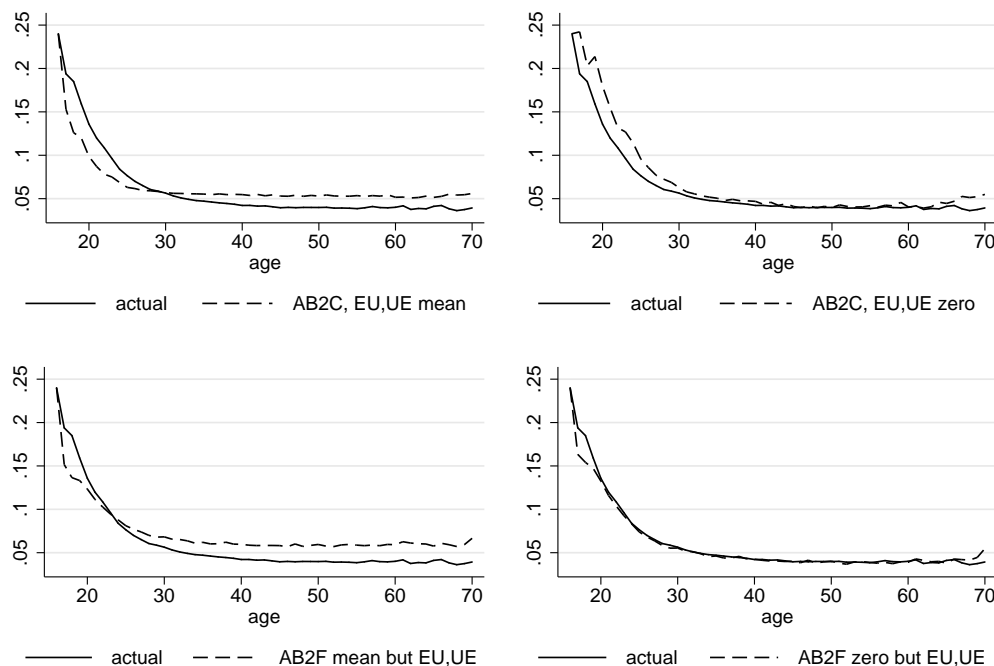
2.3 Effects of ignoring transition to and from inactivity

What happens if we abstract from inactivity in our models? To answer this question we extend the ideas regarding the AB1C decomposition, but we allow both UE and EU to be fixed at arbitrary values over the life cycle. We denote this decomposition as “all but two change” (AB2C). Alternatively we could ask how unemployment and participation profiles would look like if we only allow EU and UE transitions to occur. Using the same nomenclature we could label this procedure as AB2F, “all but two fixed”. This latter setup resembles traditional two state search models since they completely abstract from inactivity transitions. Under both decompositions we consider two scenarios: in the first one, we fix transition probabilities at their life cycle means; in the second one, we set them to zero.

However, it is more difficult to give a meaningful counterfactual interpretation to these exercises. We can think of our estimated transition probabilities as being the equilibrium outcomes from an

underlying structural model. Thus, the more probabilities we change in our empirical counterfactual analysis, the more “equilibrium conditions” are implicitly changed, leading to adjustments that we cannot account for.

Figure 9: Alternative Decomposition of Life Cycle Unemployment rates



This rationale highlights a great danger in pursuing modeling strategies that ignore inactivity. As we see in Figure 9, the fit of the model in which inactivity transition probabilities are zero is nearly perfect. We could naively think that it is appropriate to omit inactivity from structural models as a modeling strategy. Even though such a model would likely give a good fit to the data, it would be useless for running any counterfactual experiments and misleading for policy purposes. If we perform a counterfactual experiment in a model with only UE and EU transitions, we are really doing two things: changing the aspect of interest of the model, and adjusting some unknown equilibrium conditions in order to keep inactivity transitions fixed at zero. Hence, the model structure is not invariant to policy changes.

3 The model

In this section, we develop a three-state labor search model along the lines of Garibaldi and Wasmer (2005), by adding a life-cycle structure to the original framework. We then ask in the next Section if the standard model is able to replicate the aforementioned empirical findings. The conclusion is that it can qualitatively reproduce the evolution of stocks over the life cycle, but some discrepancies are also emphasized. In particular, the increase and then decrease in participation over the life cycle occur too quickly in the model. This is also true for the decline in unemployment at the beginning of the life cycle. Moreover, the gradual decrease in the transition probabilities for the young and the subsequent fall in the OU transition probability for the old are not well matched either.

3.1 Workers

Time is discrete and discounted by a factor β . All agents are risk neutral. There are three sources of heterogeneity among workers. First, workers live for T periods and differ in their age a . The size of each cohort is normalized to one and the total mass of workers in the economy equals T . Second, though all workers share the same *market productivity* \mathcal{Z} , another source of heterogeneity is *home productivity*, denoted by h . Market productivity affects the amount of output a worker produces when he is employed by a firm, while home productivity influences the amount of output he produces at home. Home production occurs only if the worker is inactive. Third, workers can be either employed, unemployed or inactive. Employed workers produce \mathcal{Z} units of output and earn a wage $w_a(h)$. Unemployed workers do not produce anything. They search for a job, which they find with probability p . Meanwhile they receive an amount of benefits b . The flow utility of being inactive is equal to $h\mathcal{H}$, the amount of home production, where \mathcal{H} is a scaling factor.

Home productivity evolves stochastically. In each period, with probability λ , all individuals draw new productivity value from the distribution G with support on $[0, 1]$. The distribution of home productivities at age 1 is denoted by G_0 .

The random evolution of home productivity over the life cycle implies that workers may choose to move out of a specific labor market state. We denote by h_a^u the reservation productivity such that a worker with age $a = 1, \dots, T$ is indifferent between inactivity and unemployment. Similarly, h_a^w is the reservation value such that a worker is indifferent between inactivity and employment. Additionally, exogenous job separations occur with probability s , forcing workers out of employment.

At any time, workers can choose to be unemployed or inactive at no cost. However, to find a new job, workers have to be unemployed first. No worker is employed at age 1.

We denote respectively by $W_a(h)$, $U_a(h)$ and $H_a(h)$ the present discounted values of being employed, unemployed or inactive for a worker of age a with home productivity h . Their mathematical formulation can be found in the Appendix.

3.2 Firms

Firms post and keep open vacancies at a flow cost c and hire one worker with probability q . Once a worker is hired, production starts.

Search is undirected in the economy. There is a unique labor market, implying that, when a firm posts a vacancy, it ignores the characteristics of the worker that may be hired in the future. Hence the whole distribution of ages and home productivities among the unemployed has to be taken into account when calculating the value of posting a vacancy. We denote by $\Psi_U(a, h)$ this distribution. Similarly, $\Psi_W(a, h)$ and $\Psi_H(a, h)$ refer to the joint distributions of age and home productivities among the employed and inactive populations respectively.

3.3 Wages

Wages are bargained *à la* Nash. The wage $w_a(h)$ is derived from the following condition:

$$(1 - \alpha) [W_a(h) - \max\{U_a(h), H_a(h)\}] = \alpha J_a(h), \quad (3)$$

where $\alpha \in (0, 1)$ denotes the bargaining power of workers.

3.4 Free entry

The description of the economy has so far disregarded the determination of the transition probabilities p and q . We now detail the assumptions yielding a solution for these probabilities.

The labor market is characterized by a matching function $m(u, v)$, which gives the mass of formed matches in each period. It is an increasing function of both the aggregate mass of posted vacancies v and the aggregate mass of unemployed u . It is also a constant-returns-to-scale and concave function that satisfies $m(u, 0) = m(0, v) = 0$.

The probability of filling a vacancy can be calculated as the share of vacancies that actually leads to a match, i.e. $q = \frac{m(u, v)}{v}$, and the probability for a worker to find a job is $p = \frac{m(u, v)}{u}$.

Given constant returns to scale, it follows that

$$p = m(1, \theta^{-1})$$

and

$$q = m(\theta, 1),$$

where $\theta \equiv \frac{v}{u}$ is the labor-market tightness.

The labor market tightness is a sufficient statistic for the determination of the probabilities and its equilibrium value is obtained by assuming free entry of firms, that is, in each period, the following restriction is imposed:

$$V = 0. \tag{4}$$

3.5 Stationary equilibrium

We study the stationary equilibrium of the economy. This equilibrium is defined as

- a set of wage rules $\{w_a(h)\}_{a=1}^T$ for all $h \in (0, 1)$ satisfying Nash bargaining, given by condition (3),
- a set of reservation productivities $\{h_a^u\}_{a=1}^T$ and $\{h_a^w\}_{a=1}^T$ that solve the Bellman equations $W_a(h)$, $U_a(h)$ and $H_a(h)$,
- a labor market tightness θ satisfying the free entry condition (4) and
- distributions $\Psi_U(a, h)$, $\Psi_W(a, h)$ and $\Psi_H(a, h)$ that are constant over time.

4 Quantitative analysis

4.1 Calibration

Our model differs from a standard search model only in the assumption that an additional source of worker heterogeneity is age. In this Section, we question the quantitative importance of this additional ingredient of the model. Age matters for two reasons. First, agents cannot be born employed. Thus a reason why age may have some quantitative implications is because initial conditions force part of the labor force out of employment. Second, age introduces a finite horizon restriction, which is absent in a standard search model where agents are infinitely lived. This restriction may induce

some workers to choose to retire before period T depending on their productivity values and how far they are from the end of their lives.¹⁶

As a consequence of these two observations, prime-age workers in our economy offer a natural benchmark to compare an economy with a life-cycle structure to an economy from a standard search model. Prime-age workers in the model are very similar to the agents in an economy with no life-cycle structure. Two elements support this claim. First, the Blackwell (1965) sufficient conditions for a contraction imply that, as we move away from the last period T , the decision rules followed by workers are approximated by the decision rules followed by infinitely lived agents. Hence, the transition probabilities between labor-market states in the life cycle model are close to the ones of an economy without life-cycle structure if workers are *young enough*. Second, because transition probabilities follow the Markov property for young agents, if the associated transition matrix has a unique unit eigenvalue, the distribution of workers between the three labor-market states in the stationary equilibrium converges to the distribution in the economy without life-cycle structure. This happens when agents are *old enough*.

Following the latter rationale, our calibration strategy chooses parameter values such that some moments of prime-age workers (40-49) are replicated. In particular, we choose parameters that match the average transition probabilities of prime-age workers in a model without life-cycle structure à la Garibaldi and Wasmer (2005) and the transition probabilities for this age group. We do not restrict the parameters to fit the behavior of young and old workers. We thus leave the moments characterizing the behavior of young and old workers to assess the quantitative relevance of the life-cycle structure of the model.

We consider a time period to be a month and focus on the population aged 18-64. Workers live for $T = (64 - 18 + 1) \times 12 = 564$ periods. Because of the discrete-time nature of the data, transitions from inactivity to employment (OE) are observed. Unfortunately our model does not consider transitions of this sort. For this reason, we follow Garibaldi and Wasmer (2005) and attribute those transitions to infra-month movements. We apply the procedure described in their paper and base the calibration on the modified rates $(\gamma_{40-49}^{OU})^{TA} = \sum_{a=40}^{49} (\gamma_a^{OU} + \gamma_a^{OE})/10$ and $(\gamma_{40-49}^{UE})^{TA} = \sum_{a=40}^{49} (\gamma_a^{UE} + \gamma_a^{OE})/10$, where the subscript TA stands for the correction for the time

¹⁶The first element can be recovered in the standard search model simply by assuming that agents die at a rate n and are immediately replaced by a mass of new-born workers. Cahuc and Zylberberg (2004) show that the steady-state rate of unemployment is an increasing function of n in this case. In our model, n is equal to T^{-1} . However this assumption does not exclude the finite horizon restriction because death is a non-anticipated shock.

aggregation bias.

We numerically analyze the stationary equilibrium by making discrete the set of home productivity values that characterize workers. Home productivity is constrained to take values in $\hat{H} = \{\hat{h}_1, \hat{h}_2, \dots, \hat{h}_{500}\}$. We assume that the new value home productivity takes after a shock occurs is drawn from a uniform distribution with support on this interval. The initial distribution of home productivities (at age 1) is also uniformly distributed on the same interval. Hence, average home productivity is constant over the life cycle. Additionally, we fix the productivity parameter \mathcal{Z} to be equal to one.

We follow [Cole and Rogerson \(1999\)](#) by noticing that the probability of finding a job p can be treated as a parameter (provided that an equilibrium exists). Indeed, the calibration chooses the parameters of the model in order to match the average probabilities to transit between the three labor market states. Hence, some parameters such as the flow cost of having a vacancy posted c and the parameters characterizing the matching function m can always be fixed such that a particular value for p is assigned. This is simply equivalent to considering p as a parameter. From this perspective, seven parameters have to be chosen: the discount factor β , the flow utility of being unemployed b , the maximum home productivity value \mathcal{H} , the transition probability p , the bargaining power α , the separation rate s , and the probability of receiving a home productivity shock λ . We fix the discount rate at a 4% yearly rate. Because it is difficult to identify the bargaining power separately from the flow utility of being unemployed, we fix it at its standard 0.5 value. The rest of the parameters are obtained by minimizing a measure of distance between the average transition probabilities in the data and the model.

Table 2 reports the results. The parameter values can be interpreted from the targets. First, the probability of finding a job for an unemployed (p) is equal to 34.02% and is similar to the probability that an unemployed becomes employed in the data after applying the [Garibaldi and Wasmer \(2005\)](#) procedure to correct for time aggregation bias. Second, the job separation probability in the model ($s = 0.0231$) is also similar to the probability that an employed worker becomes non employed, which is equal to the sum of the *EU* and *EO* probabilities. Third, the value taken by λ equals twice the probability that an inactive becomes unemployed. This is a consequence of the important size of the *UE* transition probability as compared to the probabilities of transiting between inactivity and unemployment: to obtain such a large transition flow in the model, prime-aged unemployed

workers receiving a home productivity shock are constrained to accept any job offer they receive. As a result, only half of non participants receiving a home productivity shock chooses to search for a job. Finally, one can observe that, while the UO transition probability is similar to the OU in the data, it is not the case for the model. This is a consequence of the discrete-time nature of the model as compared to [Garibaldi and Wasmer \(2005\)](#), who consider continuous time. Since we consider their model for the calibration, a discrepancy is generated by the fact that some workers, who are supposed to transit from unemployment to inactivity in their model, receive job offers, which they accept in our discrete-time model.

The values of b and \mathcal{H} can be compared to the values used in the calibration of standard search models ([Pissarides \(2009\)](#)). But, because the standard model ignores inactivity, we consider the average flow value of non employment for comparison. This value is an average of the flow values of unemployment and inactivity. The calibration of \mathcal{H} produces an average home productivity for employees equal to 0.6486. Combined with the value for b (0.2586), it implies an average flow value of non employment (for employees) equal to 0.5952. This value is a bit below the flow value of non employment used in [Pissarides \(2009\)](#) or [Hall and Milgrom \(2008\)](#), which is equal to 0.71. As a consequence, the average wage represents 92% the value of output, while it is 98% in [Pissarides \(2009\)](#) and 99% in [Hall and Milgrom \(2008\)](#).

The ratio of unemployment benefits to the paid wage is 24%, which is close to the 25% ratio that [Hall and Milgrom \(2008\)](#) fix. They also consider that it is a reasonable estimate between two bounds. On the one hand, [Hall \(2006\)](#) calculates the ratio of benefits paid to previous earnings, on the assumption that the unemployed have the same average wage as the employed and finds the ratio to be about 12%. This is a lower bound because unemployed workers receive wages that are on average lower than employed. On the other hand, [Anderson and Meyer \(1997\)](#) calculate an after-tax replacement rate of 36% from statutory provisions of the Unemployment Insurance system. This is an upper bound because a significant fraction of the unemployed do not receive any benefit.

4.2 Decision rules

We approximate the value functions $W_a(h)$, $U_a(h)$ and $H_a(h)$ with the value function iteration algorithm. We first approximate them for $a = T$, as given in equations (5), (6) and (7) in the Appendix. Then, given those approximations, we recursively approximate the rest of the functions

Table 2: Summary of the calibration

<i>Fixed parameters</i>	Parameter	Value
Discount factor	β	$(0.96)^{1/12}$
Bargaining power	α	0.5
Market productivity maximum value	\mathcal{Z}	1
<i>Calibrated parameters</i>	Parameter	Value
Home productivity maximum value	\mathcal{H}	1.3308
Flow utility of being unemployed	b	0.2586
Probability of finding a job	p	0.3402
Probability of separation	s	0.0208
Probability of home productivity change	λ	0.2783
<i>Calibration targets: transition probabilities</i>		
<i>40-49 years old workers</i>	Model	Data
EU	1.14%	1.16%
EO	0.96%	0.82%
UE	34.01%	34.02%
UO	8.60%	13.16%
OU	14.81%	14.70%

Notes: We apply the correction proposed by [Garibaldi and Wasmer \(2005\)](#) for the transition probabilities *OU* and *UE* by adding the observed *OE* transition probability (8.56%).

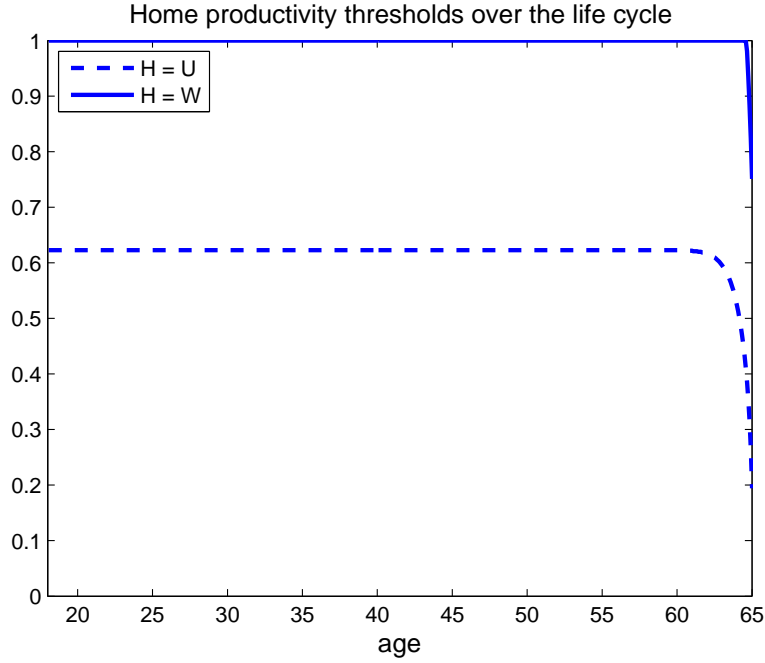
for $a < T$ by iterating backwardly.

The policy function is obtained from these value functions. In [Figure 10](#) the solid line displays the home productivity value that makes a worker indifferent between employment and inactivity. The dashed line shows the home productivity value that makes a worker indifferent between unemployment and inactivity. Both values are non increasing over the life cycle.

Workers are willing to search for a job when they anticipate a long enough stream of sufficiently high wages that compensate the initial search cost. As workers get older, the unemployed requires lower home productivity to choose inactivity. The discounted sum of wages decreases in age because little time remains and outside options decay, while the search cost does not change. This explains why the dashed line decreases on the [Figure](#).

The evolution of a firm surplus as well as Nash bargaining explains why the solid line is decreasing. As a worker becomes older, the outside option for the firm of searching for an alternative

Figure 10:



worker increases, decreasing the firm surplus. Because wages are determined under Nash bargaining, the fall in the firm surplus generates a decrease in wages. This in turn gives incentives to workers to retire earlier.

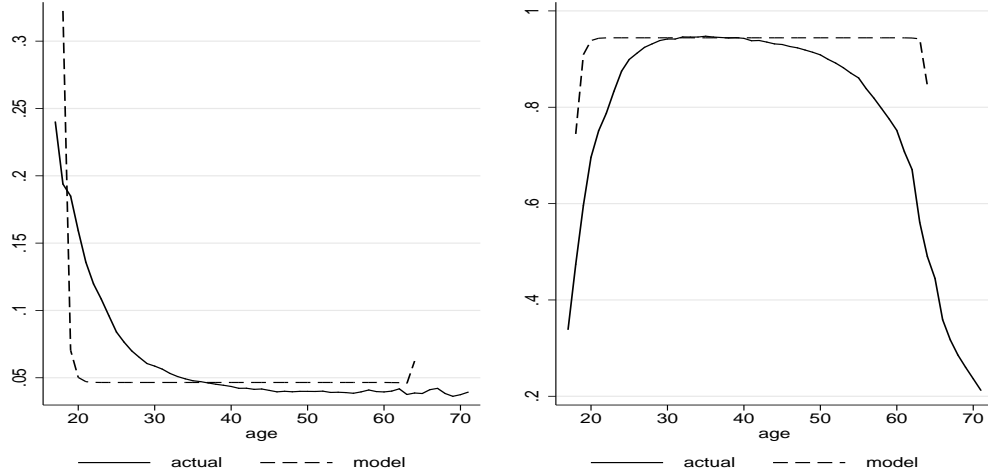
4.3 Unemployment and participation over the life cycle

The evolution of those decision rules help explain the evolution of unemployment and participation over the life cycle, which are displayed on Figure 11. On those graphs we report the evolution of the two statistics both in the model and the data.

We have two comments on the Figure. First, our model has good qualitative implications. It predicts higher unemployment for young agents and lower participation for both young and old workers, as the data also shows. Those facts are consequences of initial conditions and the finite horizon restriction imposed by the life cycle structure of the economy. They are not generated by ad-hoc assumptions on the home and market productivities of young and old workers.¹⁷ Moreover these results are not driven by our calibration strategy because we use moments related to prime-aged workers to obtain our parameters.

¹⁷See e.g. Rios-Rull (1996), Gomme, Rogerson, Rupert, and Wright (2004), Hansen and Imrohoroglu (2008), Janiak and Monteiro (2010).

Figure 11: Unemployment and Participation: Model versus Data

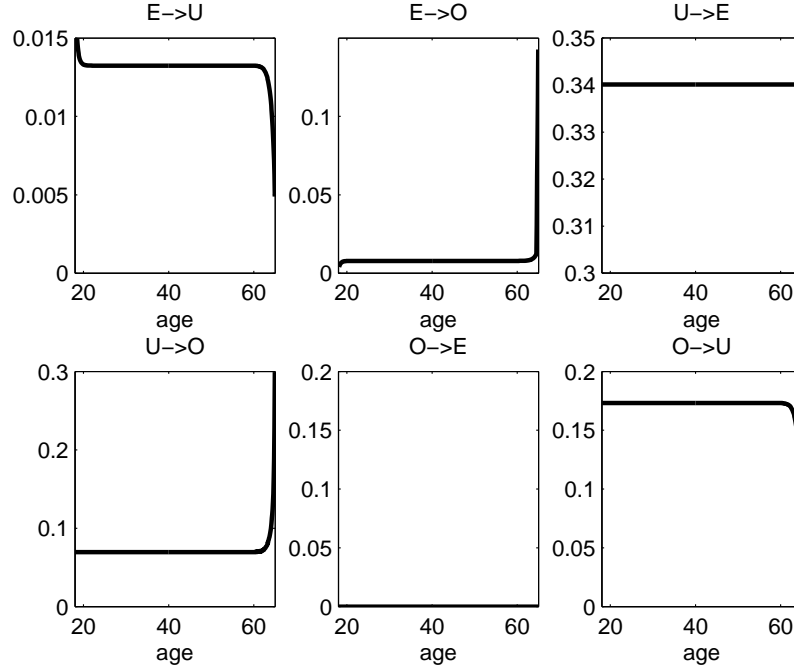


Some of the qualitative predictions of the model can be understood easily from the previous discussion. Agents are born non-employed. This fact explains the higher unemployment rate for young agents because those workers go through a time expensive search process to find a job. The lower participation rate for old workers is explained by the fall over the life cycle in the home productivity threshold values reported on Figure 10.

The somewhat surprising result in Figure 11 is the increase in the participation rate of young agents at the beginning of the cycle. It is surprising because the distribution of home market productivities for young agents is the same as the distribution for prime-aged workers. Moreover, all agents have the same market productivity. Those differences are usually thought as the main factors behind differences in participation. The reason here is the following. Due to the relatively high job finding rate, young unemployed workers quickly become employed. Once they get a job, they require a very large home productivity shock to choose inactivity as shown in Figure 10. This explains why individuals who participate with or without a job tend to remain in the labor force. This also implies that individuals who initially choose to participate tend to keep doing so.

On the other hand, workers who initially choose inactivity may face home productivity shocks so that they will choose to become unemployed. Given the relatively large job finding rate, these workers are likely to become employed soon. Hence, inactive workers at the beginning of their lives will choose participation after some time. These two combined forces imply a quick reduction of

Figure 12: Transition Probabilities implied by the Model



inactivity for the youngest workers.¹⁸

Second, though the model can generate both an inverted U-shaped pattern for participation and higher unemployment for the young, those patterns are not quantitatively pronounced. This is shown on Figure 11. The initial increase and later decrease in participation over the cycle is much more gradual in the data than in the model. The picture shows that it takes about two years for the participation rate to reach the plateau, while it takes more than ten years in the data. The difference is even more striking for the decrease in participation at the end of the cycle. The problem on the persistence also applies to the initial decrease in unemployment: though the unemployment rate is equal to 100% in the first period, it converges to the prime-age value two years after, while, again, this process takes more than ten years in the data.

The quantitative shortcomings of the life-cycle structure of the model are shown in Table 3. Here we compare the calibrated economy with another economy characterized by the same parameter values, but with no life cycle structure. It shows the aggregate participation and unemployment

¹⁸These effects are reinforced because the initial good home productivity shocks of the inactive are reversed later in life, on average. For this additional reason, a large mass of young inactive workers goes to unemployment in the model.

Table 3: Comparing economies with and without life-cycle structure

	Unemployment rate	Participation rate
Life	6.29%	90.51%
No life	5.83%	91.31%
Difference	+0.46%	-0.80%

rates of both economies. The statistics in the latter economy are actually very similar to the participation and unemployment rates for the prime-aged workers in the former economy. Confirming the discussion on the poor quantitative implications of the life-cycle structure of the model, the Table shows small differences in the participation and unemployment rates. The unemployment rate in the life-cycle economy is half a percentage point larger (6.29% vs. 5.83% in the economy without life-cycle structure). This difference is mainly driven by the initial condition that imposes 100% unemployment rate in the first period. For participation rates the difference is even smaller (in relative terms): 90.51% participation rate in the life-cycle economy vs. 91.31% in the economy without life-cycle structure.

This quantitative issue reminds the well-known discussion in [Shimer \(2005\)](#) of the limited implications of search frictions for the business cycle. The idea in [Shimer \(2005\)](#) is that search frictions alone generates a quick convergence toward steady state. To generate large difference in unemployment over time, one needs transitions probabilities to vary as well. Here we observe the same mechanics applied to the life cycle. This is confirmed by the comparison of Figure 12 with Figure 1. While the data seems to assign an important role to the decrease in the *EO* and *EU* transition probabilities at the beginning of the cycle to generate appropriate rates of participation and unemployment for young workers, this feature is absent in the model, where transition probabilities are barely constant over time. Similarly the persistent decrease in the transition probabilities out of inactivity observed in the data at the end of the life cycle appears too late in the model. Moreover, the calibrated economy assigns an important role to the increase in the *UO* transition probability, which does not appear as important from the discussion in Section 2.

We conclude that extensions of the model should focus on mechanisms leading to a persistent fall in the probability of exiting employment at the beginning of the life cycle as well as a decrease in the probability of leaving inactivity at the end of it. Absent those characteristics, it would be hard to obtain appropriate life cycle patterns in participation and unemployment. In the Appendix

we show that similar criticisms apply to the evolution of wages: the model is able to generate a hump-shaped relation between wages and age, but it lacks persistence at the beginning and end of the life cycle.¹⁹

5 Conclusion

In this paper we estimate and report life cycle transition probabilities across labor market states for male workers in the US, using data from the Current Population Survey. As in [Shimer \(2007\)](#) and [Fujita and Ramey \(2009\)](#), we construct measures of worker flows between labor force states and aggregate worker flows according to age. This procedure gives us a consistent set of facts from which we can identify age dependent job finding and job destruction rates as well as labor force exit/entry rates.

Using our estimates, we find that most differences in participation and unemployment rates over the life cycle can be attributed to the probability of leaving employment and the probability of entering unemployment from inactivity. On the other hand, the job finding rate plays a minor role. Hence, we argue that two state labor market models are not appropriate to perform counterfactual experiments for policy analysis even though they may provide a good fit to unemployment facts. The data shows a great deal of inactivity transitions whose variation greatly affects unemployment and participation life cycle profiles. If a model ignores them, any policy experiment in that economy would also involve an unknown change of general equilibrium conditions in order to maintain inactivity transition probabilities fixed at zero.

We then put forward a simple search and matching model of the job market, in the spirit of [Mortensen and Pissarides \(1994\)](#) and extended as in [Garibaldi and Wasmer \(2005\)](#) to account for three working states. Using a life cycle version of the model where all agents are homogeneous in terms of market productivity but receive individual shocks to the value of inactivity, we are able to replicate the observed profiles of life cycle participation and unemployment rates. However, we are not successful in matching the implied flows nor transition probabilities as seen in the data, which suggests new directions for further research.

¹⁹As an alternative way of calibrating the model, we have tried to match the life cycle profiles of unemployment and participation, taking the transition probabilities out of our calibration targets. In order to obtain a persistent evolution of the stocks over the life cycle, we need transition probabilities to be much smaller. This is because we need to give more importance to initial and final conditions and this can be done by reducing reallocation between the three labor-market states. Moreover, it is hard to obtain a persistent evolution of the stocks without generating a low level of participation or high unemployment.

References

- ABOWD, J. M., AND A. ZELLNER (1985): “Estimating Gross Labor-Force Flows,” *Journal of Business & Economic Statistics*, 3(3), 254–83.
- ALVAREZ, F., AND M. VERACIERTO (1999): “Labor-market Policies in an Equilibrium Search Model,” *NBER Macroeconomics Annuals*, pp. pp. 265–304.
- ANDERSON, P. M., AND B. D. MEYER (1997): “Unemployment Insurance Takeup Rates and the After-Tax Value of Benefits,” *The Quarterly Journal of Economics*, 112(3), pp. 913–937.
- BERTOLA, G., AND R. J. CABALLERO (1994): “Cross-Sectional Efficiency and Labour Hoarding in a Matching Model of Unemployment,” *The Review of Economic Studies*, 61(3), pp. 435–456.
- BLACKWELL, D. (1965): “Discounted Dynamic Programming,” *The Annals of Mathematical Statistics*, 36(1), pp. 226–235.
- BLANCHARD, O. J., AND P. DIAMOND (1990): “The Cyclical Behavior of the Gross Flows of U.S. Workers,” *Brookings Papers on Economic Activity*, 21(1990-2), 85–156.
- CAHUC, P., AND A. ZYLBERBERG (2004): *Labor Economics*, vol. 1 of *MIT Press Books*. The MIT Press.
- CHÉRON, A., J. HAIRAUT, AND F. LANGOT (2008): “Life-Cycle Equilibrium Unemployment,” Discussion paper no. 3396, IZA.
- COLE, H. L., AND R. ROGERSON (1999): “Can the Mortensen-Pissarides Matching Model Match the Business-Cycle Facts?,” *International Economic Review*, 40(4), pp. 933–959.
- DARBY, M. R., J. C. HALTIWANGER, AND M. W. PLANT (1986): “The Ins and Outs of Unemployment: The Ins Win,” UCLA Economics Working Papers 411, UCLA Department of Economics.
- DAVIS, S. J. (1987): “Fluctuations in the Pace of Labor Reallocation,” *Carnegie-Rochester Conference Series on Public Policy*, 27(1), 335–402.
- FRAZIS, H., E. ROBINSON, T. EVANS, AND M. DUFF (2006): “Estimating Gross Flows Consistent with Stocks in the CPS,” *Monthly Labor Review*, 128(9), 3–9.
- FUJITA, S., AND G. RAMEY (2009): “The Cyclicity of Separation and Job Finding Rates,” *International Economic Review*, 5(2), pp. 415–430.
- GARIBALDI, P., AND E. WASMER (2005): “Equilibrium Search Unemployment, Endogenous Participation, and Labor Market Flows,” *Journal of the European Economic Association*, 3(4), 851–882.
- GOMME, P., R. ROGERSON, P. RUPERT, AND R. WRIGHT (2004): “The Business Cycle and the Life Cycle,” Working paper, Federal Reserve Bank of Cleveland.
- HALL, R. E. (2006): “Job Loss, Job Finding and Unemployment in the U.S. Economy over the Past 50 Years,” in *NBER Macroeconomics Annual 2005, Volume 20*, NBER Chapters, pp. 101–166. National Bureau of Economic Research, Inc.

- HALL, R. E., AND P. R. MILGROM (2008): “The Limited Influence of Unemployment on the Wage Bargain,” *American Economic Review*, 98(4), 1653–74.
- HANSEN, G., AND S. IMROHOROGLU (2008): “Consumption over the Life Cycle: The Role of Annuities,” *Review of Economic Dynamics*, 11(3), 566–583.
- JAIMOVICH, N., AND H. SIU (2009): “The Young, the Old, and the Restless: Demographics and Business Cycle Volatility,” *American Economic Review*, 99(3), pp. 804–826.
- JANIAK, A., AND P. MONTEIRO (2010): “Labor Force Heterogeneity: Implications for the Relation Between Aggregate Volatility and Government Size,” Center for Applied Economics WP 272, University of Chile.
- KITAO, S., L. LJUNQVIST, AND T. SARGENT (2008): “A Life Cycle Model of Trans-Atlantic Employment Experiences,” mimeo, University of Southern California.
- KRUSSEL, P., T. MUKOYAMA, R. ROGERSON, AND A. ŞAHİN (2008): “Aggregate Implications of Indivisible labor, incomplete markets, and labor market frictions,” *Journal of Monetary Economics*, 55, 961–979.
- (2010): “Aggregate labor market outcomes: the roles of choice and chance,” *Quantitative Economics*, 1(1), 97–127.
- LEMIEUX, T. (2006): “Increasing Residual Wage Inequality: Composition Effects, Noisy Data, or Rising Demand for Skill?,” *American Economic Review*, 96(3), 461–498.
- LJUNGQVIST, L. (2002): “How Do Layoff Costs Affect Employment?,” *Economic Journal*, 112(482), 829–853.
- LOW, H., C. MEGHIR, AND L. PISTAFERRI (2010): “Wage Risk and Employment Risk over the Life Cycle,” *American Economic Review*, 100(4), pp. 1432–1467.
- MENZIO, G., I. TELYUKOVA, AND L. VISSCHERS (2010): “Directed Search over the Life-Cycle,” mimeo, University of Pennsylvania.
- MORTENSEN, D. T., AND C. A. PISSARIDES (1994): “Job Creation and Job Destruction in the Theory of Unemployment,” *Review of Economic Studies*, 64, pp. 397–415.
- MORTENSEN, D. T., AND C. A. PISSARIDES (1999): “New Developments in Models of Search in the Labor Market,” in *Handbook of Labor Economics*, ed. by O. Ashenfelter, and D. Card, vol. 3 of *Handbook of Labor Economics*, chap. 39, pp. 2567–2627. Elsevier.
- MURPHY, K. M., AND F. WELCH (1990): “Empirical Age-Earnings Profiles,” *Journal of Labor Economics*, 8(2), 202–29.
- OECD (1996): “Growing into Work,” in *Employment Outlook*, chap. 4, pp. 109–159. OECD.
- (1998): “Work-Force Ageing in OECD Countries,” in *Employment Outlook*, chap. 4, pp. 123–151. OECD.

- PISSARIDES, C. A. (1986): “Unemployment and Vacancies in Britain,” *Economic Policy*, 1(3), 499–559.
- (2009): “The Unemployment Volatility Puzzle: Is Wage Stickiness the Answer?,” *Econometrica*, 77(5), 1339–1369.
- PRIES, M., AND R. ROGERSON (2009): “Search Frictions and Labor Market Participation,” *European Economic Review*, 53, pp. 568–587.
- RAVN, M. O. (2006): “The Consumption - Tightness Puzzle,” *NBER International Seminar on Macroeconomics*, pp. pp. 9–63.
- RIOS-RULL, J.-V. (1996): “Life-Cycle Economies and Aggregate Fluctuations,” *The Review of Economic Studies*, 63(3), pp. 465–489.
- SHIMER, R. (2005): “The Cyclical Behavior of Equilibrium Unemployment and Vacancies,” *American Economic Review*, 95(1), 25–49.
- (2007): “Reassessing the Ins and Outs of Unemployment,” NBER Working Papers 13421, National Bureau of Economic Research, Inc.
- VERACIERTO, M. (2008): “On the Cyclical Behavior of Employment, Unemployment and Labor Force Participation,” *Journal of Monetary Economics*, 55(6), 1143–1157.

6 Appendix

6.1 Robustness of the Empirical Procedure

The results we report in the paper are robust to a series of methodological changes. One valid concern about our empirical results is if they remain valid once we account for compositional changes. For example, our estimates summarize the behavior of people of the same age who were born in different years. If there were systematic differences in life cycle transition profiles across generations, our unconditional results will confound life cycle profile changes and the aging effect of older generations. Hence, we introduce cohort dummy effects to account for this possibility.

Another concern is the time effect. Since our unconditional estimates do not consider these effects, it may be possible we are confounding trends in the labor market and life cycle profiles. We also compute these profiles after controlling for education compositional changes, family structure changes and state specific effects.

In Table 4 we estimate 8 alternative models using different sets of controls. To remove cohort effects we introduce a set of year-of-birth dummies. In order to control for time effects, we can choose a “saturated” model (one dummy category per period) or a trend and seasonal dummies approach (more parsimonious). Since, both ways render quite similar results, we stick to the latter to save in degrees of freedom. For the rest of the control variables, we just define categorical dummies at the individual level and we compute CPS weighted averages within each bin defined by age, cohort and period. As we control for these variables, we need to define a normalization for the estimated probabilities. Instead of picking a specific cohort, month or educational group, we normalize the effect of controls by forcing the sum of these effects to add up to zero. In other words, we redefine our control variables such that $\mathbb{E}[f_{atc}^{xz}|W_{atc}] = \gamma_a^{xz}$. In the case of the college graduate proportion or other proportions, we normalize the variable so that its sample mean equals zero.

Model 0 is the unconditional life cycle profile in which we based our analysis in the paper. Model 1 introduces cohort and time effects. Model 2 adds state effects. As stated in Table 4, the subsequent models 3-8 introduce education and family variables combined with cohort, time and state controls.

Our findings are robust to the addition of all these controls. We compute the average absolute difference over the life cycle between the unconditional profiles of unemployment, participation and transition probabilities (model 0) and the alternative conditional profiles (models 1-8). The results are shown in Table 5. We can also perform a relative comparison across models in Table 6. Both tables show that most of the results from the previous discussion are maintained. Nevertheless, some differences occur in flows from and to inactivity.

One would like to know whether these differences are due to systematic under or overestimation of transition probabilities. Visual evidence can be found in Figures 13-20. Although these figures show some recurrent patterns of over or under estimation at some ages, the overall shape of the unconditional life cycle profiles is clearly preserved. Figure 13 shows that ignoring compositional changes in the population structure leads us to underestimate the unemployment rate for the young and overestimate it for the old. The effects of these compositional effects in participation rates is less important in relative terms. However, the overall shapes look stable across models.

Looking at Figure 15, we see that not accounting for compositional changes tends to overestimate this transition for the young, and to underestimate it for the old. In the case of Figure 16, we see that this probability would be lower for young adults (20-30 years old) if we control for compositional changes. In Figure 17 we observe that the estimated *UE* probabilities tend to be higher for young

Table 4: Alternative specifications of models

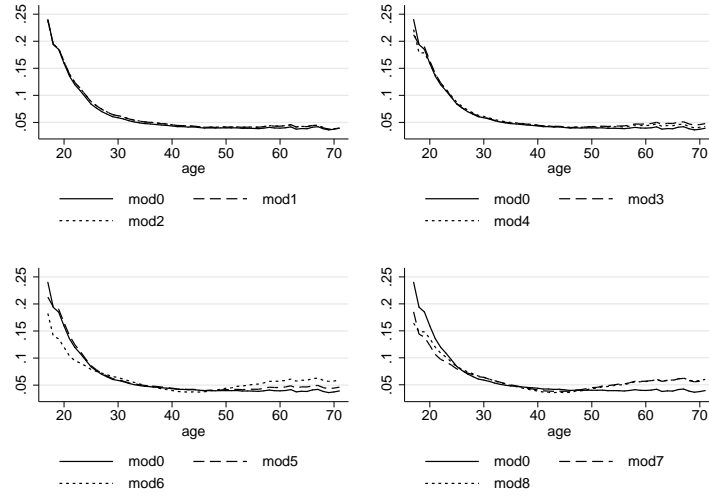
Models(1)	m0	m1	m2	m3	m4	m5	m6	m7	m8
Cohort		Y	Y	Y	Y	Y	Y	Y	Y
Time dummies		Y	Y						
Polynomial trends(2)				Y	Y	Y	Y	Y	Y
Seasonal				Y	Y	Y	Y	Y	Y
State(3)			Y		Y	Y		Y	Y
College(4)				Y	Y				Y
Educational Categories(5)						Y	Y	Y	Y
Married(6)							Y	Y	Y
Family Size(7)							Y	Y	Y
Children Number(8)							Y	Y	Y

NOTES: (1) Estimations are based in CPS weighted averages of variables grouped by age, period and cohort (2) We include five order trend polynomials to capture deterministic time trends in variables; (3) Proportion of people living in a particular state within a particular age/period/cohort; (4) Share of working-age population having college education or more per age/period/cohort; (5) Share of working-age population in each educational categories per age/period/cohort. We use [Lemieux \(2006\)](#) educational categories due to discontinuity in the CPS educational recording. These are 0-4,5-8,9,10,11,12,13-15,16,17+ years of schooling; (6) Proportion of married people per period/cohort; (7) For each age/period/cohort, we use the proportion of people living in households with 1, 2,...,10+ people; (8) For each age/period/cohort, we include the proportion of people living in households with 0, 1, 2,...,5+ children.

workers and lower for older ones, as we introduce the aforementioned control variables. The *UO* transition probability follows the exactly opposite pattern as more controls are introduced in [18](#). The conditional *OE* transition rates in [Figure 19](#) differ from the unconditional one in that the former are higher for workers younger than 30, and lower for those older than that. A somewhat more complicated pattern emerges from models depicted in [Figure 20](#). As we control for cohort and time effects, we roughly find the same pattern as in the *OE* case. Once we introduce family and education controls, we see that the conditional estimates are higher for young and old workers, but lower for middle age ones.

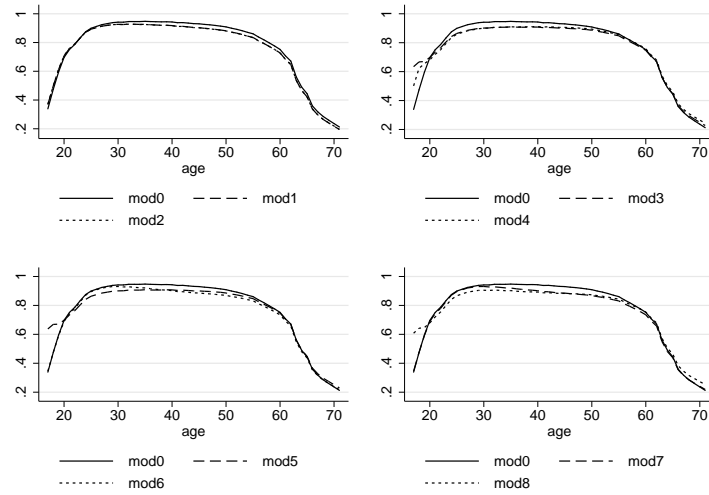
Do these differences imply a different assessment on the relative importance of these transitions as determinants of unemployment of participation rates? Not really. [Tables 7-10](#) tabulate the explanatory power of method AB1C for all the nine models (0-8). [Table 7](#) uses transition probabilities fixed at life time averages for each method. [Tables 8, 9](#) and [10](#) set transition probabilities at ages 20, 40, and 60. Even though we see some variation of relative importance for each transition in explaining unemployment and participation, we notice that the relative ranking of importance does not vary much. Albeit the explanatory power of the *UE* (job finding) probability on unemployment increases as we introduce more controls in [Table 7](#), left panel, its relative importance in our metric is always surpassed by the *EU*, *UO* and *OU* probabilities. The AB1C method consistently show that transitions from and to inactivity have a great deal of explanatory power over both unemployment and participation over the life cycle.

Figure 13: Comparing empirical models: Unemployment rate



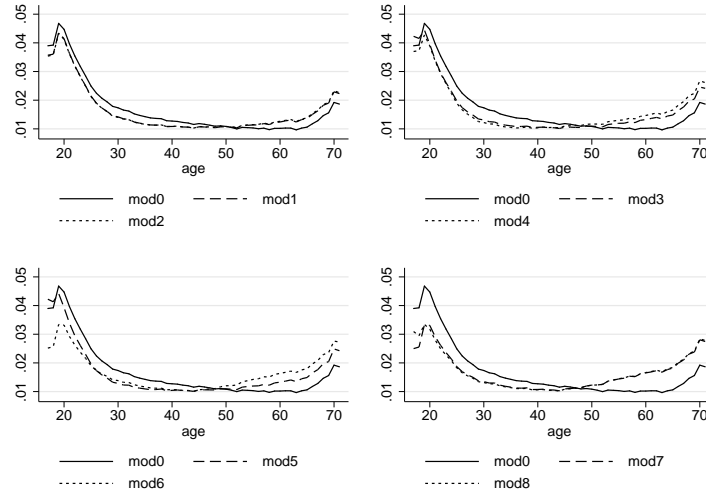
NOTE: Models' specifications are described in Table (4)

Figure 14: Comparing empirical models: Participation rate



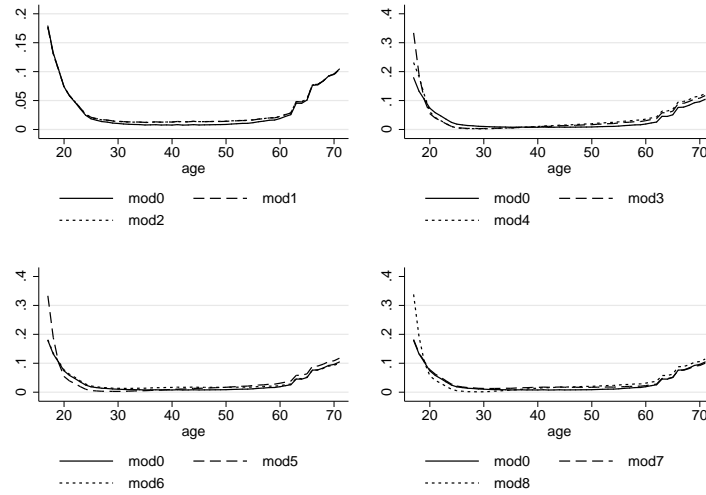
NOTE: Models' specifications are described in Table (4)

Figure 15: Comparing empirical models: *EU* flow



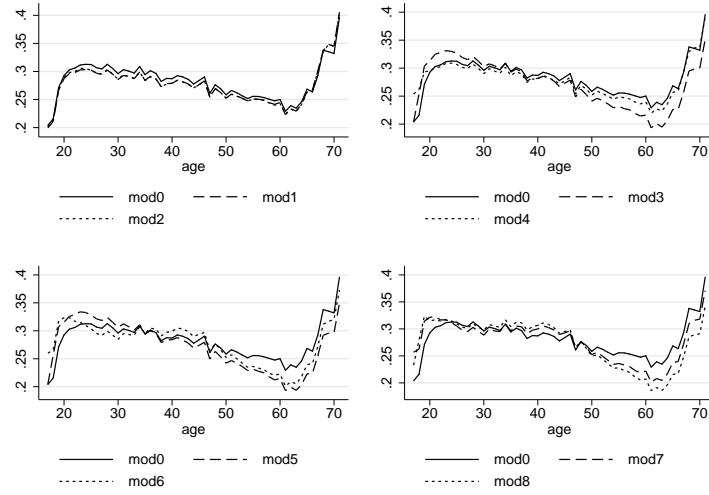
NOTE: Models' specifications are described in Table (4)

Figure 16: Comparing empirical models: *EO* flow



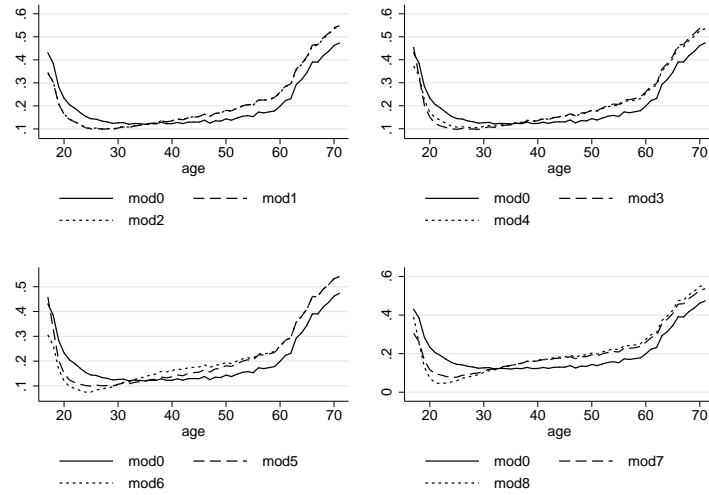
NOTE: Models' specifications are described in Table (4)

Figure 17: Comparing empirical models: *UE* flow



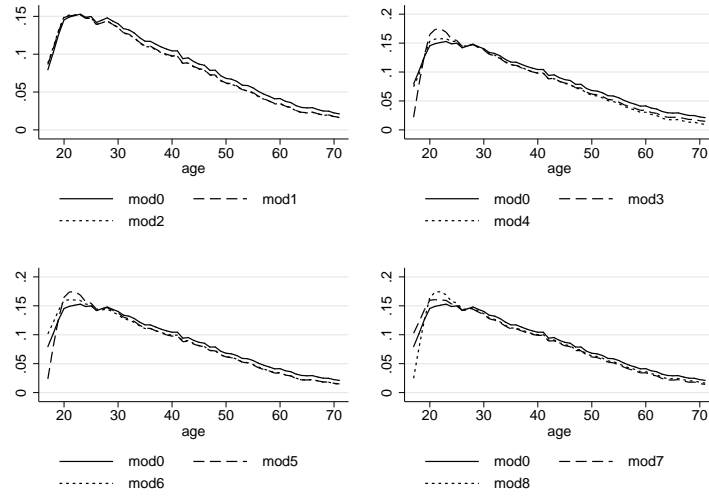
NOTE: Models' specifications are described in Table (4)

Figure 18: Comparing empirical models: *UO* flow



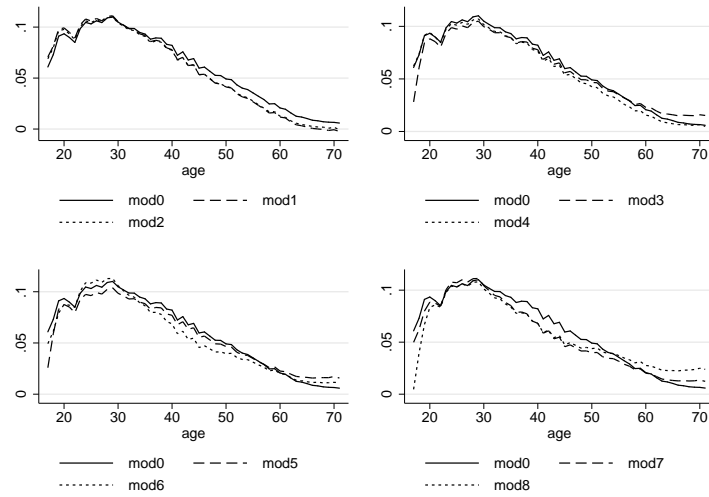
NOTE: Models' specifications are described in Table (4)

Figure 19: Comparing empirical models: *OE* flow



NOTE: Models' specifications are described in Table (4)

Figure 20: Comparing empirical models: *OU* flow



NOTE: Models' specifications are described in Table (4)

Table 5: Mean absolute difference of estimated life cycle profiles across models

	P	U	<i>EU</i>	<i>EO</i>	<i>UE</i>	<i>UO</i>	<i>OE</i>	<i>OU</i>
m1	2.06	0.27	0.23	0.35	0.74	4.34	0.54	0.58
m2	2.10	0.28	0.24	0.35	0.72	4.18	0.59	0.52
m3	3.19	0.41	0.31	1.23	1.94	4.35	0.82	0.49
m4	2.99	0.34	0.39	1.30	0.84	3.77	0.71	0.39
m5	3.26	0.38	0.32	1.26	2.04	4.21	0.82	0.56
m6	2.08	1.29	0.50	0.50	1.64	5.60	0.69	0.63
m7	2.11	1.22	0.51	0.51	1.60	5.51	0.72	0.64
m8	3.91	1.16	0.52	1.37	2.28	6.64	0.70	0.96

NOTE: The numbers correspond to $(100/A) \sum_{a=1}^A |f_a^0 - f_a^m|$, the average absolute difference between the unconditional life cycle profile of the variable f^0 and the conditional on f^m with $m = 1, 2, \dots, 8$. Models are described by Table (4).

Table 6: Mean percentage difference of estimated life cycle profiles across models

	P	U	<i>EU</i>	<i>EO</i>	<i>UE</i>	<i>UO</i>	<i>OE</i>	<i>OU</i>
m1	3.16	5.30	14.95	33.32	2.60	21.31	9.24	26.71
m2	3.19	5.57	15.65	33.17	2.52	20.63	10.01	21.98
m3	5.26	8.49	20.02	50.77	7.10	21.88	12.81	20.93
m4	4.99	6.20	25.89	64.73	3.27	18.95	14.82	10.63
m5	5.41	7.26	20.44	52.97	7.44	21.27	12.75	23.09
m6	2.48	22.24	30.99	46.99	6.18	29.56	11.33	16.31
m7	2.52	21.11	31.81	47.05	6.03	29.09	11.76	18.85
m8	6.61	20.85	32.58	67.59	8.46	35.23	9.97	43.59

NOTE: The numbers correspond to $(100/A) \sum_{a=1}^A |f_a^m / f_a^0 - 1|$, the average percentage difference between the unconditional life cycle profile of the variable f^0 and the conditional on f^m with $m = 1, 2, \dots, 8$. Models are described by Table (4).

Table 7: Percentage explained by specification, probabilities fixed at life cycle averages)

	Panel A: Unemployment					
	Method AB1C at avg					
	EU	EO	UE	UO	OE	OU
m0	12.1	23.9	2.4	6.5	3.0	46.2
m2	11.9	22.9	2.8	4.0	3.9	52.0
m3	10.1	21.8	1.8	4.8	3.4	54.3
m5	45.0	18.5	21.6	44.8	3.7	63.9
m6	28.9	21.0	15.8	33.8	6.3	81.8
m7	47.7	21.1	23.4	47.6	4.8	66.2
m8	6.4	32.1	12.5	25.8	13.6	62.1
m9	8.7	29.7	14.5	31.5	11.5	64.2
m13	57.1	43.6	36.6	67.2	20.4	64.0
	Panel B: Participation					
	Method AB1C at avg					
	EU	EO	UE	UO	OE	OU
m0	0.7	12.6	1.3	0.8	18.6	6.9
m2	1.3	16.7	0.7	0.6	13.6	3.6
m3	1.4	16.9	0.8	0.7	14.1	3.5
m5	15.2	23.0	13.8	14.2	25.9	16.9
m6	11.9	22.5	10.5	10.6	18.0	12.0
m7	15.7	23.6	14.4	14.9	26.3	17.2
m8	1.9	22.3	1.2	1.2	13.6	1.5
m9	1.9	22.7	1.3	1.3	13.9	1.5
m13	18.3	27.9	17.2	18.6	28.2	16.9

Table 8: Percentage explained by specification, probabilities fixed at age 20)

	Panel A: Unemployment Method AB1C at 20					
	EU	EO	UE	UO	OE	OU
m0	17.4	36.3	2.4	6.4	3.8	49.7
m2	24.5	36.8	2.4	5.6	2.6	55.6
m3	20.0	34.6	1.5	6.5	2.4	57.6
m5	48.3	18.7	21.9	47.2	4.6	66.6
m6	27.8	21.7	16.0	36.3	3.4	82.5
m7	50.9	21.2	23.8	50.1	5.9	68.9
m8	3.7	48.3	11.1	31.1	11.0	64.4
m9	4.1	43.1	13.4	36.4	9.2	66.3
m13	56.5	44.8	36.9	68.9	17.6	67.0
	Panel B: Participation Method AB1C at 20					
	EU	EO	UE	UO	OE	OU
m0	1.0	18.5	1.3	0.8	26.9	9.2
m2	3.3	23.2	0.7	0.4	24.5	5.4
m3	3.4	23.4	0.8	0.5	25.1	5.2
m5	17.0	23.4	13.7	13.4	35.5	17.4
m6	14.2	23.4	10.4	9.7	22.9	11.5
m7	17.6	23.9	14.3	14.1	35.8	17.6
m8	3.5	28.8	1.1	0.7	25.0	2.2
m9	3.6	29.4	1.2	0.8	25.3	2.3
m13	19.8	28.4	17.1	19.1	35.8	16.8

Table 9: Percentage explained by specification, probabilities fixed at age 40)

	Panel A: Unemployment Method AB1C at 40					
	EU	EO	UE	UO	OE	OU
m0	12.3	17.0	2.4	8.5	2.9	48.1
m2	10.0	14.1	2.6	5.5	3.2	53.7
m3	9.1	14.5	1.7	6.5	2.8	55.9
m5	44.3	16.8	21.7	46.6	3.2	65.2
m6	30.1	17.6	15.9	36.4	5.0	82.2
m7	46.9	19.0	23.5	49.5	4.4	67.5
m8	9.0	24.2	11.6	27.7	12.7	62.5
m9	11.6	23.9	13.8	33.1	10.6	64.5
m13	57.7	30.6	36.9	67.6	19.2	64.3
	Panel B: Participation Method AB1C at 40					
	EU	EO	UE	UO	OE	OU
m0	1.0	18.5	1.3	0.8	26.9	9.2
m2	3.3	23.2	0.7	0.4	24.5	5.4
m3	3.4	23.4	0.8	0.5	25.1	5.2
m5	17.0	23.4	13.7	13.4	35.5	17.4
m6	14.2	23.4	10.4	9.7	22.9	11.5
m7	17.6	23.9	14.3	14.1	35.8	17.6
m8	3.5	28.8	1.1	0.7	25.0	2.2
m9	3.6	29.4	1.2	0.8	25.3	2.3
m13	19.8	28.4	17.1	19.1	35.8	16.8

Table 10: Percentage explained by specification, probabilities fixed at age 60)

	Panel A: Unemployment Method AB1C at 60					
	EU	EO	UE	UO	OE	OU
m0	12.6	20.5	3.2	6.0	6.2	30.0
m2	10.6	18.2	5.0	3.1	9.4	28.1
m3	9.4	17.9	3.5	3.9	8.8	30.4
m5	44.6	18.2	21.1	43.4	12.7	50.2
m6	29.2	21.2	15.5	31.9	16.8	64.2
m7	47.3	20.7	22.8	46.0	14.1	52.0
m8	6.4	27.1	17.6	23.7	19.9	45.0
m9	8.6	26.0	18.7	29.5	18.2	47.7
m13	57.0	40.6	36.0	66.3	31.0	54.3
	Panel B: Participation Method AB1C at 60					
	EU	EO	UE	UO	OE	OU
m0	0.9	11.2	1.3	0.8	10.9	4.0
m2	1.0	13.5	0.8	0.7	6.2	2.5
m3	1.1	13.8	0.9	0.9	6.7	2.5
m5	14.9	22.0	14.0	14.6	21.1	16.7
m6	11.7	22.8	10.9	11.2	19.4	14.2
m7	15.4	22.6	14.6	15.3	21.8	17.1
m8	1.9	18.8	1.6	1.5	7.5	1.7
m9	2.0	19.2	1.7	1.6	7.7	1.8
m13	18.4	26.6	17.6	18.6	25.7	17.5

6.2 Workers' Bellman equations

We denote respectively by $W_a(h)$, $U_a(h)$ and $H_a(h)$ the present discounted values of being employed, unemployed or inactive for a worker aged a with home productivity h . For workers aged $a < T$, those value functions read as

$$\begin{aligned} W_a(h) = & w_a(h) \\ & + \beta \lambda (1-s) E_{h'} [\max\{W_{a+1}(h'), U_{a+1}(h'), H_{a+1}(h')\}] \\ & + \beta (1-\lambda)(1-s) \max\{W_{a+1}(h), U_{a+1}(h), H_{a+1}(h)\} \\ & + \beta \lambda s E_{h'} [\max\{U_{a+1}(h'), H_{a+1}(h')\}] \\ & + \beta (1-\lambda)s \max\{U_{a+1}(h), H_{a+1}(h)\} \end{aligned}$$

$$\begin{aligned} U_a(h) = & b \\ & + \beta p (1-\lambda) \max\{W_{a+1}(h), U_{a+1}(h), H_{a+1}(h)\} \\ & + \beta p \lambda E_{h'} [\max\{W_{a+1}(h'), U_{a+1}(h'), H_{a+1}(h')\}] \\ & + \beta (1-p)(1-\lambda) \max\{U_{a+1}(h), H_{a+1}(h)\} \\ & + \beta (1-p)\lambda E_{h'} [\max\{U_{a+1}(h'), H_{a+1}(h')\}] \end{aligned}$$

and

$$\begin{aligned} H_a(h) = & h\mathcal{H} \\ & + \beta \lambda E_{h'} (\max\{U_{a+1}(h'), H_{a+1}(h')\}) \\ & + \beta (1-\lambda) \max\{U_{a+1}(h), H_{a+1}(h)\}, \end{aligned}$$

and, in the case of a worker aged T years old,

$$W_T(h) = w_T(h), \tag{5}$$

$$U_T(h) = b, \tag{6}$$

and

$$H_T(h) = h\mathcal{H}. \tag{7}$$

6.3 Firms' Bellman equations

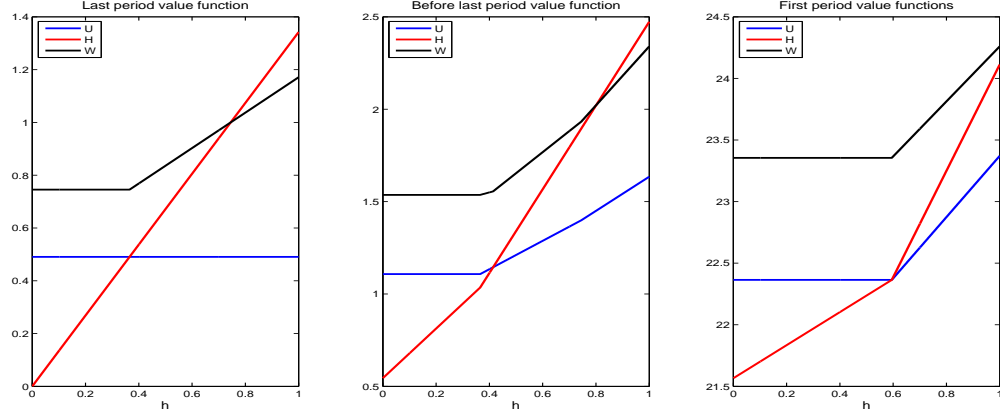
Denote by $J_a(h)$ the value of producing for a firm whose employee is aged a and whose home productivity is equal to h and by V the value of having a vacancy posted. They read as

$$V = -c + q E_{a',h'} [J_{a'}(h')] + (1-q)V$$

and

$$\begin{aligned} J_a(h) = & \mathcal{Z} - w_a(h) \\ & + \beta (1-s) \lambda E_{h'} [\max\{J_{a+1}(h'), 0\}] \\ & + \beta (1-s)(1-\lambda) \max\{J_{a+1}(h), 0\}, \end{aligned}$$

Figure 21: Value functions



in the case of an employee aged $a < T$ years old, while, if the employee is T years old, it is

$$J_T(h) = Z - w_T(h).$$

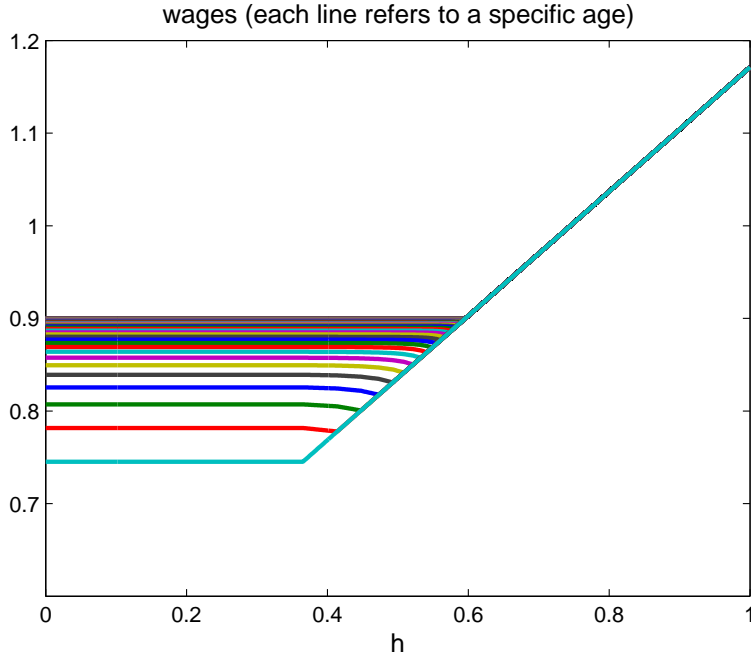
6.4 Approximated value functions

We approximate the value functions $W_a(h)$, $U_a(h)$ and $H_a(h)$ with the value function iteration algorithm. We first approximate them for $a = T$, as given in equations (5), (6) and (7). Then, given those approximations, we recursively approximate the rest of the functions for $a < T$ by iterating backwardly.

Figure 21 shows the resulting approximations for a equal to T , $T - 1$ and 1 respectively. In the case of $a = T$, we can observe the following. The value of being unemployed is independent of h , while the value of being inactive is increasing in it. This is because benefits are set independently of home productivity, while, obviously, the opposite occurs with the amount of goods produced at home. In the case of the value of being employed, the function is constant below a given threshold and increasing beyond it. This is because the relevant outside option, when wages are bargained over, differs in the two situations. Below the threshold, home productivity is so low that the agent would prefer to remain unemployed if she was forced out of employment. Hence the outside option is independent of h in this case. However, beyond this threshold, the agent would choose inactivity if she lost her job. This explains the piecewise structure of the function.

In the case of $a = T - 1$, the functions have similar shapes, but with some differences. A first difference has to do with the level: for each value of h , the values are higher when $a = T - 1$ than when $a = T$. This is because, when $a = T - 1$, the value functions correspond to the discounted sums over two periods, while only one period is considered when $a = T$. A second difference is that the value of being unemployed is now increasing in h beyond a certain threshold. This has to do with the fact that a worker unemployed at age $T - 1$ may become employed or inactive at age T . Given that the value of being employed or inactive at age T are increasing in h beyond a certain threshold, this implies that the present discounted value of being unemployed at age $T - 1$ may be increasing in h . A last difference concerns the increasing part of the value of being employed: while the slope of the increasing part is homogenous when $a = T$, this is not the case anymore when $a = T - 1$. The explanation is the following. While two sorts of outside options are

Figure 22:



considered when $a = T$ (unemployed and inactivity), when $a = T - 1$, one has to consider three cases. Specifically, unemployment as an outside option is two types: some workers unemployed at $T - 1$ may remain unemployed at T , while others will choose inactivity if they do not find a job. The marginal impact of h being larger in the second case, this explains the change in the slope of the value of being employed.

These effects of course accumulates over iterations. However, as the values for $a = 1$ shows, they tend to die out as the number of iterations gets large. This is because, for large T , the case $a = 1$ is very close to the case where agents are infinitely lived. Hence, value functions become homogenous for low values of a .

7 Wages: quantitative analysis

We now show that our observations on the evolution of participation and unemployment over the life cycle extend to the evolution of wages. On the one hand our model predicts an inverted U-shaped relation between wages and age, as the data predicts too. But on the other hand it lacks persistence in earning growth. Moreover, in addition to the lack of persistence, our calibration implies a decrease in wages for old workers stronger than the one observed in the data and generates lower wage growth for young workers than the data suggests.

Figure 22 shows the relation between wages and the two state variables, namely age and the home productivity parameter h . The latter variable is represented on the x axis and each line on the graph refers to a specific age. In particular, the light blue line at the bottom refers to the relation between the wage and the home productivity parameter h for a worker aged T , and as we move upwards, one tends to consider younger workers, e.g., the red line refers to workers aged $T - 1$, the green line considers workers aged $T - 2$, etc... Hence, for a given value of h , the model predicts a decreasing relationship between age and earnings. This is the consequence of a mechanism we

Figure 23:



already mentioned in Section 4.2: as a worker gets older, the outside option for the firm of searching for an alternative employee increases, decreasing the firm surplus and wages too.

For a given age, the relation between the paid wage and home productivity is not monotone. To understand why, three types of workers have to be considered. First, for unattached workers, i.e. workers who would choose to become inactive if they were not employed, the relation is strictly increasing. This is because home productivity increases the value of non employment, which pushes wages upwards because of Nash bargaining.²⁰ Second, attached workers (those who would choose unemployment over inactivity if they were not employed) are separated in two groups. On the one hand, there are those who chooses unemployment over inactivity today but who will choose inactivity over unemployment the next period if they will still be assigned the same h value. On the other hand, there are those who will always choose unemployment over inactivity for the h value that characterizes them. In the case of the former group, there is no relation between the paid wage and home productivity (the relevant flow value of non employment is b , which does not depend on h), while in the case of the latter, the relation appears decreasing on Figure 22. The reason for this negative impact reminds the effect of a firing tax on wages in matching models, described in Mortensen and Pissarides (1999) and Ljungqvist (2002): once the worker becomes unattached, h affects positively the negotiated wage as it increases the flow value of being non-employed, but this increase is anticipated ex ante, implying a compensation to the firm when the worker is still attached. In Appendix 8 below, the marginal impact of h on wages is derived analytically for the periods T and $T - 1$.

²⁰Figure 22 shows that wages can be above marginal productivity \mathcal{Z} (fixed to one in our calibration) for large values of h . This is because of labor hoarding (Bertola and Caballero (1994)): firms pay a wage above marginal productivity on a temporary basis in order to save on turnover costs since they anticipate mean reversion for h .

Given the analysis of Figure 22, it is possible to understand the relationship between average wages and age, as it is described on Figure 23. The graph shows an inverted U-shaped pattern, which is the consequence of the evolution of average home productivity for employed workers and, in the case of old workers, strengthened by the finite horizon restriction. Young workers experience an increase in wages on average because their home productivity increases on average too. The increase in average home productivity can be understood from Figure 10. Unemployed workers have lower productivity because the productivity threshold that describes the indifference between unemployment and inactivity is low. This explains why young employed also have low home productivity. But, once those workers are employed and hit by a λ shock, they do not choose to leave activity (though they would have done so if unemployed). This implies the increase in average productivity at the beginning of the life cycle as drawn on Figure 23.

The decrease in average wages at the end of the life cycle is explained by two elements: the finite horizon restriction and the evolution of average home productivity. As the discussion of Figure 22 emphasized, the finite horizon restriction matters because the outside option for the firm of searching for an alternative worker increases with age, decreasing the firm surplus and wages too. Moreover, home productivity decreases on average because, as shown on Figure 10, the productivity threshold that describes the indifference between being employed and out of the labor force drops at the end of the life cycle. As a consequence, workers with high home productivity values suddenly decide to retire at the end of their lives, which decreases average home productivity among the employed population. Those two elements explain the drop in average wages at the end of the life cycle.

Thus, adding a life cycle structure to an otherwise standard search model produces a hump shaped relationship between wages and age. However, comparing the wages our model generates with evidence from the literature (e.g., [Murphy and Welch \(1990\)](#)) on the empirical age-earnings profile reveals two characteristics which are not in line with the data. First, the increase in average wages at the beginning of the life cycle and the decrease at the end are more persistent in the data. The relationship between wages and years of experience described in [Murphy and Welch \(1990\)](#) shows a steady increase over twenty to thirty years of experience, while the maximum value is reached after two years in the model. The same occurs at the end of the life cycle: average wages start to decrease once workers turn 62 years old, while this occurs before in the data. Second, our model predicts lower wage growth for young workers than the data suggests and a larger fall for old workers. [Murphy and Welch \(1990\)](#) shows that the yearly wage growth rate is about 10 percent at the beginning of a worker's career and converges toward zero until twenty years of experience have been filled. But this growth rate is five percent in the model and positive wage growth lasts only two years. Similarly, wages decrease at a two percent yearly rate for individuals with forty years of experience in [Murphy and Welch \(1990\)](#), while this rate can reach 17% in the model.

8 Why are wages not monotone in h ?

We show the comparative statics for workers aged T and $T - 1$.

The general formulation for wages reads

$$\begin{aligned} w_a(h) = & (1 - \alpha) \max\{U_a(h), H_a(h)\} + \alpha \mathcal{Z} \\ & - \beta(1 - \lambda)[s + (1 - s)(1 - \alpha)] \max\{U_{a+1}(h), H_{a+1}(h)\} \\ & - \beta\lambda[s + (1 - s)(1 - \alpha)] \int_0^1 \max\{U_{a+1}(h'), H_{a+1}(h')\} dG(h') \end{aligned}$$

The wage paid to a worker aged T is straightforward to calculate and follows:

$$w_T(h) = \alpha \max\{b, h\mathcal{H}\} + (1 - \alpha)\mathcal{Z}.$$

The above equation explains why the wage as a function of h , as drawn on Figure 22 for workers aged T , is first flat and then increasing.

The effect of h on the wage paid to workers aged $T - 1$ requires some additional calculus. We first need to calculate the value of unemployment for a worker aged $T - 1$, as follows:

$$U_{T-1}(h) = \begin{cases} [1 + \beta\{1 - (1 - \alpha)p(1 - \lambda[1 - G(h_T^u)]) - \lambda[1 - G(h_T^u)]\}]b \\ + \beta p(1 - \alpha)(1 - \lambda[1 - G(h_T^w)])\mathcal{Z} + \beta\lambda \int_{h_T^w}^1 h' \mathcal{H} dG(h') \\ + \beta\lambda[1 - p(1 - \alpha)] \int_{h_T^u}^{h_T^w} h' \mathcal{H} dG(h') & \text{if } h \leq h_T^u \\ \\ [1 + \beta\lambda G(h_T^u)(1 - p + p\alpha)]b + \beta p(1 - \alpha)[1 - \lambda + \lambda G(h_T^w)]\mathcal{Z} \\ + \beta(1 - \lambda)(p\alpha + 1 - p)h\mathcal{H} + \beta\lambda \int_{h_T^w}^1 h' \mathcal{H} dG(h') \\ + \beta\lambda(p\alpha + 1 - p) \int_{h_T^u}^{h_T^w} h' \mathcal{H} dG(h') & \text{if } h_T^u < h \leq h_T^w \\ \\ b + \beta(1 - \lambda)h\mathcal{H} + \beta\lambda \int_{h_T^w}^1 h' \mathcal{H} dG(h') \\ + \beta\lambda(p\alpha + 1 - p) \int_{h_T^u}^{h_T^w} h' \mathcal{H} dG(h') & \text{if } h_T^w < h \end{cases}$$

and the value of inactivity:

$$H_{T-1}(h) = \begin{cases} h\mathcal{H} + \beta[1 - \lambda + \lambda G(h_T^u)]b + \beta\lambda \int_{h_T^u}^1 h' \mathcal{H} dG(h') & \text{if } h \leq h_T^u \\ [1 + \beta(1 - \lambda)]h\mathcal{H} + \beta\lambda G(h_T^u)b + \beta\lambda \int_{h_T^u}^1 h' \mathcal{H} dG(h') & \text{if } h_T^u < h \end{cases}$$

Hence, the marginal effect of h on the wage paid to a worker aged $T - 1$ is

$$w'_{T-1}(h) = \begin{cases} 0 & \text{if } h \leq h_{T-1}^u \text{ and } h \leq h_T^u \\ -\beta(1 - \lambda)[\alpha s + (1 - \alpha)^2 p]\mathcal{H} & \text{if } h \leq h_{T-1}^u \text{ and } h \in (h_T^u, h_T^w) \\ -\beta(1 - \lambda)\alpha s\mathcal{H} & \text{if } h \leq h_{T-1}^u \text{ and } h \geq h_T^w \\ (1 - \alpha)\mathcal{H} & \text{if } h > h_{T-1}^u \text{ and } h \leq h_T^u \\ [1 - \alpha + \beta(1 - \lambda)(1 - s)\alpha]\mathcal{H} & \text{if } h > h_{T-1}^u \text{ and } h > h_T^w \end{cases}$$

We see that the effect is positive for unattached workers and can be negative or null for attached workers. It is negative when workers will become unattached in the future if their h value will not change.

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