A Note on the Optimality of the Cash Flow Tax

PABLO SERRA*AND DANIEL HOJMAN†
CEA, Department of Industrial Engineering, University of Chile‡

ABSTRACT. This paper analyzes the optimal tax policy within an endogenous growth model with productive government spending. We consider a one-factor (human capital) one-good economy, with the latter serving both as a final and an intermediate good. The government levies taxes in order to finance the provision of the intermediate good. Within this framework we show a highly intuitive result: the optimal tax structure is a 100 percent tax on cash flows and no tax on labor income. As a consequence, the consumption tax causes a deadweight loss, which increases with the intensity of use of the intermediate good.

Keywords: endogenous growth, optimal taxing, cash flow.

JEL classification: H21; O41

*Tel: +562 678 4058; Fax: +562 689 7895; e-mail: pserra@dii.uchile.cl
†Tel: +562 678 4030; e-mail: dhojman@dii.uchile.cl
‡DII- U. de Chile, República 701, 6521122 Santiago, Chile.

1. Introduction

The purpose of this note is to show that Ramsey's taxation principle holds within the framework of an endogenous growth model where public expenditure enhances private productivity à la Barro (1990). We consider a one-factor one-good closed economy, where the single good serves both as a final and an intermediate good. The intermediate good, which is freely provided by the government, is tax financed. We assume competition among numerous firms, each of which receives a fraction of the intermediate good. The free supply of the intermediate good generates positive cash flows to firms. On the consumption side, a representative infinite-lived household who maximizes his/her intertemporal utility function, which includes the "consumption" of "raw" leisure time. The representative agent's income includes both labor income and cash flows from firms.

Within this framework we derive the optimal tax policy obtaining a highly intuitive result: the optimal tax structure is a 100 percent tax on cash flows and no tax on labor income. Cash flows are economic rents as they are associated with an inelastically supplied productive factor: the intermediate good provided by the government. This result is similar to the one obtained by Corsetti and Roubini (1996). In a three-sector, two-factor (human and physical capital) endogenous growth model with productive government spending, they show that the optimal policy is to tax income earned by the factor whose productivity is directly affected by the government spending externality. A difference between their work and ours is that we consider explicit taxation on firms' cash flows rather than taxation on the agent's income from factor rental. Moreover, we believe that our results extend to cases where the economy's cash flows arise from an externality associated with the public expenditure.

Gentry and Hubbard (1997) distinguish three components of cash flows: expected risk-premium on investment (return to risk taking), inframarginal returns to investments (economic profits), and ex-post benefits of risky investments (actual outcome exceeding the ex-ante expected risk premium). Inframarginal utilities correspond to rents on inelastically supplied productive factors, one of which is public expenditure. Ex-post profits on risky investments, lucky outcomes that exceed expectations, are equivalent to rents. On the question of risk premium, these authors argue that the "market value" of risk premium is zero both for the investor and the government, since it only compensates for the risk taken by each of them. Accordingly, failure to tax risk premium implies that a risky investment would be under-taxed relative to a risk-free investment. Since cash flows are rents, a straightforward application of the Ramsey principle implies that taxing them does not distort the economy, though a formal demonstration is not attempted in this paper.

In a very different formal framework, Judd (1997) obtained a result in the same spirit. Using a dynamic general equilibrium model with monopolistic competition, where cash flows arise from market power, his solution to the optimal tax problem establishes that cash flows in final goods production should be fully taxed, while intermediate or capital goods purchases by consumable goods producers should be subsidized. In our model, the only intermediate good is financed by tax revenues and freely delivered to final good producers, so subsidized by definition, and the optimal tax policy we derive is precisely the 100% taxation of final good firms' cash flows. Also, Fullerton and Metcalf (1997) demonstrate that it is optimal to extract all rents associated with government actions. Hence the 100% tax on cash flows arising from externalities associated with the publicly supplied intermediate good, is in keeping with their result.

This paper also analyzes the deadweight loss associated with the optimal consumption tax.[1] Previously, we show that the optimal consumption tax rate is equal to the intensity of use of the intermediate good in production. As it should be expected, the higher the intensity of use of the intermediate good provided by the government, the higher its optimal supply, requiring a higher tax rate. When the rates of the consumption tax and the cash flow tax are set at their optimal levels, they collect the same percentage of the economy's product, which equals the economy's cash flows. Hence the superiority of the cash flow tax over the consumption tax is entirely due to the fact that the latter includes labor income in its tax base, while the former does not. Taxing labor affects both its supply and the accumulation of human capital, and thereby growth. In fact, human capital has an alternative use, namely leisure. Thus, a distortion occurs even when investment in human capital is deducted from the tax base.

We also find that the deadweight loss associated with the consumption tax grows with the weight of leisure in consumers' preferences, but decreases with the discount rate. In fact, when leisure is more attractive, the disincentive to work and to accumulate human capital caused by the labor tax is higher. On the other hand, since the consumption tax reduces the economy's steady state growth rate, when the consumer discounts the future more heavily, its impact on welfare becomes less significant.

The rest of the paper is organized as follows. Section 2 introduces the model, and section 3 solves it. Section 4 derives the optimal tax policy, while section 5 analyzes the deadweight loss arising from the consumption tax. The final section draws conclusions.

2. The Model

2.1. Production. We consider a one-good, one-factor economy. Human capital, which we denote as H, is the single factor. The good Y serves as both a final and as an intermediate product. Following Barro (1990), that the intermediate good, denoted as G, is a publicly-provided private good.[2] We assume competition among numerous firms, each of which receives a fraction of total G. All firms share the same Cobb-Douglas technology, so the production of firm j in period t is given by:

$$Y_{jt} = (\mu_j G_t)^{\alpha} (\nu_j N_t)^{1-\alpha}$$

where μ_j is the fraction of G received by firm j, N_t is aggregate human capital used in production, ν_j is the fraction of N_t hired by firm j, and $0 < \alpha < 1$. Perfect competition, along with constant-return-to-scale technology, implies that firm j will hire a fraction $\nu_j = \mu_j$ of N_t. Aggregate output is then given by

$$Y_t = G_t^{\alpha} N_t^{1-\alpha} \tag{1}$$

and the payment to human capital equals its marginal product:

$$w_t = (1-\alpha)\left(\frac{G_t}{N_t}\right)^{\alpha} = (1-\alpha)\frac{Y_t}{N_t}, \tag{2}$$

or equivalently,

$$w_t N_t = (1-\alpha) Y_t. \tag{3}$$

[1] Recent debate on so-called "fundamental tax reform" has focused on the advantages of introducing consumption taxes. See Hall and Rabushka (1985), Auerbach (1996), Gentry and Hubbard (1997), among others. A critical view is presented in Judd (1997).

[2] As discussed in Barro and Sala-i-Martin (1992), and Sala-i-Martin (1994), the model can be extended to consider non-excludable and non-rival (pure) public goods too. Here, as in Barro (1990), we assume individual firm's property rights to a specified quantity of public services G as given.

The good financed by the government generates cash flows in t equal to,

$$F_t = \alpha Y_t. \tag{4}$$

We also assume that human capital formation is a non-market activity. Individuals use their own human capital to produce more human capital with a linear technology as in Lucas' (1988) model.[3] So, if H_t denotes the stock of human capital at time t, the human capital accumulation process is described by

$$H_{t+1} - H_t = H_t - N_t - z_t H_t - \delta H_t, \tag{5}$$

where z_t corresponds to leisure time in period t, and δ is the human capital depreciation rate. Rearranging (5) we obtain

$$H_{t+1} = (2 - \delta - z_t)H_t - N_t. \tag{6}$$

2.2. Representative Consumer. In what follows we assume the existence of a single dynastic representative agent, whose overall utility is given by:

$$\sum_{t=0}^{\infty} \beta^t u(C_t, z_t),$$

where C_t is her/his consumption in period t and $0 < \beta < 1$ (the discount rate is $\frac{1}{\beta} - 1$). Note that the individual's utility depends on his/her consumption of "raw" leisure time z_t. We assume that the instantaneous utility function has constant intertemporal elasticity of substitution (CIES), i.e.

$$u(C_t, z_t) = \begin{cases} \dfrac{(C_t z_t^{\eta})^{(1-\theta)} - 1}{(1-\theta)} & \theta \neq 1 \\ \\ \log C_t + \eta \log z_t & \theta = 1 \end{cases}$$

where $\theta > 0$ is the inverse of the intertemporal elasticity of substitution. This functional form is standard in endogenous growth models with leisure.[4]

The income of the representative agent includes both labor income and cash flows from firms. The government can tax either labor income, or cash flows, or both. Let τ_h denote the labor income tax rate, and τ_f the cash flow tax rate. Then, the budget constraint of the representative consumer is:

$$C_t = (1 - \tau_h) w_t N_t + (1 - \tau_f) F_t. \tag{7}$$

Equations (2) through (4) and (7) imply

$$C_t = \frac{(1-\xi)}{(1-\alpha)} w_t N_t, \tag{8}$$

[3] The assumption of human capital formation as a non-market activity is not necessary for the main points pursued in this paper. Neither a cash flow tax nor a consumption tax (if properly defined) tax investment in accumulable factors. Extension to the case in which human capital formation is a market activity is analyzed in Milessi-Ferretti and Roubini (1995).

[4] See Lucas (1988), Jones et al (1993), Milessi-Ferretti and Roubini (1995), Corsetti and Roubini (1996), among others.

where

$$\xi\left(\tau_h, \tau_f\right) \equiv (1-\alpha)\tau_h + \alpha\tau_f. \tag{9}$$

We assume that the government balances its budget in each period and that tax revenues only finance the intermediate good G. Hence

$$G_t = \tau_h w_t N_t + \tau_f F_t = \xi Y_t.$$

Thus ξ is the share of product collected by government, which may be interpreted as the public sector's size. Substituting the previous expression for G_t in (1) gives $Y_t = \xi^{\frac{\alpha}{1-\alpha}} N_t$, which, together with equation (2), results in

$$w(\xi) \equiv (1-\alpha)\xi^{\frac{\alpha}{1-\alpha}}. \tag{10}$$

Thus, the payment to human capital w is constant over time and independent of consumer preferences. Moreover, w is an increasing function of $(1-\alpha)$, which measures the relative importance of human capital in production.

3. Solving the Model

3.1. The Steady State. The representative consumer chooses a path that solves his/her intertemporal optimization problem:

$$\max_{H_t, N_t, C_t, z_t} \sum_{t=0}^{\infty} \beta^t u(C_t, z_t) \tag{11}$$

subject to:

$$0 \leq n_t + z_t \leq 1$$

and

$$\lim_{t\to\infty} \frac{\partial u(C_t, z_t)}{\partial H_t} H_t = 0,$$

where $n_t \equiv N_t/H_t$ is the fraction of time the individual works. The first condition states that working time plus leisure cannot exceed the individual's total available time, and that the same condition holds for the time devoted to accumulating human capital. The second one is the transversality condition. The human capital accumulation equation (6) and the budget constraint (7) are used to solve for variables C_t and N_t. Assuming that the representative consumer sees cash-flow income as unaffected by her/his decisions and that he/she accumulates capital, the first Euler equation, which is derived by differentiating equation (11) with respect to z_t, is:

$$\frac{C_t}{z_t} - (1-\tau_h)\frac{w}{\eta} H_t = 0. \tag{12}$$

From (8) and (12), we conclude that

$$n_t = \lambda z_t,$$

where

$$\lambda(\tau_f, \tau_h) \equiv \left(\frac{1-\alpha}{\eta}\right)\left[\frac{1-\tau_h}{(1-\alpha)(1-\tau_h) + \alpha(1-\tau_f)}\right]. \tag{13}$$

Hence, the representative agent's working time is proportional to her/his leisure time. The proportionality factor λ increases with both $(1-\alpha)$ and the cash flow tax rate τ_f, but decreases with the labor tax rate τ_h. Given that $n = \lambda z$, equation (8) can be rewritten as:

$$C_t = \frac{\lambda(1-\xi)w(\xi)z}{1-\alpha} H_t = \lambda(1-\xi)z\xi^{\frac{\alpha}{1-\alpha}} H_t. \tag{14}$$

The expression for the second Euler equation, which results from differentiating (11) with respect to H_t, is

$$\beta(2-\delta-z_t)C_t^{-\theta} z_t^{\eta(1-\theta)} - C_{t-1}^{-\theta} z_{t-1}^{\eta(1-\theta)} = 0. \tag{15}$$

In steady state, growth rate and leisure time are both constant over time,[5] i.e.,

$$g \equiv \frac{C_{t+1}}{C_t} = \frac{H_{t+1}}{H_t} \quad \text{and} \quad z \equiv z_t$$

and equation (15) can be rewritten as:

$$A(z) = \frac{g^\theta}{\beta} \tag{16}$$

where

$$A(z) \equiv 2 - \delta - z. \tag{17}$$

Also, (6) implies

$$g = 2 - \delta - (z+n) = 2 - \delta - z(1+\lambda). \tag{18}$$

Combining (17) and (18) we obtain

$$A(z) = \frac{\lambda}{1+\lambda}(2-\delta) + \frac{1}{1+\lambda} g. \tag{19}$$

Hence, (16) and (19) imply that the steady state growth rate $g-1$ is characterized by the following equation:

$$\frac{g^\theta}{\beta} = \frac{\lambda}{1+\lambda}(2-\delta) + \frac{1}{1+\lambda} g. \tag{20}$$

Equation (20) can be expressed as a fixed point equation:

$$g = f(g; \theta, \beta, \lambda) \tag{21}$$

where

$$f(g; \theta, \beta, \lambda) \equiv \frac{(1+\lambda)}{\beta} g^\theta - \lambda(2-\delta). \tag{22}$$

Equation (18) determines that the domain of function $f(g; \theta, \beta, \lambda)$ is $[1-\delta, 2-\delta]$.

As in most endogenous growth models, a restriction on the parameters is needed to ensure the existence and uniqueness of a steady state solution for this economy, as follows:

Assumption (a1) $\beta(2-\delta)^{1-\theta} < 1$.

This assumption implies that the transversality condition is satisfied even for parameters such that the economy grows at its maximum rate $g = 2-\delta$. In fact, the transversality condition can be expressed in terms of g as $\beta g^{1-\theta} < 1$. Observe that, since $(2-\delta) > 1$ and $\beta < 1$, Assumption (a1) is trivially satisfied for all $\theta \geq 1$. Empirical evidence supports values of θ satisfying the later. For instances, Lucas (1990) uses $\theta = 2.2$.

[5] In a on factor endogenous growth model there are no transitional dynamics and the economy is always in a steady state.

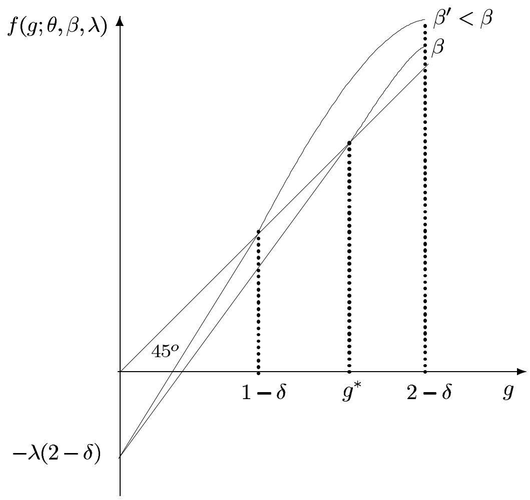

Figure 1a: Steady state growth rate, $\theta < 1$.

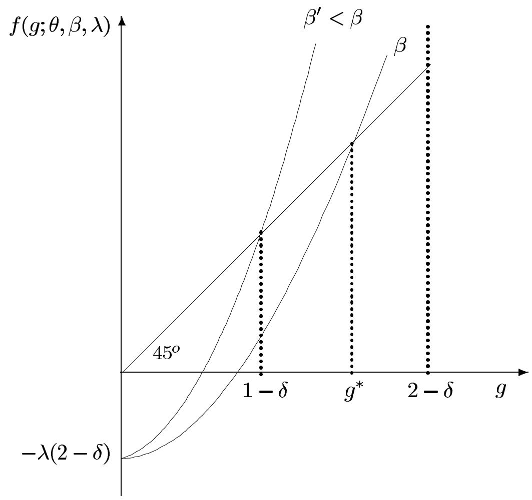

Figure 1b: Steady state growth rate, $\theta > 1$.

Figure 1: Steady state growth rate.

Lemma 1. *If Assumption (a1) holds, then there is a unique fixed point g in the range $[0, 2-\delta]$. Moreover at this point $f'(g;\theta,\beta,\lambda) > 1$.*

Proof. For any given $\theta, \lambda > 0$, and substituting for g in equation (22),

$$f(g = 2-\delta;\theta,\beta,\lambda) - 2 - \delta = (1+\lambda)\left[\frac{(2-\delta)^\theta}{\beta} - (2-\delta)\right].$$

hence

$$f(g = 2-\delta;\theta,\beta,\lambda) - 2 - \delta = \frac{(1+\lambda)(2-\delta)^\theta}{\beta}\left[1 - \beta\,(2-\delta)^{1-\theta}\right].$$

Thus, from Assumption (a.1), it follows

$$f(g = 2-\delta;\theta,\beta,\lambda) > 2-\delta.$$

Since for any θ, $\lambda > 0$, function $f(g;\theta,\beta,\lambda)$ is strictly increasing in g, and given that $f(g = 0;\theta,\beta,\lambda) = -\lambda(2-\delta)$, it necessarily cuts the 45 degree line once and from below; this concludes the proof. ■

Notice that as β diminishes, function f shifts upwards and the value of the fixed point decreases (see Figure 1). Eventually, if the representative agent strongly discounts the future, i.e., for a small enough β, he/she will prefer not to accumulate human capital and the initial stock will diminish at the depreciation rate δ. It can be shown that if the fixed point of f is smaller than $(1-\delta)$, then the equilibrium growth rate is simply $g = 1-\delta$, corresponding to the corner solution of the agent's intertemporal optimization problem in which $n_t + z_t = 1$ (i.e. no time is devoted to human capital accumulation).

Lemma 2. *If Assumption (a1) holds, then an increase in variable λ leads the economy to a higher growth rate, but less leisure time.*

Proof. Implicit differentiation in (21) results in

$$\frac{dg}{d\lambda} = f'(g;\theta,\beta,\lambda)\frac{dg}{d\lambda} + \frac{\partial f}{\partial \lambda}. \Rightarrow \frac{dg}{d\lambda} = \frac{-\frac{\partial f}{\partial \lambda}}{f'(g;\theta,\beta,\lambda)-1}.$$

Differentiating equation (22) and introducing equations (16) and (17) leads to

$$\frac{df}{d\lambda} = \frac{g^\theta}{\beta} - (2-\delta) = A(z) - (2-\delta) = -z.$$

Hence,

$$\frac{dg}{d\lambda} = \frac{z}{f'(g;\theta,\beta,\lambda)-1},$$

which is strictly positive since $f'(g;\theta,\beta,\lambda) > 1$ (Lemma 1). Furthermore, from equation (18) it follows that

$$\frac{dg}{d\lambda} = -(1+\lambda)\frac{dz}{d\lambda} - z,$$

which entails

$$\frac{dz}{d\lambda} = -\frac{\frac{dg}{d\lambda}+z}{(1+\lambda)} = -\frac{zf'}{(1+\lambda)(f'-1)} < 0. \tag{23}$$

■

Proposition 3. *If Assumption (a1) holds, then an increase (decrease) in the labor tax rate lowers (raises) the rate of growth, but increases (decreases) leisure time. Changes in the cash flow tax have opposite effects.*

Proof. The proof is immediate from lemma 2 and the fact that λ is increasing in the cash flow tax rate and decreasing in the labor tax rate. ■

4. THE OPTIMAL TAX STRUCTURE

When the transversality condition is satisfied, the representative agent's welfare for any $\theta \neq 1$ is given by

$$\overline{v} = \sum_{t=0}^{\infty} \beta^t u(C_t, z_t) = \frac{\overline{u} - \frac{1}{(1-\beta)}}{1-\theta}, \tag{24}$$

where

$$\overline{u} = \left(\frac{A(z)}{A(z)-g}\right)(C_0 z^\eta)^{1-\theta}.$$

Substituting C_0 (equation (14)) and recalling that $A(z) - g = n = \lambda z$, then

$$\overline{u} = \frac{A(z)}{\lambda z}\left[\lambda(1-\xi)\xi^{\frac{\alpha}{1-\alpha}} z^{(1+\eta)} H_0\right]^{1-\theta}.$$

Rearranging terms:

$$\overline{u} = A(z) z^{\eta(1-\theta)-\theta} \lambda^{-\theta}\left[(1-\xi)\lambda(1-\xi)\xi^{\frac{\alpha}{1-\alpha}} z^{(1+\eta)} H_0\right]^{(1-\theta)} (H_0)^{(1-\theta)}$$

Defining,

$$J(\lambda) \equiv A(z(\lambda))z(\lambda)^{\eta(1-\theta)-\theta}\,\lambda^{-\theta}, \tag{25}$$

and

$$M(\xi) \equiv \left[(1-\xi)\xi^{\frac{\alpha}{1-\alpha}}\right]^{(1-\theta)},$$

it follows that

$$\overline{u} = J(\lambda)M(\xi)\left(H_0\right)^{(1-\theta)},$$

from which equation (24) becomes

$$\overline{v} = \frac{J(\lambda)M(\xi)\left(H_0\right)^{(1-\theta)}}{1-\theta} - \frac{1}{(1-\theta)(1-\beta)}. \tag{26}$$

We now examine the problem faced by a benevolent planner who chooses the tax structure (τ_f, τ_h) that maximizes the representative agent's welfare given by (26).

Lemma 4. *If $\theta < 1$, function $M(\xi)$ attains its maximum at $\xi = \alpha$. For $\theta > 1$, $\xi = \alpha$ is a minimum of $M(\xi)$.*

Proof. Straightforward differentiation of $M(\xi)$ shows has an extreme point at $\xi = \alpha$. In fact,

$$M'(\xi) = \left[\frac{(1-\theta)}{(1-\alpha)(1-\xi)\xi}\right](\alpha-\xi)M(\xi)$$

has a zero at $\xi = \alpha$. Moreover, $\forall\theta \neq 1$, $\dfrac{M'(\xi)}{1-\theta}$ is positive for $\xi < \alpha$ and negative for $\xi > \alpha$, which concludes the proof. ■

Lemma 5. *If $\theta < 1$, function $J(\lambda)$ attains its maximum at $\lambda = 1/\eta$. For $\theta > 1$, $\lambda = 1/\eta$ is a minimum of $M(\xi)$.*

Proof. See the Appendix. ■

Proposition 6. *For the economy described above, the optimal tax structure is $\tau_h^* = 0$ and $\tau_f^* = 1$.*

Proof. Functions $\xi(\tau_f, \tau_h)$ and $\lambda(\tau_f, \tau_h)$ define a regular correspondence between the pairs (τ_f, τ_h) and (ξ, λ). We notice that $\xi(\tau_f = 1, \tau_h = 0) = \alpha$ and $\lambda(\tau_f = 1, \tau_h = 0) = \frac{1}{\eta}$. Given this change of variables and observing from equation (26) that $\frac{J(\lambda)M(\xi)(H_0)^{(1-\theta)}}{1-\theta}$ is positive (negative) for $\theta < 1$ ($\theta > 1$), the result follows from Lemmas 4 and 5. ■

As the base of the cash flow tax consists of rents, the tax does not distort the economy. Moreover, since the rents are generated by public expenditure, it is optimal to extract them fully from firms. Taxing labor, on the other hand, distorts the work-leisure decision, causing a deadweight loss.

Case 7. *When $\theta = 1$, Assumption (a1) is immediately satisfied and g can be explicitly solved from (16):*

$$g(\lambda) = \frac{(2-\delta)\beta\lambda}{(1-\beta)+\lambda} \tag{27}$$

and

$$z(\lambda) = \frac{(2-\delta)(1-\beta)}{(1-\beta)+\lambda}. \tag{28}$$

It can be shown that for $\theta = 1$, the representative agent's welfare is given by

$$\overline{v} = \sum_{t=0}^{\infty} \beta^t u(C_t, z_t) = \frac{\log\left(g^\beta (C_0 z^\eta)^{1-\beta}\right)}{(1-\beta)^2}. \tag{29}$$

Using equations (14), (27) and (28), it follows that

$$\overline{v} = \frac{\log(J_\beta(\lambda) M_\beta(\xi) B)}{(1-\beta)^2}$$

where $B = (H_0)^{(1-\beta)}(2-\delta)^{\eta(1-\beta)+1}(1-\beta)^{(1+\eta)(1-\beta)}\beta^\beta$, does not depend on tax rates, $M_\beta(\xi)$ is exactly the same as $M(\xi)$ substituting β for θ, and

$$J_\beta(\lambda) = \lambda\left[(1-\beta)+\lambda\right]^{-[1+\eta(1-\beta)]}.$$

Since

$$J'_\beta(\lambda) = J_\beta(\lambda)\frac{(1-\beta)(1-\eta\lambda)}{\lambda\left[(1-\beta)+\lambda\right]},$$

$J_\beta(\lambda)$ now attains its maximum at $\lambda = \frac{1}{\eta}$, just as $J(\lambda)$. Therefore, Proposition 6 also holds when $\theta = 1$.

5. The deadweight of the consumption tax

We examine the deadweight loss associated with the consumption tax in this economy by comparing it to the cash flow tax. An immediate corollary of Proposition 6 is the superiority of the cash flow tax over the consumption tax, independently of the rate of the latter. In fact, the consumption tax implies a common rate for both labor income and cash flows, i.e. $\tau_c \equiv \tau_h = \tau_f$. Thus the optimal tax structure cannot be achieved for any value of τ_c.

Proposition 8. *The optimal consumption tax rate is $\tau_c^* = \alpha$.*

Proof. For the consumption tax $\tau_c \equiv \tau_h = \tau_f$. Then from equations (9) and (13) $\lambda^c \equiv \lambda(\tau_c, \tau_c) = \frac{1-\alpha}{\eta}$, and $\xi^c \equiv \xi(\tau_c, \tau_c) = \tau_c$. Thus both $\lambda^c$and $\xi^c$are independent of the tax rates, which implies that $J(\lambda)$ is independent of the tax structure. Accordingly, in this case tax optimization reduces to finding τ_c that maximizes $M(\tau_c)$, and the result follows from Lemma 4. ■

This result establishes that the optimal consumption tax rate is equal to the intensity of use of factor G in production. For higher values of α, the publicly supplied good has a larger impact on the economy, and optimality requires higher consumption tax rates to finance larger supplies of the intermediate good. Again, this result highlights the importance of the production externality associated with public expenditure G.

Moreover, tax revenue collection with the optimal consumption tax equals the economy's cash flows αY, the same collection attained with the optimal cash flow tax. In other words, with both taxes the efficient size of the public sector $\xi = \frac{G}{Y}$ equals α, as in Barro and Sala-i-Martin (1992). The intuition of this result is direct. The private marginal productivity of the public good is $\frac{\partial Y}{\partial G} = \alpha\frac{Y}{G} = \frac{\alpha}{\xi}$. As argued by these authors, the efficiency condition is simply $\frac{\partial Y}{\partial G} = 1$, because "producing" one unit of public services G requires one unit of output. From another perspective, if the economy were entirely private and there existed a competitive market for good G, its unit price would be one.

While with both taxes the optimality of the tax rate ensures that the private marginal productivity of the intermediate good G coincides with its social marginal productivity, this is not necessarily true for the accumulable factor. In fact, the private return to investment in human capital differs from the social return by a factor $(1 - \tau_h)$. Thus, with a consumption tax $\tau_f = \tau_h = \alpha$, the private return to investment is lower than the social return, and henceforth he human capital accumulation falls short the socially efficient level In contrast, with a cash-flow tax $\tau_h = 0$, and Pareto optimality is achieved in a decentralized economy.[6]

Finally, we are interested in analyzing what variables the deadweight loss caused by the optimal consumption tax depend on. For reasons of space, we focus on the simplest case which corresponds to $\theta = 1$. However, the conclusions drawn in the analysis are valid for any admissible value of θ.

For the optimal cash-flow tax rate $\tau_f^* = 1$ $(\tau_h \equiv 0)$, $\lambda^f \equiv \lambda(0,1) = \frac{1}{\eta}$. Substituting these values in equations (27) and (28) gives

$$g^f = \frac{(2-\delta)\beta\eta}{1+\eta(1-\beta)} \text{ and } z^f = \frac{(2-\delta)(1-\beta)\eta}{1+\eta(1-\beta)}.$$

For the optimal consumption tax $\tau_c^* = \alpha$, $\lambda^c \equiv \lambda(\alpha,\alpha) = \frac{1-\alpha}{\eta}$. Then the corresponding values are:

$$g^c = \frac{(2-\delta)\beta(1-\alpha)\eta}{1-\alpha+\eta(1-\beta)} \text{ and } z^c = \frac{(2-\delta)(1-\beta)\eta}{1-\alpha+\eta(1-\beta)}.$$

Replacing the expressions for g and z in (29), we can compute the welfare differential between the cash flow tax and the consumption tax $\Delta\overline{v} \equiv \overline{v}^f - \overline{v}^c$, to obtain

$$\Delta\overline{v} = \frac{\left\{\log\left(\frac{1}{1-\alpha}\left[\frac{1-\alpha+\eta(1-\beta)}{1+\eta(1-\beta)}\right]^{\eta(1-\beta)+1}\right)\right\}}{(1-\beta)^2}.$$

It can be seen that $\Delta\overline{v}$ is always positive. Let x denote the expression $\eta(1-\beta)$; then when $x = 0$, $\Delta\overline{v} = 0$. Moreover $\Delta\overline{v}$ rises with x. Thus the greater the weight of leisure in the utility function the greater deadweight loss arising from the consumption tax. The reason is that the greater the weight of leisure in the consumer's preferences, the disincentives to both work and accumulate capital, caused by a labor tax, are higher. In addition,

$$\frac{\partial\Delta\overline{v}}{\partial\alpha} = \frac{\alpha\eta}{(1-\beta)} > 0.$$

Parameter α measures the intensity of the intermediate good G in production, indicating the magnitude of the externality generated by the public expenditure. Thus, the previous result states that the deadweight loss caused by the consumption tax increases with the externality associated with public expenditure.

---

[6] This result reflects an important difference between our setting and Barro´s original model. Barro (1990, 1992) does not distinguish between labor income and cash flows, and consequently considers a unique income tax rate. Therefore, in his model, even if the public sector's size is optimal, the decentralized economy cannot achieve the efficient solution.

Figure 2: The deadweigth loss of consumption as function of β.

Finally, as illustrated by Figure 2, the deadweight loss associated with the consumption tax is greater when the representative consumer discounts the future less heavily, i.e. when β is closer to 1. In fact, the limit of $\Delta\overline{v}$ when β tends to zero is infinity. The reason for this is that the consumption tax reduces the economy's steady state growth rate, so when the consumer discounts the future less its impact on welfare becomes more significant.

6. Conclusion

This paper finds that the optimal labor and cash flows tax rates are 0% and 100%, respectively. A corollary of this result is the superiority of the optimal cash flow tax over a consumption tax. The former result is not surprising since cash flows derive from rents, while the consumption tax distorts the work/leisure decision by taxing labor income.

These results have been proved for an specific model, in which cash flows arise from productive government spending; but they should also extend to more general settings. It would be interesting to prove the result when cash flows arise from the existence of market power or ex-post returns to risky investments. Generalizing the model to include a second accumulable factor of production (e.g. physical capital) is also left for future research.

Acknowledgment. This research was financially supported by the Chilean National Science Foundation (CONICYT). We have benefitted from comments by the participants at the Public Economics Seminar, Universitat de Pompeu Favre.

REFERENCES

AUERBACH, A., 1997. "The Future of Fundamental Tax Reform." *American Economic Review* 87(2):143-146.

BARRO, R., 1990. "Government Spending in a Simple Model of Endogenous Growth", *Journal of Political Economy* 98(5): 103-125.

BARRO, R. and X. SALA-I-MARTIN, 1992. "Public Policy in Economic Growth Models". *Review of Economic Studies* 59: 645-661.

CORSETTI, G. and N. ROUBINI, 1996. "Optimal Government Spending and Taxation in Endogenous Growth models", *NBER Working Paper 5851*, December.

FULLERTON, D. and G.E. METCALF, 1997. "Environmental Taxes and the Double-Dividend Hypothesis: Did You Really Expect Something Out of Nothing", *NBER Working Paper 6199*.

GENTRY, W. and R.G. HUBBARD, 1997. "Distributional Implications of Introducing a Broad-Based Consumption Tax" in J. James Poterba (Ed.), Tax Policy and the Economy, Vol.11, Cambridge. MA: MIT Press.

HALL, R. and A. RABUSHKA, 1985. "The Flat Tax", Hoover Institution Press, Stanford, California.

JUDD, K.L., 1997. "The Optimal Tax Rate for Capital Income is Negative", *NBER Working Paper* 6004, April.

LUCAS, R., 1990. "Supply-side economics: An analytical review", *Oxford Economic Papers* 42: 293-316.

LUCAS, R., 1988. "In the Mechanics of Economic Development", *Journal of Monetary Economics* 22(1):3-42.

MILESSI-FERRETI, G.M. and N. ROUBINI, 1995. "Growth Effect of Income and Consumption Taxes: Positive and Normative Analysis", *NBER Working Paper* 5317, October.

RAMSEY, F.P., 1927. "A contribution to the theory of taxation", *Economic Journal* 37: 47-61.

REBELO, S., 1991. "Long-Run Policy Analysis and Long-Run Growth," *Journal of Political Economy* 94: 1002-1037.

SALA-I-MARTIN, X., 1994. "Apuntes de crecimiento económico". Antoni Bosch Editor, Barcelona.

APPENDIX: PROOF OF LEMMA.

Lemma 5 *If $\theta < 1$, function $J(\lambda)$ attains its maximum at $\lambda = 1/\eta$. For $\theta > 1$, $\lambda = 1/\eta$ is a minimum of $M(\xi)$.*

Lemma 9. ***Proof.*** *We can rewrite equation (25) as:*

$$J(\lambda) \equiv L\left(z(\lambda)\right)\lambda^{-\theta},$$

where

$$L(z) \equiv A(z)z^{\eta(1-\theta)-\theta}.$$

Differentiating $J(\lambda)$ results in:

$$J'(\lambda) = \frac{\partial L}{\partial z}\frac{\partial z}{\partial \lambda}\lambda^{-\theta} - \theta L\lambda^{-\theta-1} = \lambda^{-\theta-1}\left[\lambda\frac{\partial L}{\partial z}\frac{\partial z}{\partial \lambda} - \theta L\right].$$

From the definition of $L(z)$ and equation (17)

$$\frac{\partial L}{\partial z} = -\left(\frac{\theta - \eta + \theta\eta}{z} + \frac{1}{A}\right)L.$$

Now, differentiating equation (22) and recalling equation (16), we obtain

$$f'(g) = (1+\lambda)\theta\frac{g^{\theta-1}}{\beta} = (1+\lambda)\theta\frac{A}{g}. \tag{30}$$

Substituting the above expression for $f'(g)$ in equation (23), results in:

$$\frac{\partial z}{\partial \lambda} = -\frac{\theta A z}{g(f'-1)}.$$

Hence

$$\lambda\frac{\partial L}{\partial z}\frac{\partial z}{\partial \lambda} = \frac{\lambda\left[(\theta-\eta+\theta\eta)A + z\right]}{g(f'-1)}\theta L,$$

from which it follows that

$$J'(\lambda) = \theta L\lambda^{-\theta-1}\left[\frac{\lambda\left[(\theta-\eta+\theta\eta)A + z\right]}{g(f'-1)} - 1\right] = \frac{\theta L\lambda^{-\theta-1}}{g(f'-1)}\left[\lambda(\theta-\eta+\theta\eta)A + \lambda z + g - gf'\right].$$

From equations (17) and(18) $A(z) = \lambda z + g$ and from equation (30) $gf' = (1+\lambda)\theta A$, hence we can rewrite $J'(\lambda)$ as follows:

$$J'(\lambda) = \frac{\theta AL\lambda^{-\theta-1}}{g(f'-1)}\left\{\lambda(\theta-\eta+\theta\eta) + 1 - (1+\lambda)\theta\right\} = \frac{(1-\theta)\theta AL\lambda^{-\theta-1}}{g(f'-1)}\left[(1+\lambda) - (1+\eta)\lambda\right]$$

and finally obtain

$$J'(\lambda) = \frac{(1-\theta)\theta AL\lambda^{-\theta-1}}{g(f'-1)}(1-\lambda\eta).$$

Then function $J(\lambda)$ has an extreme point at $\lambda = \frac{1}{\eta}$. Lemma 1 ensures that $f'(g;\theta,\beta,\lambda) > 1$; therefore, $\forall\theta \neq 1$, $\dfrac{J'(\lambda)}{1-\theta}$ is positive for $\lambda < \frac{1}{\eta}$, and negative for $\lambda > \frac{1}{\eta}$, which concludes the proof. ■