

## INGENIERÍA INDUSTRIAL · UNIVERSIDAD DE CHILE

# **DOCUMENTOS DE TRABAJO** Serie Economía



 № 340
 CENTRALIZED COURSE ALLOCATION

 ANTONIO ROMERO-MEDINA Y MATTEO TRIOSSI



# Centralized Course Allocation

Antonio Romero-Medina<sup>\*</sup> Matteo Triossi<sup>†</sup>

August 2, 2018

#### Abstract

We present the renegotiable acceptance mechanism in the context of the multi-unit assignment problem. This mechanism combines features of the immediate and deferred acceptance mechanisms and implements the set of stable matchings in both Nash and undominated Nash equilibria under substitutable priorities. In addition, we prove that under slot-specific priorities, the immediate acceptance mechanism also implements the set of stable matchings in Nash and undominated Nash equilibria. Finally, we present modifications of both mechanisms and show that we can dramatically reduce the complexity of the message space when preferences are responsive.

Economic Literature Classification Numbers: C71; C78; D71.

<sup>&</sup>lt;sup>\*</sup>Departamento de Economía, Universidad Carlos III de Madrid, Calle Madrid 126, 28903 Getafe, Madrid, Spain. E-mail: aromero@eco.uc3m.es.

<sup>&</sup>lt;sup>†</sup>Corresponding Author. Centro de Economía Aplicada, Departamento de Ingeniería Industrial, Universidad de Chile, Calle Beaucheff 851, 8370456, Santiago, Chile. E-mail: mtriossi@dii.uchile.

Keywords: renegotiable acceptance, immediate acceptance, multi-unit assignment, stability.

# 1 Introduction

We are interested in multi-unit assignment problems for which multiple objects are assigned to agents on the basis of priorities. This is the case, for example, of the course allocation problem (see Sönmez and Ünver, 2010; Budish, 2011; Kojima, 2013). The time scheduling problem or the assignment of landing slots (see Schummer and Vohra, 2013; Schummer and Abizada, 2017) are also examples of multi-unit assignment problems.

We focus our attention on fair (or stable) allocations. Under multi-unit demand, no stable and strategy-proof mechanism exists. Additionally, the deferred acceptance mechanism can produce unstable matchings as Nash equilibrium outcomes (see Roth and Sotomayor, 1990; Haeringer and Klijn, 2009). To overcome this difficulty, we concentrate on mechanisms that are able to achieve stable matchings as a result of a strategic interaction. To achieve this objective, we introduce the renegotiable acceptance mechanism. Similar to the immediate acceptance mechanism, the renegotiable acceptance mechanism assigns seats at courses to students who rank them first and then to those who rank them second, and so on.<sup>1</sup> However, similar to the deferred acceptance mechanism, seats are not definitively assigned to students. The allocation can be renegotiated, and a student can lose a (tentatively) assigned course. Our mechanism allows for students to express the intensity of their

<sup>&</sup>lt;sup>1</sup>We use the terminology of immediate acceptance introduced in Thomson (2018), but the many-to-one version of this mechanism is also known as the "Boston mechanism" (see Abdulkadiroğlu and Sönmez, 2003) and was first analyzed by Alcalde (1996), who calls it the "now-or-never mechanism".

preferences. By ranking a course higher, a student increases her chances of being admitted. Therefore, students have incentives to act strategically. The effects of these manipulations cancel out at equilibrium and the renegotiable acceptance mechanism is able to implement the set stable matchings in both Nash equilibrium and undominated Nash equilibrium under substitutable priorities.

As best as we know, this is the first paper to consider substitutable priorities in a course allocation problem (see Marutani, 2018, for the use of substitutable priorities in the school choice problem). Substitutable priorities account for situations in which admission criteria are slot-specific (see Kominers and Sönmez, 2016). These are situations in which a subgroup of students is given priority for a portion of the seats that are otherwise assigned according to a given criterion. Slot-specific priorities allow for the designer, for example, to introduce diversity in the classroom (see Dur et al. 2016, 2018 for applications to school choice).<sup>2</sup> Restricting our attention to slot-specific priorities, we show that the immediate acceptance mechanism is able to implement the set stable of matchings in Nash equilibrium and undominated Nash equilibrium. In situations in which more general substitutable priorities are required, for example, when students are expected to work in teams of a given size, the immediate acceptance mechanism can result in unstable matchings. Thus, our results extend to the multi-unit assignment problem

<sup>&</sup>lt;sup>2</sup>Slot-specific priorities also encompass approaches such as majority quotas as defined in Kojima (2012) and minority reserves introduced by Hafalir et al. (2013).

the previous implementation results by Alcalde (1996) and Ergin and Sönmez (2006) for the marriage and school admission problems, respectively.

Finally, note that in the multi-unit assignment problem, describing the entire preference profile requires listing up to  $2^n$  subsets of courses, where n is the number of courses (see Budish et al., 2017). We show that if students' preferences are responsive, then it is possible to simplify the strategy space.<sup>3</sup> We introduce simplified versions of the renegotiable and immediate acceptance mechanisms for which students' strategies are the rankings over individual courses and the maximum number of courses they are willing to take. Then, we run either the renegotiable acceptance or the immediate acceptance mechanism with any responsive extension of the submitted profiles. We prove that the simplified mechanisms preserve the incentive properties of the full mechanisms.

#### 1.1 Related Literature

Our strategy is to relax the equilibrium requirements from dominant strategy to Nash equilibrium to implement the set of stable allocations. Two other approaches have been used to tackle the course allocation problem.

The first approach is to insist on implementing stable allocations in dominant strategies and restrict the set of admissible priorities. Kojima (2013) shows that dominant strategy implementation of stable allocations is possible

<sup>&</sup>lt;sup>3</sup>The assumption of responsive preferences is common in the literature on course allocation and is used, among others, in Kojima (2013), Kojima and Ünver (2014), and Dogal and Klaus (2018).

if and only if priorities satisfy essential homogeneity. Under essential homogeneity, the mechanism required to implement stable allocations is equivalent to a serial dictatorship in which the order of choice depends on the distribution of seats among courses. If we require the mechanism to be independent of the seat distribution, a stable mechanism that makes true preferences a dominant strategy exists if and only if priorities are acyclical (see Romero-Medina and Triossi, 2018). Essential homogeneity and acyclicity impose severe restrictions on the design of priorities. It is worth noting that both Kojima (2013) and Romero-Medina and Triossi (2018) consider only responsive priorities.

In a model without priorities, Budish (2011) focuses on the efficiency of the final allocation and introduces the approximate competitive equilibrium from equal incomes. This mechanism is efficient and approximately strategyproof in large markets. However, unstable allocations can survive even in large markets, maintaining the tension between efficiency and fairness (see Budish and Cantillon, 2012).<sup>4</sup> Also, the implementation of the approximate competitive equilibrium from equal incomes is complex and computationally intensive (see Budish et al., 2017).

The paper is organized as follows. Section 2 introduces the model and

<sup>&</sup>lt;sup>4</sup>Budish (2011) bounds absolute envy in a weak way. More precisely, student a can envy student b if this envy can be removed by kicking student b out of at most one of his assigned courses without altering his bundle. This weak fairness concept is compatible with the existence of multiple blocking pairs. Consider a model in which the preferences of the students are monotonic in the number of assigned courses, and consider any assignment in which all students are assigned the same number of courses. This assignment satisfies Budish's weak no-envy condition but can fail to eliminate justified envy.

notation. Section 3 presents our results. Section 4 concludes.

# 2 The Model

There are a finite set of courses C and a finite set of students S, with  $C \cap S = \emptyset$ . Each course c has priorities over the subsets of students,  $Ch_c$ . Priorities are described by a choice function  $Ch_c: 2^S \to 2^S$ , where  $Ch_{c}(S') \subseteq S'$  for all  $S' \subseteq S^{5}$ . We assume that the choice function is **substitutable**. Formally, if  $S' \subseteq S$ ,  $s, s' \in S \setminus S'$  and  $s \notin Ch_c(S' \cup \{s\})$ , then  $s \notin Ch_c(S' \cup \{s, s'\})$ . In other words,  $Ch_c$  is substitutable if, whenever course c rejects a student from a given subset of students, it rejects her when more students become available. We also assume that  $Ch_c$  satisfies the irrelevance of rejected students condition.<sup>6</sup> Formally, we assume that if  $S' \subseteq S$  and  $s \notin Ch_c(S' \cup \{s\})$ , then  $Ch_c(S' \cup \{s\}) = Ch_c(S')$ . In other words,  $Ch_c$  satisfies the irrelevance of rejected students condition if rejected students do not affect courses' choices. If  $Ch_c$  is substitutable and satisfies the irrelevance of rejected students condition, then they are rationalizable by a linear order on  $2^{S}$ ,  $P_{c}$ , which is  $Ch_{c}(S') = \max_{P_{c}} \{S'' \mid S'' \subseteq S'\}$  for all  $S' \subseteq S$  (see Alva, 2018). A priority structure is given by  $Ch_C = (Ch_c)_{c \in C}$ or, equivalently, by  $P_C = (P_c)_{c \in C}$ , where  $P_c$  rationalizes  $Ch_c$  for all  $c \in C$ . A particular class of substitutable priorities is the class of slot-specific pri-

<sup>&</sup>lt;sup>5</sup>Given a set X, by  $2^X$ , we denote the set of the subsets of X.

<sup>&</sup>lt;sup>6</sup>The condition has been previously studied as "irrelevance of rejected contracts" in Aygün and Sönmez (2013) for models of matching with contracts and as "irrelevance of rejected items" in Alva (2018) for general choice models.

orities introduced by Kominers and Sönmez (2016) in a matching model with contracts. Under slot-specific priorities, each course  $c \in C$  has a finite set of slots,  $\sigma \in \Sigma_c$ . Each slot  $\sigma$  has a priority order  $\succ_{\sigma}$ , which is a strict, complete, and transitive binary relation over  $S \cup \{\emptyset\}$ . The higher a student is ranked under  $\succ_{\sigma}$ , the stronger the claim that she has for slot  $\sigma$  in course c. If  $\emptyset \succ_{\sigma} s$ , student s is not acceptable for slot  $\sigma$ . The total supply of course c is  $q_c = |\Sigma_c|$ . Let us define q as the vector of supply for each course  $q = (q)_{c \in C}$ . We assume that the slots in C are ordered according to a linear order of precedence  $\triangleright_c$ . Given two slots  $\sigma, \sigma' \in \Sigma_c, \sigma \triangleright_c \sigma'$  means that slot  $\sigma$ is to be filled before slot  $\sigma'$  whenever possible. For each course c, we assume that slots in  $\Sigma_c$  are ordered in such a way that  $\sigma^1 \triangleright_c \sigma^2 \triangleright_c \dots \triangleright_c \sigma^{q_c}$ . Let  $S' \subseteq S$ . The choice of school c from S', denoted by  $Ch_{c}(S')$ , is obtained as follows: slots at school c are filled one at a time following the order of precedence. The highest-priority acceptable student in S' under  $\succ_{\sigma^1}$ , for example, student  $s^1$ , is chosen for slot  $\sigma^1$  of school c; the highest-priority acceptable student in  $S' \setminus \{s^1\}$  under  $\succ_{\sigma^1}$ , for example, student  $s^2$ , is chosen for slot  $\sigma^2$ of school c, and so on. The choice function  $Ch_c$  satisfies substitutability (see Kominers and Sönmez, 2016) and the irrelevance of rejected students condition. A slot-specific priority structure is a tuple  $\left(q, \left(\Sigma_c, (\succ_{\sigma})_{\sigma \in \Sigma_c}, \rhd_c\right)_{c \in C}\right)$ . Throughout the paper, we assume that priorities are fixed, substitutable, and satisfy the irrelevance of rejected students conditions.

Each student  $s \in S$  has a strict preference relation  $P_s$  over the set of subsets of  $C, 2^C$ . For each  $C' \subseteq C$  and each  $s \in S$ , we denote by  $Ch_s(C')$  the **choice set** of student *s*, which is the favorite combination of courses among the ones belonging to *C'*. Formally,  $Ch_s(C') = \max_{P_s} \{D \mid D \subseteq C'\}$ . A subset of courses  $C' \subseteq C$  is not acceptable to student *s* when  $\emptyset P_s C'$ . We assume that the choice set induced by each  $P_s$  is **substitutable** as previously defined for the case of courses' priorities. Let  $\mathcal{P}$  be the set of substitutable preferences on  $2^C$ . A more restrictive condition is responsiveness. We say that  $P_s$  is **responsive** (see Roth 1985), with **demand**  $q_s$  if, for each  $C' \subseteq$ C and for all  $c, c' \in C \setminus C'$ , the following holds: (1) if  $|C'| < q_s$ , then  $C' \cup \{c\} P_s C' \cup \{c'\}$  if and only if  $\{c\} P_s\{c'\}, (2)$  if  $|C'| < q_s$ , then  $C' \cup$  $\{c\} P_s C'$  if and only if  $\{c\} P_s\emptyset$ , and (3) if  $|C'| > q_s$ , then  $\emptyset P_s C'$ .

For each  $S' \subseteq S$ , set  $P_{S'} = (P_s)_{s \in S'}$ . For each  $s \in S$ , set  $P_{-s} = P_{S \setminus \{s\}}$ . Given a preference relation P on  $2^C$ , the restriction of P to  $C' \subseteq C$ , denoted by  $P_{|C'}$  is a preference that ranks all subsets in  $2^{C'}$  exactly as P does and ranks all other subsets of courses as not acceptable. Formally  $P_{|C'}$  is such that, for all  $Q, T \subseteq C', QP_{|C'}T$  if and only if QPT and, for all  $Q \not\subseteq C'$ ,  $\emptyset P_{|C'}Q$ .

A matching is a function  $\mu : C \cup S \to 2^C \cup 2^S$  such that, for each  $s \in S$  and each  $c \in C$ ,  $\mu(s) \in 2^C$ ,  $\mu(c) \in 2^S$  and  $c \in \mu(s)$  if and only if  $s \in \mu(c)$ . The set of all matchings is denoted by  $\mathcal{M}$ . Matching  $\mu$  is individually rational for  $x \in C \cup S$  if  $Ch(\mu(x)) = \mu(x)$ . Matching  $\mu$  is blocked by a pair  $(c,s) \in C \times S$  if  $s \notin \mu(c)$ ,  $c \in Ch_s(\mu(s) \cup \{c\})$ , and  $s \in Ch_c(\mu(c) \cup \{s\})$ . Finally, a matching  $\mu$  is stable for  $(S, C, P_S, Ch_C)$  if it is individually rational for all  $x \in C \cup S$  and there exists no pair blocking it.

If  $P_S$  and  $Ch_C$  are substitutable and  $Ch_C$  satisfies the irrelevance of rejected students conditions, then a stable matching exists (see Echenique y Oviedo, 2006).

A mechanism is a function that associates a matching to every preference profile for students,  $P = (P_s)_{s \in S}$ ,  $\varphi : \mathcal{P}^{|S|} \to \mathcal{M}$ . A mechanism is stable if  $\varphi(P)$  is a stable matching for each P. A mechanism is strategyproof if  $\varphi(P) R_s \varphi(P'_s, P_{-s})$  for each P,  $s \in S$ , and  $P'_s$ , where  $R_s$  denotes the weak preferences associated to  $P_s$ . Given a priority structure  $Ch_C$  and preference profile  $P \in \mathcal{P}^{|S|}$ , a mechanism  $\varphi$  induces a normal form game  $\mathcal{G}(P) = (S, \mathcal{P}^{|S|}, \varphi, P)$ , where S is the set of players,  $\mathcal{P}^{|S|}$  is the strategy space,  $\varphi$  is the outcome function, and P is the profile of students' preferences. Let  $\Phi : \mathcal{P}^{|S|} \rightrightarrows \mathcal{M}$  be a correspondence. We say that mechanism  $\varphi$  implements  $\Phi$  in Nash equilibrium (NE from now on) if, for each  $P \in \mathcal{P}^{|S|}$ , the set of Nash equilibria of  $\mathcal{G}(\mathcal{P}) = (S, \mathcal{P}^{|S|}, \varphi, P)$ , NE(P) coincides with  $\Phi(P)$ . We say that mechanism  $\varphi$  implements  $\Phi$  in undominated Nash equilibrium (UNE from now on) if, for each  $P \in \mathcal{P}^{|S|}$ , the set of undominated Nash equilibria of  $\mathcal{G}(\mathcal{P}) = (S, \mathcal{P}^{|S|}, \varphi, P)$ , UNE(P) coincides with  $\Phi(P)$ .

# 3 The renegotiable acceptance mechanism

In this section, we introduce the renegotiable acceptance mechanism. The new mechanism has characteristics of both the immediate and the deferred acceptance mechanisms. As in the immediate acceptance mechanism, students are accepted by courses at most once along the mechanism. As in the deferred acceptance mechanism, a course can replace previously accepted students with new ones.

The message space for students in the renegotiable acceptance mechanism is the set of preference profiles on the subsets of courses. In the first stage, only the favorite set of courses of each student is considered. Among the students demanding a given course, the group with the highest priority is chosen. At the end of this stage, all students assigned to at least one course are removed, jointly with the students not demanding any course. At the  $r^{th}$  step of the procedure, only the  $r^{th}$  choices of the remaining students are considered. Each course considers the students already assigned to it and the new students claiming a seat, and chooses the subset of highest priority. All students who have been assigned at least one course at this stage are removed, jointly with the students not demanding any course. The procedure stops when all students have been removed.

Let  $P = (P_s)_{s \in S}$  be a preference profile. Let  $s \in S$  and let r be an integer such that  $1 \leq r \leq 2^{|S|}$ , and let  $C_{P_s}^r$  be the  $r^{th}$  ranked acceptable set of courses according to  $P_s$ , when one exists. Let  $C_{P_s}^r$  be empty otherwise.

Given a priority system  $(P_c)_{c\in C}$  and a preference profile for students  $(P_s)_{s\in S}$ , the following procedure describes the **renegotiable acceptance** mechanism.

Step 1: Only the top acceptable choices of students are considered. For each

course c, let  $S_c^1$  be the set of students who selected c among their first choices. Formally,  $S_c^1 = \{s \in S \mid c \in C_{P_s}^1\}$ . Define  $\mu^1(c) = Ch_c(S_c^1)$ . Every student in  $\mu^1(c)$  is enrolled in course c. Every student in  $\mu^1(c)$ and every student s such that  $C_{P_s}^1 = \emptyset$  is removed from the market. Set  $T^1 = S$ . Let  $T^2$  be the set of remaining students.

Step  $\mathbf{r}, r \geq 2$ : Only the  $r^{th}$  choices of students in  $T^r$  are considered. For each course c, let  $S_c^r = \mu^{r-1}(c) \cup \{s \in T^r \mid c \in C_{P_s}^r\}$  be the set of students enrolled at c at the end of stage r and of the remaining students ranking a set containing c in the  $r^{th}$  place. Let  $\mu^r(c) = Ch_c(S_c^r)$ . Every student in  $\mu^r(c)$  and every student s such that  $C_{P_s}^r = \emptyset$  is removed from the market. Let  $T^{r+1}$  be the set of remaining students.

The procedure stops when all students have been removed. Formally, it stops at  $r^* = \min \{r \mid T^{r+1} = \emptyset\}$ . Let  $RA(P) = \mu^{r^*}$  be the final outcome. Note that the procedure produces an outcome even when preferences are not substitutable.

We first show that in the renegotiable acceptance mechanism, students can obtain any attainable set of courses by ranking them in the first place.

**Lemma 1** Let  $P = (P_s)_{s \in S}$  be a preference profile for students, and let  $\mu = RA(P)$ . If priorities are substitutable, for each  $s \in S$  and  $C' \subseteq \mu(s)$ ,  $C' = RA(P_{s|C'}, P_{-s})(s)$ .

**Proof.** Let  $s \in S$  and let  $C' \subseteq \mu(s)$ . Let  $c \in C'$ , let r(c) be the step of the renegotiable acceptance mechanism when c has been assigned to s for the first

time along the mechanism, and formally let  $r(c) = \min_{r \le r^*} \{r \mid s \in \mu^r(c)\}$ . Note that r(c) = r(c') for all  $c, c' \in \mu(s)$  and that  $\mu^r(s) = \emptyset$  for all r < r(c). The substitutability of  $Ch_c$  implies that  $C_{P_s}^{r(c)} P_s C'$ ; otherwise,  $s \in \mu^r(c)$  for some r < r(c). For all  $i \le r(c)$ , let  $P_s^{r(c)}$  be a preference profile over  $2^C$  such that  $C_{P_s}^{r(c)} = C'$ , and for  $j \ne r(c)$ :  $C_{P_s}^{r(c)} = C_{P_s}^j$  if  $C_{P_s}^j \ne C'$  and  $C_{P_s}^{j}r(c) = C_{P_s}^{r(c)}$  if  $C_{P_s}^j = C'$ . Note that  $RA\left(P_s^{r(c)}, P_{-s}\right)(s) = C'$ . For all i, i < r(c), let  $P_s^i$  be a preference profile over  $2^C$ , such that  $C_{P_s}^{i} = C'$ , and for  $j \ne i$ :  $C_{P_s}^j = C_{P_s}^{j+1}$  if  $C_{P_s}^{j+1} \ne C'$  and  $C_{P_s}^j = C_{P_s}^{j+1}$  if  $C_{P_s}^{j+1} = C'$ . Intuitively, each  $P_s^j$  lifts C' at place j in the preference profile of s without altering the ranking above the  $j^{th}$  place.

We prove by contradiction that  $RA(P_s^{i-1}, P_{-s})(s) = RA(P_s^i, P_{-s})(s) = C'$  for all  $i, 1 \leq i < r(c)$ . For every preference profile on  $2^C$ ,  $Q_s$ , let  $\mu_{Q_s}^j$  be the outcome at stage j of the mechanism when preferences are  $(Q_s, P_{-s})$ . Note that  $\mu_{P_s}^i = \mu_{P_s^j}^i$  for all  $i, j, 2 \leq i < j \leq r(c)$ . Thus, to prove that  $RA(P_s^{i-1}, P_{-s})(s) = RA(P_s^i, P_{-s})(s)$  for all i < r(c), it suffices to show that  $s \in Ch_c\left(\mu_{P_s}^{i-1}(c) \cup \left\{s \in S \mid c \in \bigcup_{s' \neq s} C_{P_{s'}}^{i-1}\right\} \cup \{s\}\right)$  for all  $i, 2 \leq i \leq r(c)$ . By contradiction, assume that it is not the case, and let j be the maximum integer such that

 $s \notin Ch_c \left( \mu_{P_s}^{j-1}(c) \cup \left\{ s \in S \mid c \in \bigcup_{s' \neq s} C_{P_{s'}}^{i-1} \right\} \cup \{s\} \right) \text{ and}$   $s \in Ch_c \left( \mu_{P_s}^j(c) \cup \left\{ s \in S \mid c \in \bigcup_{s' \neq s} C_{P_{s'}}^{i-1} \right\} \cup \{s\} \right). \text{ Because } P_c \text{ is substitutable}, s \in Ch_c \left( \mu_{P_s}^j(c) \cup \{s\} \right). \text{ The } j^{th} \text{ step of the mechanism when preferences are } (P_s^j, P_{-s}) \text{ yields } \mu_{P_s^j}^j(c) \text{ to course } c. \text{ We have}$   $s \notin Ch_c \left( \mu_{P_s}^{j-1}(c) \cup \left\{ s \in S \mid c \in \bigcup_{s' \neq s} C_{P_{s'}}^{i-1} \right\} \cup \{s\} \right) = \mu_{P_s^j}^j(c). \text{ The irrele-}$ 

vance of rejected students condition implies that

 $Ch_{c}\left(\mu_{P_{s}}^{j-1}\left(c\right)\cup\left\{s\in S\mid c\in\bigcup_{s'\neq s}C_{P_{s'}}^{i-1}\right\}\cup\left\{s\right\}\right)=Ch_{c}\left(\mu_{P_{s}}^{j}\left(c\right)\cup\left\{s\right\}\right)=\mu_{P_{s}}^{j}\left(c\right).$ In particular,  $s\notin Ch_{c}\left(\mu_{P_{s}}^{j}\left(c\right)\cup\left\{s\right\}\right)$ , which yields a contradiction. Thus, we have  $RA\left(P_{s}^{1},P_{-s}\right)\left(s\right)=C'$ . It follows that  $RA\left(P_{s|C'},P_{-s}\right)\left(s\right)=C'$ , which concludes the proof of the claim.  $\blacksquare$ 

Lemma 1 implies that each student can obtain her favorite attainable set of courses by listing a reduced amount of options. Thus, we can prove that every Nash equilibrium outcome of the renegotiable acceptance mechanism is stable, and every stable allocation is a Nash equilibrium outcome of the renegotiable acceptance mechanism.

**Theorem 1** The renegotiable acceptance mechanism implements the set of stable matching in NE in the domain of substitutable preferences if priorities are substitutable.

**Proof.** (i) We first prove that any NE outcome is a stable matching. Let  $P^*$  be a NE of the games induced by the renegotiable acceptance mechanism, and let  $\mu = RA(P^*)$ . Matching  $\mu$  is individually rational for each course by definition. We prove by contradiction that  $\mu$  is individually rational for students. Assume  $Ch_s(\mu(s)) \neq \mu(s)$  for some  $s \in S$ . Let  $P'_s = P_{s|Ch_s(\mu(s))}$ . Because  $P_s$  is substitutable,  $P'_s$  is substitutable as well. By Lemma 1:  $RA(P'_s, P^*_{-s})(s) = Ch_s(\mu(s))$ . Thus, the deviation is profitable to s, which yields a contradiction. Assume that there exists a pair blocking  $\mu$ ,  $(c, s) \in C \times S$ . Let  $P' = P_{s|Ch_s(\mu(s)\cup\{c\})}$ . Because  $s \in Ch_c(\mu(c)\cup\{s\})$ ,

the deviation is profitable to s, which yields a contradiction. It follows that matching  $\mu$  is individually rational and cannot be blocked by any coursestudent pair; thus, it is stable.

(ii) Let  $\mu$  be a stable matching. For each s, let  $P_s^* = P_{s|\mu(s)}$ . Set  $P^* = (P_s^*)_{s \in S}$ . We have  $RA(P^*) = \mu$ . We prove by contradiction that  $P^*$  is a Nash equilibrium. Assume that  $s \in S$  has a profitable deviation,  $P'_s$ , and let  $\mu' = RA(P'_s, P^*_{-s})$ . Let  $c \in Ch_s(\mu(s) \cup \mu'(s)) \setminus \mu(s)$ . Because  $P_s$  is substitutable,  $c \in Ch_s(\mu(s) \cup \{c\})$ . Let  $P''_s = P_{s|Ch_s(\mu(s) \cup \{c\})}$ , then  $RA(P''_s, P^*_{-s})(s) = Ch_s(\mu(s) \cup \{c\})$ . It follows that (c, s) blocks  $\mu$ , which yields a contradiction.

The renegotiable acceptance mechanism yields unstable matchings with respect to stated preferences. However, unstable matchings are ruled out by strategic behavior. From Lemma 1, it follows that if pair (c, s) blocks an outcome matching  $\mu$ ,  $P_{s|Ch_s(\mu(s)\cup\{c\})}$  is a profitable deviation for s.

Note that the equilibrium strategies defined in part (ii) of the proof of Theorem 1 are undominated. Thus, we have the following result.

**Corollary 1** The renegotiable acceptance mechanism implements the set of stable matching in UNE in the domain of substitutable preferences if priorities are substitutable.

#### 3.1 Slot-specific priorities

The class of slot-specific priorities is a strict subset of the set of substitutable priorities that allows for a flexible matching of students to courses. Let  $\left(q, \left(\Sigma_c, (\succ_{\sigma})_{\sigma \in \Sigma_c}, \triangleright_c\right)_{c \in C}\right)$  be a slot-specific priority structure. Let  $P_C = (P_c)_{c \in C}$  be a profile of linear orders that rationalize the respective choice functions. The hypothesis of Theorem 1 are satisfied by slot-specific priorities. It follows that under these priorities, the renegotiable acceptance mechanism implements the set of stable matching in NE when students' preferences are substitutable.

Under slot-specific priorities, we can adapt the immediate acceptance mechanism to allocate courses, which works as follows. First, all students submit a preference profile. In the first stage, the favorite acceptable set of courses of each student is considered. Among the students claiming a course, those with the highest priorities for any given course are assigned to it. At the end of this stage, all students assigned to at least one course and all assigned seats are removed from the procedure. At the  $n^{th}$  stage of the mechanism, only the  $n^{th}$  choices of the remaining students are considered, and we repeat the procedure until no more slots or students are remaining.

Given a priority structure  $(P_c)_{c\in C}$  and a preference profile for students  $(P_s)_{s\in S}$ , the following procedure describes the **immediate acceptance mechanism**.

• Step 1: Only the top acceptable choices of students are considered.

For each course c, let  $S_c^1$  be the set of students who selected c among their first choices. Formally,  $S_c^1 = \{s \in S \mid c \in C_{P_s}^1\}$ .<sup>7</sup> Define  $\mu^1(c) = Ch_c(S_c^1)$ . Every student in  $\mu^1(c)$  is definitively enrolled in course c. Every student in  $\mu^1(c)$  and every student s such that  $C_{P_s}^1 = \emptyset$  is removed from the market. Set  $T^1 = S$ . Let  $T^2$  be the set of remaining students.

Step r, r ≥ 2: Only the r<sup>th</sup> choices of students in T<sup>r</sup> are considered. For each course c let S<sup>r</sup><sub>c</sub> = {s ∈ T<sup>r</sup> | c ∈ C<sup>r</sup><sub>Ps</sub>} be the set of students in T<sup>r</sup> who selected c among their r<sup>th</sup> choices. Let μ<sup>r</sup>(c) = max<sub>Pc</sub> {μ<sup>r-1</sup>(c) ∪ S' | S' ⊆ S<sup>r</sup><sub>c</sub>}. Every student in μ<sup>r</sup>(c) and every student s such that C<sup>r</sup><sub>Ps</sub> = Ø is removed from the market. Let T<sup>r+1</sup> be the set of remaining students.

The procedure stops when all students have been removed. Formally, it stops at  $r^* = \min \{r \mid T^{r+1} = \emptyset\}$ . Let  $IA(P) = \mu^{r^*}$  be the final outcome. Note that a student never loses the seat at a course she has been assigned to at some step of the mechanism, but she can be moved to slots of different precedence along the mechanism. Furthermore, all matchings are individually rational for courses.

Under substitutable preferences, all stable matchings are Nash equilibrium outcomes of the immediate acceptance mechanism. However, not all Nash equilibrium outcomes are stable matchings. This is because not all

<sup>&</sup>lt;sup>7</sup>For each *i* and each  $P_s$ ,  $C_{P_s}^i$  is defined as for the renegotiable acceptance mechanism.

outcomes of the mechanism are individually rational for courses as can be seen in Example 1.

**Example 1** There are two courses,  $C = \{c_1, c_2\}$  and four students,  $S = \{s_1, s_2, s_3, s_4\}$ . Each student wants to enroll in exactly one course. The maximal number of students  $c_1$  can enroll is three but the ideal number is two. Preferences and priorities are as follows:

$$\begin{split} P_{s_1} : \{c_2\}, \{c_1\}; \\ P_{s_2} : \{c_1\}; \\ P_{s_3} : \{c_1\}; \\ P_{s_4} : \{c_2\}; \\ P_{c_1} : \{s_1, s_3\}, \{s_1, s_2, s_3\}, \{s_2, s_3\}, \{s_1, s_2\}, \{s_1\}, \{s_3\}, \{s_2\}; \\ P_{c_2} : \{s_4\}, \{s_1\}, \{s_2\}, \{s_3\}. \\ All \ priorities \ are \ substitutable. \ Truth \ telling \ results \ in \ matching \ \mu, \ where \\ \mu(c_1) &= \{s_1, s_2, s_3\} \ and \ \mu(c_2) &= \{s_4\}, \ which \ is \ not \ individually \ rational \\ because \ Ch_{c_1}(\mu(c_1)) \neq \mu(c_1). \ However, \ truth \ telling \ is \ a \ Nash \ equilibrium \\ of \ the \ immediate \ acceptance \ mechanism \ because \ any \ agent \ but \ s_1 \ is \ assigned \end{split}$$

to her preferred course, and  $s_1$  has no profitable deviations.

The instability of NE allocations comes from the fact that acceptances are definitive. In Example 1, when  $s_1$ 's application comes, course  $c_1$ 's priorities prescribe the rejection of the student's application, but it cannot. Unlike the renegotiable acceptance mechanism, the immediate acceptance mechanism does not allow for courses to reject previously accepted students. When priorities are slot-specific, this is not a concern because all outcomes of the immediate acceptance mechanism are individually rational for courses. Even if the executions of the two mechanisms do not coincide under slotspecific priorities, we can replicate the strategy of the proof of Theorem 1 and prove an analogous of Lemma 1: students can obtain any attainable set of courses by ranking them in the first place when the immediate acceptance mechanism is employed.

**Lemma 2** Let  $P = (P_s)_{s \in S}$  be a preference profile for students, and let  $\mu = IA(P)$ . For each  $s \in S$  and  $C' \subseteq \mu(s)$   $C' = IA(P_{s|C'}, P_{-s})$ .

**Proof.** Let  $s \in C$  and let  $C' \subseteq \mu(s)$ . Let  $c \in C'$ , let r(c) be the step of the immediate acceptance mechanism when c has been assigned to s, and formally let  $r(c) = \min_{r \leq r^*} \{r \mid s \in \mu^r(c)\}$ . Let  $\sigma$  be the slot to which s is assigned at stage r(c). Thus, student s is the highest priority student for slot  $\sigma$  among the ones in  $\mu^r(c)$  and who are not assigned to a slot preceding  $\sigma$ . Formally, for each  $r \leq r(c)$ , if  $s' \in \mu^r$  and  $s' \succ_{\sigma} s \succ_{\sigma}$ , there exists a slot  $\sigma' \in \Sigma_c, \sigma' \succ_c \sigma$  such that  $s' \succ_{\sigma'} \sigma$ . Thus,  $C' = IA(P_{s|C'}, P_{-s})$ .

This result allows us to prove that every Nash equilibrium outcome of the immediate acceptance mechanism is stable and every stable allocation is a Nash equilibrium outcome of the immediate acceptance mechanism under slot-specific priorities.

**Theorem 2** The immediate acceptance mechanism implements the set of stable matching in NE in the domain of substitutable preferences if priorities are slot-specific.

**Proof.** (i) We first prove that any NE outcome is a stable matching. Let  $P^*$  be a NE of  $(S, \mathcal{P}^{|S|}, IA, P)$  and let  $\mu = IA(P^*)$ . As observed,  $\mu$  is individually rational for each course. We prove by contradiction that  $\mu$  is individually rational for students. Assume  $Ch_s(\mu(s)) \neq \mu(s)$  for some  $s \in S$ . Let  $P'_s = P_{s|Ch_s(\mu(s))}$ , by Lemma 2:  $IA(P'_s, P^*_{-s})(s) = Ch_s(\mu(s))$ . Thus, the deviation is profitable to s, which yields a contradiction. Assume that there exists a pair blocking  $\mu$ ,  $(c, s) \in C \times S$ . Let  $P' = P_{s|Ch_s(\mu(s)\cup\{c\})}$ . Because  $s \in Ch_c(\mu(c) \cup \{s\})$ , the deviation is profitable to s, which yields a contradiction belocked by a pair,  $\mu$  is stable.

(ii) Let  $\mu$  be a stable matching. For each s, let  $P_s^* = P_{s|\mu(s)}$ . Set  $P^* = (P_s^*)_{s \in S}$ . We have  $IA(P^*) = \mu$ . We prove by contradiction that  $P^*$  is a Nash equilibrium. Assume that  $s \in S$  has a profitable deviation,  $P'_s$ , and let  $\mu' = IA(P'_s, P^*_{-s})$ . Let  $c \in Ch_s(\mu(s) \cup \mu'(s)) \setminus \mu(s)$ . Because  $P_s$  is substitutable,  $c \in Ch_s(\mu(s) \cup \{c\})$ . Let  $P''_s = P_{s|Ch_s(\mu(s) \cup \{c\})}$ , then  $IA(P''_s, P^*_{-s})(s) = Ch_s(\mu(s) \cup \{c\})$ . It follows that (c, s) blocks  $\mu$ , which yields a contradiction.

The cost of introducing substitutable priorities is to allow for courses to renegotiate their assigned group of students to preserve the individual rationality of the outcome. Theorem 2 proves that this is no longer the case under slot-specific priorities: Nash implementation of stable matchings does not require students to lose their positions along the mechanism. Note that the equilibrium strategies defined in the part (ii) of the proof of Theorem 2 are undominated. Thus, we obtain the following result.

**Corollary 2** The immediate acceptance mechanism implements the set of stable matching in UNE in the domain of substitutable preferences if priorities are slot-specific.

#### 3.2 Simplifying the strategy space

The renegotiable and immediate acceptance mechanisms perform well under substitutable and slot-specific priorities, respectively. However, the complexity of the strategy space might hinder its practical implementation (see Budish et al., 2017). We prove that if the preferences of the students are responsive, the message space can be simplified.<sup>8</sup> Our findings can be applied to situations in which course schedules do not overlap, and students have only one possible group to attend to for each course. This is often the case for the courses organized by neighborhood associations and local libraries, and for elective courses at small community colleges and universities.

We next introduce two mechanisms derived from the renegotiable and immediate acceptance mechanisms for which students have to reveal their preferences for individual courses and demands, instead of their full profile of preferences for all possible subsets of courses.

For each  $s \in S$ , let  $M_s = \mathcal{L}(C) \times (\mathbb{N} \cap [0, |C|])$ , where  $\mathcal{L}(C)$  is the set of

 $<sup>^{8}\</sup>mathrm{As}$  mentioned in the introduction, the assumption of responsive preferences is standard in the course assignment literature.

linear order on  $C \cup \{\emptyset\}$  and  $\mathbb{N}$  is the set of non-negative integers. For each  $s \in S$ , let  $(\geq_s, q_s) \in M_s$  and let  $P_s = P_s(\geq_s, q_s)$  be a profile of responsive preferences with demand  $q_s$ , which coincides with  $\geq_s$  on the set of individual courses.<sup>9</sup>

Given a priority system  $(Ch_c)_{c\in C}$  and  $(\geq_s, q_s)_{s\in S}$ , the simplified renegotiable acceptance mechanism is defined by the following outcome function  $SRA((\geq_s, q_s)_{s\in S}) = RA(P_s(\geq_s, q_s)_{s\in S})$ . In other words, in a simplified mechanism, students play the game induced by the corresponding mechanism with preferences that are responsive to the revealed ones.

**Proposition 1** Assume that students preferences are responsive and priorities are substitutable. The simplified renegotiable acceptance mechanism implements the set of stable matchings in Nash equilibrium.

**Proof.** (i) We first prove that any NE outcome is a stable matching. Let  $(>_s^*, q_s^*)_{s \in S}$  be a NE of the game induced by the simplified renegotiable acceptance mechanism when students' preferences are given by  $(P_s)_{s \in S}$  and let  $\mu = SRA((>_s^*, q_s^*)_{s \in S})$ . Matching  $\mu$  is individually rational for each course. We prove by contradiction that  $\mu$  is individually rational for students. Assume that  $\mu$  is not individually rational for students  $s \in S$ , which assumes that there exists a course  $c \in \mu(s)$  such that  $\emptyset P_s c$  or  $|\mu(s)| > q_s$ , where  $q_s$  is the offer of course s according to  $P_s$ . Let  $>_s$  be the restriction of  $P_s$  to individual courses. By Lemma 1,  $(>_s, q_s)$  is a profitable deviation for student that

<sup>&</sup>lt;sup>9</sup>This means that  $c \ge_s c'$  if and only if  $\{c\} P_s \{c'\}$  for all  $c, c' \in C$ .

 $\mu$  is not blocked by any pair. Assume that there exists a pair blocking  $\mu$ ,  $(c,s) \in C \times S$ . Let  $>_s$  be the restriction of  $P_s$  to the individual courses in  $\mu(s) \cup \{c\}$ . Because  $s \in Ch_c$  ( $\mu(s) \cup \{c\}$ ), the deviation ( $>_s, q_s$ ) is profitable to s, which yields a contradiction.

(ii) Let  $\mu$  be a stable matching. For each s, let  $>_s$  be the restriction of  $P_s$  to the individual courses in  $\mu(s)$ . Note that  $(>_s, q_s)_{s \in S}$  yields  $\mu$  as outcome. We prove by contradiction that  $(>_s, q_s)_{s \in S}$  is a Nash equilibrium. Assume that student s has a profitable deviation,  $(>'_s, q'_s)$ , and let  $\mu'$  be the outcome of such a deviation. Let  $c \in Ch_s(\mu(s) \cup \mu'(s), P_s(>_s, q_s)) \setminus \mu(s)$ . Because  $P_s$  is responsive,  $c \in Ch_s(\mu(s) \cup \{c\})$ . Let  $>''_s$  be the restriction of  $P_s$  to the individual courses of  $\mu(s) \cup \{c\}$ . Then,  $(>'_s, q_s)$  is a profitable deviation as well, yielding  $Ch_s(\mu(s) \cup \{c\}, P_s)$ . Thus, the pair (c, s) blocks matching  $\mu$ , which yields a contradiction.

We can also define a simplified version of the immediate acceptance mechanism as follows. Given a priority system  $(Ch_c)_{c\in C}$  and  $(\geq_s, q_s)_{s\in S}$ , the simplified immediate acceptance mechanism is defined by the following outcome function  $SIA((\geq_s, q_s)_{s\in S}) = IA(P_s(\geq_s, q_s)_{s\in S}).$ 

**Proposition 2** Assume that student preferences are responsive and priorities are slot-specific. The simplified immediate acceptance mechanism implements the set of stable matchings in Nash equilibrium.

The proof of Proposition 2 is similar to the proof of Proposition 1 and is omitted.

# 4 Conclusions

In this paper, we present the renegotiable acceptance mechanism to allocate courses to students on the basis of priorities. Under substitutable preferences and priorities, the renegotiable acceptance mechanism implements the set of stable matching in Nash equilibrium and in undominated Nash equilibrium. The mechanism produces matchings that are fair, and its practical implementation is not computationally demanding. The renegotiable acceptance mechanism is based on the immediate acceptance mechanism but allows for courses to reject previously accepted students. During the procedure, courses are only tentatively assigned, and the readjustments preserve individual rationality. This makes our new procedure a hybrid between the immediate and the deferred acceptance mechanisms. We also analyze the immediate acceptance mechanism under the assumption of slot-specific priorities and find that it implements the set of stable matching in Nash equilibrium and in undominated Nash equilibrium. The results depend on the fact that both mechanisms provide each student with incentive to top-rank the best achievable subset of courses given the preferences submitted by the other students. This property helps to rule out unstable matchings as equilibrium outcomes.

Finally, we study the possibility of reducing the complexity of the strategy space. We show that this is possible when courses are not complements. In this case, a mechanism that asks each student a ranking on individual courses and the number of courses that she is willing to take implements the set of stable matchings in Nash equilibria.

# Aknowledgements

We thank María Haydée Fonseca and the participants to the Workshop in Game Theory and Social Choice for their useful comments. Both authors acknowledge financial support from Ministerio Economía y Competitividad (Spain) under projects ECO2014\_57442\_P and ECO2017\_87769\_P and from Fondecyt under project No. 1151230. Romero-Medina acknowledges financial support from Ministerio Economía y Competitividad (Spain) MDM 2014-0431 and Comunidad de Madrid, MadEco-CM (S2015/HUM-3444). Triossi acknowledges financial support from the Institute for Research in Market Imperfections and Public Policy, ICM IS130002, Ministerio de Economía, Fomento y Turismo.

# References

- [1] Alcalde, J., 1996.Implementation of Stable Solutions to Marriage Problems. J. Econ. Theory 69, 240 - 254, https://doi.org/10.1006/jeth.1996.0050.
- [2] Alva, S., 2018. WARP and combinatorial choice. J. Econ. Theory 173, 320–333, https://doi.org/10.1016/j.jet.2017.11.007.

- [3] Abdulkadiroğlu, A., Sönmez, T., 2003. School Choice: A Mechanism Design Approach. Amer. Econ. Rev. 93, 729-747, https://doi.org/10.1257/000282803322157061
- Т., [4] Aygün, O., Sönmez, 2013. Matching with Con-Econ. 2050-2051, tracts: Comment. Amer. Rev. 103,https://doi.org/10.1257/aer.103.5.2050.
- Budish, E., 2011. The Combinatorial Assignment Problem: Approximate Competitive Equilibrium from Equal Incomes. J. Polit. Economy 119, 1061–1103, https://doi.org/10.1086/664613.
- [6] Budish, E., Cachon, G., Kessler, J., and Othman, A., 2017. Course Match: A Large-Scale Implementation of Approximate Competitive Equilibrium from Equal Incomes for Combinatorial Allocation. Oper. Res. 65(2), 314–336, https://doi.org/10.1287/opre.2016.1544.
- Budish, E., Cantillon, E., 2012. The Multi-Unit Assignment Problem: Theory and Evidence from Course Allocation at Harvard. Amer. Econ. Rev. 102(5), 2237-2271, https://doi.org/10.1257/aer.102.5.2237.
- [8] Dogal, B., Klaus B., 2018. Object allocation via immediate-acceptance: Characterizations and an affirmative action application. J. Math. Econ. In Press. https://doi.org/10.1016/j.jmateco.2018.04.001.

- [9] Dur, U., Kominers, S.D., Pathak, P.A. and Sönmez, T., 2016. Explicit vs. Statistical Targeting in Affirmative Action: Theory and Evidence from Chicago's Exam Schools. NBER Working Paper No. 22109.
- [10] Dur, U., Kominers, S.D., Pathak, P.A., Sönmez, T., 2018. Reserve Design: Unintended Consequences and The Demise of Boston's Walk Zones. Forthcoming, J. Polit. Economy.
- [11] Echenique, F. and Oviedo, J., 2006. A theory of stability in many-tomany matching markets. Theor. Econ. 1(2), 233-273.
- [12] Ergin, H., Sönmez, T., 2006. Games of School Choice under the Boston Mechanism. J. Public. Econ. 90, 215–237, https://doi.org/10.1016/j.jpubeco.2005.02.002.
- [13] Hafalir, I.E., Yenmez, M.B., Yildirim, M.A., 2013, Effective affirmative action in school choice. Theor. Econ. 8, 325–363, https://doi.org/10.3982/TE1135.
- [14] Haeringer, G., Klijn, F., 2009, Constrained school choice, J. Econ Theory, Volume 144, 1921–1947, https://doi.org/10.1016/j.jet.2009.05.002.
- 2012.Impossibilities [15] Kojima, F., School choice: for affirmative action, Econ. Behav. 75.685-693, Games https://doi.org/10.1016/j.geb.2012.03.003.

- [16] Kojima, F., 2013.Efficient Resource Allocation under Multi-unit Demand. Games Econ. Behav. 82, 1 - 14, https://doi.org/10.1016/j.geb.2013.06.005.
- Kojima, F., Ünver, U., 2014. The "Boston" school-choice mechanism: an axiomatic approach. Econ. Theory 55, 515–544, https://doi.org/10.1007/s00199-013-0769-8.
- [18] Marutani, K., 2018. Gaming the deferred acceptance when message spaces are restricted. Math. Soc. Sci. 93, 153–158, https://doi.org/10.1016/j.mathsocsci.2018.03.007.
- S.D., Τ., 2016. Matching [19] Kominers, Sönmez, with Slot-Specific **Priorities**: Theory. Theor. Econ. 11(2),683 - 710, https://doi.org/10.3982/TE1839.
- [20] Romero-Medina, A., Triossi, M., 2018. Two-Sided Strategy-Proofness in Many-to-Many Matching Markets. Mimeo, https://ssrn.com/abstract=2983158.
- [21] Roth, A.E., 1985. The College Admissions Problem is not Equivalent to the Marriage Problem. J. Econ. Theory 36, 277–288, https://doi.org/10.1016/0022-0531(85)90106-1.
- [22] Roth, A.E., Sotomayor, M., 1990. Two-Sided Matching: A Study in Game-Theoretic Modeling and Analysis. Cambridge, Cambridge University Press.

- [23] Schummer, J., Abizada, A., 2017. Incentives in landing slot problems.
   J. Econ. Theory 170, 29-55, https://doi.org/10.1016/j.jet.2017.04.003.
- [24] Schummer, J., Vohra, R.V., 2013. Assignment of Ar-Slots. Amer. Econ. J.: Microeconomics rival 5,164 - 185, https://doi.org/10.1257/mic.5.2.164.
- [25] Sönmez, T., Ünver, U., 2010. Course Bidding at Business Schools. Int. Econ. Rev. 51(1), 99-123, https://doi.org/10.1111/j.1468-2354.2009.00572.x.
- [26] Thomson, W., 2018. On the terminology of economic design: a critical assessment and some proposals. Rev. Econ. Des. 22(1-2), 67-99, https://doi.org/10.1007/s10058-018-0210-7.

## Centro de Economía Aplicada Departamento de Ingeniería Industrial Universidad de Chile

## 2018

- 340. Centralized Course Allocation Antonio Romero-Medina y Matteo Triossi
- 339. Identifying Food Labeling Effects on Consumer Behavior Sebastián Araya, Andrés Elberg, Carlos Noton y Daniel Schwartz
- 338. Cooperatives vs Traditional Banks: The impact of Interbank Market Exclusion Raphael Bergoeing y Facundo Piguillem
- 337. Sorting On-line and On-time Stefano Banfi, Sekyu Choi y Benjamín Villena-Roldán
- 336. Investment Opportunities and Corporate Credit Risk Eugenia Andreasen, Patricio Valenzuela

### 2017

- 335. Efectos de la Reforma del Código de Aguas Ronald Fischer
- 334. Returns to Higher Education: Vocational Education vs College Ana Maria Montoya, Carlos Noton y Alex Solis
- 333. Group strategy-proof stable mechanisms in priority-based resource allocation under multi-unit demand: a note Antonio Romero-Medina y Matteo Triossi
- 332. (Group) Strategy-proofness and stability in many-to-many matching markets Antonio Romero-Medina y Matteo Triossi
- 331. Longevity, Human Capital and Domestic Investment Francisco Parro y Francisco Szederkenyi y Patricio Valenzuela
- The Inequality-Credit Nexus Ronald Fischer, Diego Huerta y Patricio Valenzuela
- 329. Inequality, Finance, and Growth Matías Braun, Francisco Parro y Patricio Valenzuela
- 328. Take-it-or-leave-it contracts in many-to-many matching markets Antonio Romero-Medina y Matteo Triossi

## 2016

327. Do High-Wage Jobs Attract more Applicants? Directed Search Evidence from the Online Labor Market Stefano Banfi y Benjamín Villena-Roldán

- 326. Economic Performance, Wealth Distribution and Credit Restrictions with Continuous Investment Ronald Fischer y Diego Huerta
- 325. Motivating with Simple Contracts Juan F. Escobar y Carlos Pulgar
- 324. Gone with the wind: demographic transitions and domestic saving Eduardo Cavallo, Gabriel Sánchez y Patricio Valenzuela

### 2015

- 323. Colaboración Público-Privada en infraestructuras: Reforma del sistema concesional español de autopistas de peaje Eduardo Engel, Ronald Fischer, Alexander Galetovic y Ginés de Rus
- 322. The Joy of Flying: Efficient Airport PPP contracts Eduardo Engel, Ronald Fischer y Alexander Galetovic
- 321. On the welfare cost of bank concentration Sofía Bauducco y Alexandre Janiak
- 320. Banking Competition and Economic Stability Ronald Fischer, Nicolás Inostroza y Felipe J. Ramírez
- 319. Persistent Inequality, Corruption, and Factor Productivity Elton Dusha
- 318. Reputational Concerns in Directed Search Markets with Adverse Selection Elton Dusha
- 317. Soft Budgets and Renegotiations in Public-Private Partnerships: Theory and Evidence Eduardo Engel Ronald Fischer Alexander Galetovic
- Inequality and Private Credit Diego Huerta, Ronald Fischer y Patricio Valenzuela
- 315. Financial Openness, Domestic Financial Development and Credit Ratings Eugenia Andreasen y Patricio Valenzuela
- 314. The Whole is Greater than the Sum of Its Parts: Complementary Reforms to Address Microeconomic Distortions
  (Por aparecer en The World Bank Economic Review)
  Raphael Bergoeing, Norman V. Loayza y Facundo Piguillem
- 313. Economic Performance, Wealth Distribution and Credit Restrictions under variable investment: The open economy Ronald Fischer y Diego Huerta
- 312. Destructive Creation: School Turnover and Educational Attainment Nicolás Grau, Daniel Hojman y Alejandra Mizala

- 311. Cooperation Dynamic in Repeated Games of Adverse Selection Juan F. Escobar y Gastón Llanes
- 310. Pre-service Elementary School Teachers' Expectations about Student Performance: How their Beliefs are affected by Mathematics Anxiety and Student Gender Francisco Martínez, Salomé Martínez y Alejandra Mizala
- 309. The impact of the minimum wage on capital accumulation and employment in a large-firm framework Sofía Bauducco y Alexandre Janiak

### 2014

306. Assessing the extent of democratic failures. A 99%-Condorcet's Jury Theorem. Matteo Triossi

### 2013

- 305. The African Financial Development and Financial Inclusion Gaps Franklin Allen, Elena Carletti, Robert Cull, Jun "Qj" Qian, Lemma Senbet y Patricio Valenzuela
- 304. Revealing Bargaining Power through Actual Wholesale Prices Carlos Noton y Andrés Elberg
- 303. Structural Estimation of Price Adjustment Costs in the European Car Market Carlos Noton
- 302. Remedies for Sick Insurance Daniel McFadden, Carlos Noton y Pau Olivella
- 301. Minimum Coverage Regulation in Insurance Markets Daniel McFadden, Carlos Noton y Pau Olivella
- 300. Rollover risk and corporate bond spreads Patricio Valenzuela
- 299. Sovereign Ceilings "Lite"? The Impact of Sovereign Ratings on Corporate Ratings Eduardo Borensztein, Kevin Cowan y Patricio Valenzuela
- 298. Improving Access to Banking: Evidence from Kenya F. Allen, E. Carletti, R. Cull, J."Qj" Qian, L. Senbet y P. Valenzuela
- 297. Financial Openness, Market Structure and Private Credit: An Empirical Investigation Ronald Fischer y Patricio Valenzuela
- 296. Banking Competition and Economic Stability Ronald Fischer, Nicolás Inostroza y Felipe J. Ramírez
- 295. Trust in Cohesive Communities Felipe Balmaceda y Juan F. Escobar
- 294. A Spatial Model of Voting with Endogenous Proposals: Theory and Evidence from Chilean Senate Matteo Triossi, Patricio Valdivieso y Benjamín Villena-Roldán

### 2012

- 293. Participation in Organizations, Trust, and Social Capital Formation: Evidence from Chile Patricio Valdivieso Benjamín Villena-Roldán
- 292. Neutral Mergers Between Bilateral Markets Antonio Romero-Medina y Matteo Triossi
- 291. On the Optimality of One-size-fits-all Contracts: The Limited Liability Case Felipe Balmaceda
- 290. Self Governance in Social Networks of Information Transmission Felipe Balmaceda y Juan F. Escobar
- 289. Efficiency in Games with Markovian Private Information Juan F. Escobar y Juuso Toikka
- 288. EPL and capital-labor ratios Alexandre Janiak y Etienne Wasmer
- 287. Minimum wages strike back: the effects on capital and labor demands in a large-firm framework Alexandre Janiak y Sofía Bauducco
- 286. Comments on Donahue and Zeckhauser: Collaborative Governance Ronald Fischer
- 285. Causal Effects of Maternal Time-Investment on Children Cecilia Ríos-Aguilar y Benjamín Villena-Roldan
- 284. Towards a quantitative theory of automatic stabilizers: the role of demographics Alexandre Janiak y Paulo Santos Monteiro
- 283. Investment and Environmental Regulation: Evidence on the Role of Cash Flow Evangelina Dardati y Julio Riutort
- 282. Teachers' Salaries in Latin America. How Much are They (under or over) Paid? Alejandra Mizala y Hugo Ñopo
- 281. Acyclicity and Singleton Cores in Matching Markets Antonio Romero-Medina y Matteo Triossi
- 280. Games with Capacity Manipulation: Incentives and Nash Equilibria Antonio Romero-Medina y Matteo Triossi
- 279. Job Design and Incentives Felipe Balmaceda
- 278. Unemployment, Participation and Worker Flows Over the Life Cycle Sekyu Choi, Alexandre Janiak y Benjamín Villena-Roldán
- 277. Public-Private Partnerships and Infrastructure Provision in the United States Eduardo Engel, Ronald Fischer y Alexander Galetovic

\* Para ver listado de números anteriores ir a http://www.cea-uchile.cl/.