EXTREMAL DEPENDENCE IN EXCHANGE RATE MARKETS

Viviana Fernandez[1]

Abstract

Exchange rate markets exhibit correlation in the short run, but the issue is whether such correlation lingers over long periods of time, and under extreme events (i.e., either large appreciations or depreciations). In this paper, we analyze dependence between nominal exchange rates under extreme events for a sample of ten countries with dirty/free float regimes over the period 1998-2002. In addition, we investigate whether currencies have exhibited extremal dependence on the Euro, since its adoption in 1999. Our findings are the following. First, in general, there is no evidence of extremal dependence between returns pairs. Second, the degree of dependence is stronger under large appreciations than under large depreciations. These conclusions are robust to filtering out the data for serial correlation and heteroscedasticy.

JEL classification: C22, G10 Keywords: extremal dependence, DVEC models.

[1] Department of Industrial Engineering at the University of Chile. Postal: Avenida Republica 701, Santiago-Chile. E-mail: vfernand@dii.uchile.cl. Funds provided by Grant DID No. Soc-01/19-2 (Department of Research and Development of the University of Chile), and by an institutional grant of the Hewlett Foundation are greatly acknowledged. All remaining errors are the author's.

I Introduction

There is a bulk of literature devoted to market co-movements. In a recent article, Forbes and Rigobon (2002) distinguish between contagion and interdependence. Contagion is defined as a significant increase in cross-market linkages after a shock to one country or to a group of countries takes place. If the co-movement does not increase significantly, but a high level of correlation persists in all periods, then the correct definition for this situation is interdependence.

One of the simplest ways to measure association between assets returns is by the Pearson correlation coefficient. However, this presents several flaws. First, it is only appropriate for detecting linear association between two random variables. Second, as Forbes and Rigobbon op cit. discuss, tests of contagion are biased under the presence of heterocedastic returns. Finally, given that the Pearson correlation coefficient is constructed from deviations from the sample mean, the weight given to extreme observations is the same as that given to all the other observations. Therefore, it is not an accurate measure of dependence if extreme observations present different patterns of dependence from the rest of the sample.

An alternative approach can be found in the extreme value theory (EVT). EVT studies the stochastic behavior of a process at unusually large or small levels. In particular, extremal dependence focuses on assets returns association under events which are more extreme than any other that has been previously observed. This concept differs from the definition of contagion or interdependence in that only extreme observations are taken into account. That is to say, the EVT approach does not look for a global measure of association between returns, but for a measure of dependence under unlikely scenarios over a long time-horizon (i.e. in large samples or asymptotically). However, one could analyze the existence of contagion under EVT by testing whether extremal dependence between two financial markets significantly increases after a shock occurs.

There are two types of extreme-value dependence: asymptotic dependence and asymptotic independence. Both forms of dependence allow dependence between relatively large values of each variable, but the largest values from each variable can take place jointly only when the variables are asymptotically dependent (see, for example, Coles, Heffernan and Twan, 1999).

The existing literature in extreme value theory has primarily focused on the case of asymptotic dependence. However, if the series are asymptotically independent, this approach will overestimate extremal dependence and, therefore, the extent of financial risk. The degree of overestimation will depend upon the degree of asymptotic independence. This issue is discussed in a recent article by Poon, Rockinger, and Tawn (2003). Poon et al. control for asset returns heterocedasticity before testing for extremal dependence. Their estimation results show that tail dependence decreases when filtering out heteroscedasticity by univariate and bivariate GARCH models. In addition, Poon et al. find that extremal dependence is usually stronger in bear markets (left tails) than in bull markets (right tails).

This study focuses on dependence under extreme events in exchange rates markets under dirty/free float regimes—Argentina, Brazil, The United States, Chile, Japan, Mexico, New Zealand, Peru, South Korea and Thailand—over the sample period 1998-2002. In addition, we analyze whether these countries' currencies have exhibited extremal dependence on the Euro, since its adoption in 1999. Our findings show, in general, no evidence of extremal dependence between returns pairs, and that the degree of dependence is stronger under large appreciations than under large depreciations. These conclusions are robust to filtering out the data for serial correlation and heteroscedasticy by multivariate GARCH models.

This paper is organized as follows. Section II presents a theoretical background on dependence under extreme events. Section III presents descriptive statistics of the data and the estimation results. Finally, Section IV presents our main conclusions.

II Theoretical Background: Returns Dependence using EVT

Modeling only the probability distribution of the maximum or the minimum of a sample is inefficient if other data on extreme values are available. Therefore, an alternative approach consists of modeling the behavior of extreme values above a high threshold ("Peaks over threshold" or POT). Let us define the excess distribution above the threshold u as the conditional probability

$$F_u(y) = \Pr(X - u \le y \mid X > u) = \frac{F(y+u) - F(u)}{1 - F(u)}, \quad y > 0 \qquad (1)$$

Under some regularity conditions, there exists a positive function $\beta(u)$, for large enough u, such that (1) is well approximated by the generalized Pareto distribution (GPD)

$$H_{\zeta,\beta(u)}(y) = \begin{cases} 1 - \left(1 + \dfrac{\zeta y}{\beta(u)}\right)^{-1/\zeta} & \zeta \neq 0 \\ 1 - \exp\left(-\dfrac{y}{\beta(u)}\right) & \zeta = 0 \end{cases} \qquad (2)$$

where $\beta(u)>0$, and $y\geq 0$ when $\zeta\geq 0$, and $0\leq y\leq -\beta(u)/\zeta$ when $\zeta<0$ (see, for example, Coles, 2001, or Embrechts, Klüpperberg and Mikosch, 1997). If $\zeta>0$, F is in the Fréchet family and $H_{\zeta,\beta(u)}$ is a Pareto distribution; if $\zeta=0$, F is in the Gumbell family and $H_{\zeta,\beta(u)}$ is an exponential distribution; finally, if $\zeta<0$, F is in the Weibull family and $H_{\zeta,\beta(u)}$ is a Pareto type II distribution. In most applications of risk management, the data comes from a heavy-tailed distribution, so that $\zeta>0$.

Poon, Rockinger, and Tawn (2003) introduce a special case of threshold modeling connected with the generalized Pareto distribution, for the Fréchet case. For this particular case, the tail of a random variable Z above a (high) threshold u can be approximated as

$$1 - F(z) = \Pr(Z > z) \sim z^{-1/\eta} L(z) \qquad \text{for } z>u \qquad (3)$$

where L(z) is a slowly varying function of z,[2] and $\eta>0$. If treated as a constant for all z>u, that is L(z)=c, and under the assumption of n independent observations, the maximum-likelihood estimators for η and c are

$$\hat{\eta} = \frac{1}{n_u}\sum_{j=1}^{n_u} \log\left(\frac{z_{(j)}}{u}\right) \qquad \hat{c} = \frac{n_u}{n} u^{1/\hat{\eta}}, \qquad (4)$$

where $z_{(1)},\ldots, z_{(n_u)}$, are the n_u observations above the threshold u. $\hat{\eta}$ is known as the Hill estimator.

The asymptotic variance of $\hat{\eta}$ is given by $\operatorname{avar}(\hat{\eta}) = \frac{\eta^2}{n_u}$, whereas the asymptotic variance of $\hat{c}$ can be obtained by the delta method, avar($\hat{c}$)=$\frac{n_u}{n^2}\frac{u^{2/\eta}\log^2(u)}{\eta^2}$.

The first step is to transform the original variables to a common marginal distribution. Let (X,Y) be bivariate returns with corresponding cumulative distribution functions F_X and F_Y. The bivariate returns are transformed to unit Fréchet marginals (S, T) using the transformation

$$S = -\frac{1}{\ln F_X(X)} \qquad T = -\frac{1}{\ln F_Y(Y)} \qquad S>0,\ T>0. \qquad (5)$$

Under this transformation, $\Pr(S>s)=\Pr(T>s)\sim s^{-1}$. As both S and T are on a common scale, the events {S>s} and {T>s}, for large values of s, correspond to equally extreme events for each one. Given that $\Pr(S>s)\to 0$ as $s\to\infty$, it is natural to consider the conditional probability $\Pr(T>s|S>s)$ for large s. If (S,T) are perfectly dependent, $\Pr(T>s|S>s)=1$. By contrast, if (S,T) are exactly independent, $\Pr(T>s|S>s)=\Pr(T>s)$, which tends to zero as $s\to\infty$. Let us define

$$\chi = \lim_{s\to\infty} \Pr(T > s \mid S > s) \qquad 0\le\chi\le 1\ . \qquad (6)$$

Two random variables are called asymptotically dependent if $\chi>0$, and asymptotically independent if $\chi=0$. In other words, χ measures the degree of dependence that lingers in the limit. Nonetheless, random variables, which are asymptotically independent, may show different degrees of dependence for finite levels of s. Based on this fact, Coles, Heffernan and Twan (1999) proposed the following measure of dependence

[2] A function on L on (0, ∞) is slowly varying if $\lim_{x\to\infty} L(tz)/L(z)=1$ for t>0.

$$\bar{\chi} = \lim_{s\to\infty} \frac{2\log(\Pr(S>s))}{\log(\Pr(S>s,T>s))} - 1 \qquad -1 < \bar{\chi} \le 1. \tag{7}$$

This is a well-defined measure of asymptotic independence as it gives the rate at which Pr(T>s|S>s)→0. Values of $\bar{\chi}$ >0, $\bar{\chi}$ =0 and $\bar{\chi}$ <0 are approximate measure of positive dependence, exact independence, and negative dependence.

The pair $(\chi, \bar{\chi})$ provides all the necessary information to characterize both the form and degree of extreme dependence. For asymptotically dependent variables, $\bar{\chi}$ =1 and the degree of dependence is measured by χ>0. For asymptotic independent variables, χ=0 and the degree of dependence is measured by $\bar{\chi}$. Therefore, one should first test if $\bar{\chi}$ =1 before reaching any conclusion about the dependence based on χ.

It can be shown that

$$\Pr(S>s,\ T>s) \sim L(s)s^{-1/\zeta} \qquad \text{as } s\to\infty, \tag{8}$$

where 0<ζ≤1 and L(s) is a slowly varying function. Given that $\Pr(S>s) \sim s^{-1}$, $\bar{\chi}$ boils down to $\bar{\chi} = 2\zeta - 1$.

The test of dependence is implemented by letting Z=min(S,T), and noting that

$$\begin{aligned} \Pr(Z>z) &= \Pr\{\min(S,T)>z\} \\ &= \Pr(S>z,\ T>z) \\ &= L(z)\ z^{-1/\zeta} \\ &= d\ z^{-1/\zeta} \qquad \text{for } z>u, \end{aligned} \tag{9}$$

for some high threshold u. The above equation shows that ζ is the tail index of the univariate random variable Z. Therefore, it can be obtained by the Hill estimator, constrained to the interval (0, 1]. The scale parameter d can be computed as explained earlier.

Under the assumption of independent observations on Z, we have

$$\hat{\chi} = 2\hat{\zeta} - 1 = \frac{2}{n_u}\left(\sum_{j=1}^{n_u} \log\left(\frac{z_{(j)}}{u}\right)\right) - 1 \qquad \mathrm{Var}(\hat{\chi}) = \frac{(\hat{\chi}+1)^2}{n_u}, \tag{10}$$

where $\hat{\chi}$ is asymptotically normal.

The decision rule is: if $\hat{\chi}$ is significantly less than 1, that is, if $\hat{\chi} + 1.96\sqrt{\mathrm{Var}(\hat{\chi})} < 1$, we conclude that the variables are asymptotically independent and take χ=0. In case there is no enough evidence to reject the null hypothesis $\bar{\chi} = 1$, we estimate χ under the assumption that $\bar{\chi} = \xi = 1$. In such case, $\hat{\chi} = \frac{un_u}{n}$ and $\mathrm{Var}(\hat{\chi}) = \frac{un_u(n - n_u)}{n^3}$.

III Description of the Data and Estimation Results

3.1 The data

The section deals with extremal dependence of exchange rates in a group of ten countries, which are characterized by dirty/free float regimes.[3] The sample is comprised by Brazil, Chile and Peru from South America, Mexico and The United States from North America, Japan, South Korea, and Thailand from Asia, and Australia and New Zealand from Australia/Oceania. Table 1 gives some development indicators of these ten nations for 2001. Australia, The United States and Japan stand out as the most developed with high per-capita incomes, high penetration of technology—measured by the number of personal computers per 1,000 people and internet users over population, and high-technology exports as a percentage of manufactured exports. On the other extreme are Peru and Thailand, both of which have per-capita incomes below $2,000 a year, relatively high infant mortality rates (per 1,000 live birth), and low use of technology. However, Thailand's exports are high-technology oriented when compared with the whole sample of countries.

All countries are characterized by relatively low inflation rates, as measured by the annual growth of the GDP implicit price deflator. In particular, Brazil, which suffered from chronic hyperinflation in the 1980's, had a one-digit inflation rate in 2001.

[Table 1 about here]

Table 2 in turn shows descriptive of the data series. All of them were obtained from the web site of the Bank of Canada. The sample period is January 1998-December 2002, and the data frequency is daily. When analyzing the dependence on the Euro, the sample shortens to January 1999-December 2002. The exchange rates are expressed in U.S. dollars, and the returns are continuously compounded. Mean returns are zero for all countries, except for Brazil for which the sample mean is slightly negative. All returns series strongly reject the assumption of normality, according to the Jarque-Bera test. Brazil, South Korea and Thailand stand out for the high kurtosis of their returns. Autocorrelation coefficients are not statistically significant in general, which suggests that returns are close to white noise from one day to the next. The evolution of the return series is shown in Figure 1.

[Table 2 and Figure 1 about here]

Exchange rate markets exhibit correlation in the short run, but the issue is whether such correlation lingers over long periods of time, and under extreme events (i.e., either large appreciations or depreciations). For instance, the Chilean peso is very sensitive to the evolution of the Brazilian real, and the Pearson correlation coefficient between the two

[3] A dirty float is a type of floating exchange rate that is not completely free because Central Banks interfere occasionally to alter the rate from its free-market level.

currencies is very high over some time periods. However, this shows an erratic pattern over long time horizons, as Figure 2 shows.

[Figure 2 about here]

The question we address in this article is whether there exists asymptotic dependence between currencies (that is, dependence in large samples), particularly under extreme events. In doing so, let us first consider independent observations of negative returns (X_t,Y_t), t=2,…,T, with unknown distribution F. Large negative returns correspond, in this case, with large depreciations of currencies against the U.S. dollar. The random variables $u_t = F_X(X_t)$ and $v_t = F_Y(Y_t)$ are both distributed as uniform, where F_X and F_Y are the marginal distribution functions. An informal procedure to detect extremal dependence (in the left tail, in this case) consists of examining the large values of u_t and v_t (see, for example, Coles, Heffernan, and Tawn, 1999).[4] Since F_X and F_Y are unknown, estimates are obtained from the empirical distribution functions.

Figure 3 shows left-tail dependence for selected returns pairs by geographic regions, and for selected large economies and the Euro. Except for the New Zealand dollar and the Australian dollar, the series do not exhibit strong patterns of dependence under high depreciations. Moreover, the dependence looks fairly low in some cases, like for the Japanese yen/Australian dollar pair.

[Figure 3 about here]

3.2 Estimation results

The next step consists of testing tail dependence formally by using the machinery described in Section 2. In doing so, one has to choose an appropriate threshold u to compute the tail index using the Hill estimator. The simplest approach is to plot its evolution against u and find a proper u, such that the Hill estimator appears to be stable (see, for instance, Tsay, 2001, chapter 7). Figure 4 shows the Hill estimator for the tail index ζ in equation (9) for the left tail of selected return pairs, evaluated at different values of the threshold u. As we see, simple inspection of the graphs does not shed much light on the optimal threshold to be selected in each case.

[Figure 4 about here]

Matthys and Beirlant (2000) discuss several methods of adaptive threshold selection methods that have been developed in recent years. The authors distinguish two approaches to estimating the optimal threshold u. One consists of constructing an estimator for the asymptotic mean-squared error (AMSE) of the Hill estimator, and choosing the threshold that minimizes it. This approach includes a bootstrap method, and an exponential regression model. The latter is studied in detailed in Beirlant, Diercks, Goegebeur, and Matthys (1999). The second approach derives estimators directly for u, based on the representation of the AMSE of the Hill estimator.

---

[4] As a convention in the literature of extreme value theory, large negative returns are negated. So they correspond with large positive returns.

Given that the exponential regression approach is easy to implement, we use it to find the optimal threshold. Specifically, Feuerverger and Hall (1999) and Beirlant et al (1999) derive an exponential regression model for the log-spacings of upper statistics

$$j(\log(X_{n-j+1,n}) - \log(X_{n-j,n})) \sim \left(\gamma + b_{n,k}\left(\frac{j}{k+1}\right)^{-\rho}\right) f_j \qquad 1 \le j \le k \qquad (11)$$

where $X_{1,n} \le X_{2,n} \le \ldots \le X_{n,n}$, $b_{n,k} = b\left(\frac{n+1}{k+1}\right)$, $1 \le k \le n-1$, $(f_1, f_2, \ldots, f_k)$ is a vector of independent standard exponential random variables, and $\rho \le 0$ is a real constant.

If we fix the threshold u at the $(k+1)^{th}$ largest observation, the Hill estimator can be rewritten as

$$H_{n,k} = \frac{1}{k}\sum_{j=1}^{k} j(\log(X_{n-j+1,n}) - \log(X_{n-j,n})). \qquad (12)$$

The Hill estimator written this way is the maximum likelihood estimator of γ in the reduced model

$$j(\log(X_{n-j+1,n}) - \log(X_{n-j,n})) \sim \gamma f_j \qquad 1 \le j \le k.$$

Given that the Hill estimator is an average of independent exponential random variables, its variance can be approximated by

$$\mathrm{Var}(H_{k,n}) \sim \frac{\gamma^2}{k}, \qquad (13)$$

while its bias arises from neglecting the second term in the right-hand side of equation (11)

$$E(H_{n,k} - \gamma) \sim \frac{b_{n,k}}{1-\rho}. \qquad (14)$$

For $n \to \infty$, $k \to \infty$, and $k/n \to 0$, the Hill estimator is asymptotically normal

$$\sqrt{k}\left(H_{k,n} - \gamma - \frac{b_{n,k}}{1-\rho}\right) \xrightarrow{d} N(0, \gamma^2)$$

From above, the AMSE of the Hill estimator is given by

$$\mathrm{AMSE}\, H_{k,n} = \left(\frac{b_{n,k}}{1-\rho}\right)^2 + \frac{\gamma^2}{k}. \qquad (15)$$

Therefore, the optimal threshold k_n^{opt} is defined as the one that minimizes (15)

$$k_n^{opt} \equiv \arg\min_k (\text{AMSE}\, H_{k,n}) = \arg\min_k \left(\left(\frac{b_{n,k}}{1-\rho} \right)^2 + \frac{\gamma^2}{k} \right). \quad (16)$$

The algorithm for the exponential regression goes as follows

- In model (11) fix ρ at ρ_0=–1 and calculate least-squares estimates $\hat{\gamma}_k$ and $\hat{b}_{n,k}$ for each $k \in \{3,\dots, n\}$.
- Determine $\overline{\text{AMSE}\ H_{k,n}} = \left(\frac{\hat{b}_{n,k}}{1-\hat{\rho}_k} \right)^2 + \frac{\hat{\gamma}_k^2}{k}$ for $k \in \{3,\dots, n\}$, with $\hat{\rho}_k \equiv \rho_0$. [5]
- Determine $\hat{k}_n^{opt} = \arg\min_{3\le k\le n} (\overline{\text{AMSE}\, H_{k,n}})$ and estimate γ by $H_{\hat{k}_n^{opt}}$.

The first step of the algorithm boils down to running a linear regression of $j(\log(X_{n-j+1,n}) - \log(X_{n-j,n}))$ on a constant term and $\frac{j\,(n+1)}{(k+1)^2}$, for each $k \in \{3,\dots, n\}$.

In order to control for both heteroscedasticity and serial correlation of returns, we use a diagonal VEC model or DVEC (1, 1)

$$\mathbf{r}_t = \mathbf{c} + \boldsymbol{\beta}\mathbf{r}_{t-1} + \boldsymbol{\varepsilon}_t \qquad t=2, \dots, T, \qquad (17)$$

where $\mathbf{r}_t$ is a k x 1 vector of returns, $\mathbf{c}$ is a k x 1 vector of constant terms, $\mathbf{r}_{t-1}$ is a k x 1 vector containing the first lag of $\mathbf{r}_t$, $\boldsymbol{\beta}$ is a k x 1 vector, and $\boldsymbol{\varepsilon}_t$ is a k x 1 white noise vector with zero mean. The matrix variance-covariance of $\boldsymbol{\varepsilon}_t$ is given in this case by

$$\boldsymbol{\Sigma}_t = \mathbf{A}_0 + \mathbf{A}_1 \otimes (\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}_{t-1}') + \mathbf{B} \otimes \boldsymbol{\Sigma}_{t-1} \qquad t=2, \dots, T, \qquad (18)$$

where $\mathbf{A}_0$, $\mathbf{A}_1$, $\mathbf{B}$, $\boldsymbol{\Sigma}_t$ and $\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}_{t-1}'$ are k x k matrices, for t=2, …,T, and ⊗ denotes the Hadamar product (e.g., Bollerslev, Engle, and Wooldridge, 1988; Zivot and Wang, 2003, chapter 13). In order to obtain the elements of $\boldsymbol{\Sigma}_{t}$,, only the lower part of the system in (18) is considered.

We tested extremal dependence for the returns series (raw data) and for the data filtered by the DVEC(1,1) model. In order to estimate DVEC models, we split the sample by geographic regions, and, when analyzing the dependence on the Euro, we selected some of the largest economies in the sample. Such an approach relies on previous inspection of the data. In the raw data, the extremal dependence between Japan and Chile's currencies,

[5] Matthys and Beirlant point out that for many distributions the exponential regression method works better, in MSE-sense, if the nuisance parameter ρ is fixed at some value ρ_0 rather than estimated.

for example, was fairly low not to include Chile in the Asia/Oceania model. The same applied to other pairs of currencies.

Regarding the Euro and large economies, both Brazil and Mexico were excluded from the corresponding DVEC model because of the low dependence of their currencies on the Euro. In addition, a practical reason to estimate separate DVEC models rather than only one was not to have an excessive number of parameters to estimate. It is well-known that both efficiency and stability is reduced when having many parameters to estimate at once. All estimation was carried out with the statistical package S-Plus 6.1.

Specification tests for the different DVEC models are presented in Table 3. None of the equations of the DVEC models for Asia/Oceania and the selected large economies and the Euro show evidence of either serial correlation or missing ARCH effects using 12 lags. For South and North America, all country equations accept the null hypothesis of no missing ARCH effect, and, except for Peru, there is no evidence of serial correlation left over.[6] In other words, the DVEC model works fairly well in filtering out both heterocedasticity and autocorrelation.

[Table 3 about here]

The estimation results to test the null hypothesis of asymptotic dependence under extreme events are reported in Table 4. All the returns pairs strongly reject the existence of asymptotic dependence in both tails. The only exception is the New Zealand/Australia pair, for which the null cannot be rejected at the 4-percent level in either case. One interesting feature is that some pairs show more dependence in the right tail than in the left tail (e.g., Mexican peso/U.S. dollar, Japanese Yen/Australian dollar, Japanese Yen/Euro). That means that currencies are more dependent under large appreciations than under large depreciations. Poon et al, who work with stock indices, find exactly the opposite: bear stock markets (left tail) exhibit a higher degree of extremal dependence than bull stock markets (right tail).

When filtering out the data, the dependence between all pairs of currencies is substantially reduced. However, our finding about the existence of more dependence in the right tail still holds, in general.

[Table 4 about here]

IV Conclusions

This article presents an alternative approach to the literature of contagion/interdependence, which is based on the extreme value theory (EVT). EVT studies the stochastic behavior of a process at unusually large or small levels. In particular, extremal dependence focuses on assets returns association under events which are more extreme than any other that has been previously observed. This concept differs from the definition of contagion or interdependence in that only extreme observations are taken into

[6] Given that we are working with daily data, it is not surprising to find some lingering serial correlation in some equation. We also tested the presence of serial correlation in Peru's equation using more than 12 lags, and, as expected, this tends to fade away with time.

account. This implies that the EVT approach does not pursue a global measure of association between returns, but a measure of dependence under extreme events over a long period of time.

Our application deals with the extreme-value dependence between nominal exchange rates for a sample of ten countries with dirty/free float regimes—Argentina, Brazil, The United States, Chile, Japan, Mexico, New Zealand, Peru, South Korea and Thailand—over the sample period 1998-2002. In addition, we investigate whether these countries' currencies have exhibited extremal dependence on the Euro, since its adoption in 1999. In order to measure tail dependence, we use mathematical results that have been recently derived in the statistics field. In addition, we present a simple way to determine the optimal threshold to compute the Hill estimator, a non-parametric measure of the tail index of a probability distribution.

Our findings show no evidence of extremal dependence between returns pairs, and suggest that the degree of dependence is stronger under large appreciations than under large depreciations. Our results are robust to filtering out the data for serial correlation and heteroscedasticy by multivariate GARCH models.

References

Beirlant, J., G. Dierckx, Y. Goegebeur, and G. Matthys (2000), "Tail Index Estimation and an Exponential Regression Model," *Extremes* 2:2, 177-200.

Bollerslev, T., R.F. Engle, and J.M. Wooldridge (1998), "A Capital-Asset Pricing Model with Time-Varying Covariances," *Journal of Political Economy* 96, 116-131.

Coles S., J. Heffernan, and J. Tawn (1999), "Dependence Measures for Extreme Value Analyses," *Extremes* 2(4), 339-365.

Coles, S. (2001), *An Introduction to Statistical Modeling of Extreme Values*. Springer series in statistics, Springer-Verlag London Limited.

Embrechts P., C. Klüppelberg, and T. Mikosch (1997), *Modelling Extremal Events for Insurance and Finance*. Springer-Verlag Berlin Heidelberg.

Feuerverger, A. and P. Hall (1999), "Estimating the tail exponent by modeling departure from a Pareto distribution," *Annals of Statistics* 27, 760-781.

Forbes, K. and R. Rigobon (2002), "No Contagion, only Interdependence: Measuring Stock Market Comovements," *The Journal of Finance* vol. LVII, No. 5, 2223-2261.

Matthys G., and J. Beirlant (2000), "Adaptive Threshold Selection in Tail Index Estimation," Working paper University Center for Statistics at the Catholic University Leuven.

Poon, S., M. Rockinger, and J. Tawn (2003), “Nonparametric Extreme Value Dependence Measures and Finance Applications,” *Statistica Sinica*. Forthcoming.

Tsay, R. (2001), *Analysis of Financial Time Series*. John Wiley & Sons; 1st edition.

Zivot, E., and J. Wang (2003), *Modeling Financial Times Series with S-Plus*. Insightful Corporation.

Table 1 Development Indicators for 2001

	Australia	Brazil	United States	Chile	Mexico
People					
Population, total (millions)	19.4	172.4	285.3	15.4	99.4
Population growth (annual %)	1	1.2	1.1	1.1	1.5
Life expectancy (years)	79.1	68.3	77.5	75.8	73.4
Infant mortality rate (per 1,000 live births)	6	31	2.1	10	24
Economy					
GNI, Atlas method (US$ billions)	385.9	528.9	9,800	70.6	550.2
GNI per capita, Atlas method (US$)	19,900	3,070	34,400	4,590	5,530
GDP (US$ billions)	368.7	502.5	10,100	66.5	617.8
GDP growth (annual %)	3.9	1.5	0.3	2.8	-0.3
GDP implicit price deflator (annual % growth)	2.4	7.4	2.3	1.5	5.5
Technology and infrastructure					
Fixed lines and mobile telephones (per 1,000 people)	1,095	385.1	1,118	574.8	354.0
Personal computers (per 1,000 people)	515.8	62.9	625	106.5	68.7
Internet users (millions)	7.2	8	143	3.1	3.6
Internet users/Population (%)	37.11	4.6	50	20.1	3.6
Aircraft departures	388,700	654,100	8,500,000	83,100	291,000
Trade and finance					
Trade in goods as a share of GDP (%)	34.5	23.2	19	52.2	54.2
High-technology exports (% of manufactured exports)	10	17.9	32.1	0.8	21.7
Foreign direct investment, net inflows (US$ billions)	4.4	22.6	130.8	4.5	24.7

	Japan	Peru	South Korea	Thailand
People				
Population, total (millions)	127	26.3	47.3	61.2
Population growth (annual %)	0.1	1.5	0.6	0.7
Life expectancy (years)	81.1	69.6	73.6	69
Infant mortality rate (per 1,000 live births)	3	30	5	24
Economy				
GNI, Atlas method (US$ billions)	4,522.5	52.2	447.6	118.5
GNI per capita, Atlas method (US$)	35,610	1,980	9,460	1,940
GDP (US$ billions)	4,245	54	422.2	114.7
GDP growth (annual %)	-0.6	0.2	3	1.8
GDP implicit price deflator (annual % growth)	-1.4	1.3	1.3	2.2
Technology and infrastructure				
Fixed lines and mobile telephones (per 1,000 people)	1,184.8	136.7	1,106.4	221.9
Personal computers (per 1,000 people)	348.8	47.9	256.5	27.8
Internet users (millions)	55.9	3	24.4	3.5
Internet users/Population (%)	44.0	11.4	51.6	5.7
Aircraft departures	641,300	30,000	225,700	102,400
Trade and finance				
Trade in goods as a share of GDP (%)	18.2	29.1	69.1	110.9
High-technology exports (% of manufactured exports)	26	1.5	29.1	31.1
Foreign direct investment, net inflows (US$ billions)	6.2	1.1	3.2	3.8

<u>Source</u>: World Development indicators database, April 2003. The World Bank.

Table 2 Descriptive statistics of Exchange Rates Returns

	Australia	Brazil	United States	Chile	Japan
# observations	1,253	1,253	1,253	1,253	1,253
Average	0.000	–0.001	0.000	0.000	0.000
Median	0.000	–0.001	0.000	0.000	0.000
Standard deviation	0.007	0.013	0.004	0.006	0.008
Interquartile range	0.008	0.009	0.004	0.007	0.010
Minimum	–0.033	–0.129	–0.016	–0.038	–0.038
Maximum	0.035	0.095	0.016	0.025	0.051
Kurtosis	4.71	19.30	4.25	5.98	5.87
Skewness	0.10	–0.74	0.01	–0.37	0.48
ρ_1	–0.049	0.042	–0.009	0.044	0.008
ρ_2	0.002	0.005	0.000	–0.075*	0.023
ρ_3	–0.024	0.058*	0.009	0.052	–0.055
ρ_4	0.007	0.018	–0.006	0.007	0.012
ρ_{13}	0.048	–0.043	–0.041	–0.030	0.001
ρ_{26}	–0.011	–0.016	–0.015	–0.031	–0.015
ρ_{60}	0.018	0.001	–0.041	–0.025	0.030
p-value JB test	0.00	0.00	0.00	0.00	0.00

	Mexico	New Zealand	Peru	South Korea	Thailand
# observations	1,253	1,253	1,253	1,253	1,253
Average	0.000	0.000	0.000	0.000	0.000
Median	0.000	0.000	0.000	0.001	0.000
Standard deviation	0.006	0.005	0.008	0.009	0.008
Interquartile range	0.007	0.005	0.009	0.007	0.007
Minimum	–0.033	–0.025	–0.031	–0.047	–0.048
Maximum	0.039	0.028	0.039	0.092	0.061
Kurtosis	6.30	5.38	4.87	20.65	16.26
Skewness	–0.17	–0.12	0.06	1.40	1.13
ρ_1	0.054	0.030	–0.034	0.051	0.084*
ρ_2	–0.021	–0.015	–0.052	–0.045	–0.023
ρ_3	-0.016	0.029	0.033	–0.063	–0.088*
ρ_4	0.032	0.008	–0.020	0.001	0.041
ρ_{13}	0.009	–0.025	0.046	–0.045	0.052
ρ_{26}	–0.013	0.046	–0.039	0.039	–0.015
ρ_{60}	0.007	0.012	0.004	0.016	0.051
p-value JB test	0.00	0.00	0.00	0.00	0.00

Notes: (1) The sample period is 1998-2002. The data is on a daily frequency, and was obtained from the Bank of Canada. (2) ρ_j represents the autocorrelation coefficient of order j. ‘*’ indicates significant at the 5 percent level. (3) JB test stands for the Jarque-Bera test to detect departures from normality. (4) Data source: The Bank of Canada.

Table 3 Specification Tests

(a) South and North America

Equation	Ljung-Box test for autocorrelation (12 lags)		Lagrange multiplier test for ARCH effects (12 lags)	
	Statistic	p-value	Statistic	p-value
Chilean peso	9.266	0.680	6.105	0.911
Brazilian real	13.903	0.307	8.976	0.705
U.S. dollar	15.358	0.224	10.652	0.559
Peruvian new sol	41.17	0.000	17.31	0.138
Mexican peso	14.35	0.279	10.31	0.589

(b) Asia and Oceania

Equation	Ljung-Box test for autocorrelation (12 lags)		Lagrange multiplier test for ARCH effects (12 lags)	
	Statistic	p-value	Statistic	p-value
Australian dollar	12.537	0.404	14.033	0.299
Japanese yen	4.617	0.970	10.730	0.552
Thai Baht	17.304	0.139	9.335	0.674
South Korean won	20.48	0.059	12.824	0.382
New Zealand dollar	12.35	0.418	5.774	0.927

(c) Selected Large Economies and the Euro

Equation	Ljung-Box test for autocorrelation (12 lags)		Lagrange multiplier test for ARCH effects (12 lags)	
	Statistic	p-value	Statistic	p-value
Japanese yen	5.939	0.919	7.688	0.809
U.S. dollar	12.277	0.424	7.189	0.881
Euro	3.028	0.995	8.038	0.782
Australian dollar	9.299	0.677	13.470	0.336
South Korean won	21.322	0.046	6.240	0.904

Table 4 Tail dependence

(a) North and South America

Raw data

	Left Tail						Right Tail				
	ρ	k*	$\bar{\chi}$	s.e	t-test	p-value	k*	$\bar{\chi}$	s.e	t-test	p-value
Chilean $/ Real	0.376	194	0.399	0.100	–5.987	0.000	73	0.110	0.130	–6.586	0.000
Chilean $/Sol	0.529	120	0.459	0.133	–4.054	0.000	184	0.535	0.113	–4.109	0.000
Mexican $/Real	0.277	183	0.515	0.112	–4.325	0.000	196	0.285	0.092	–7.798	0.000
Mexican $ /Can $	0.362	180	0.625	0.121	–3.100	0.000	164	0.553	0.121	–3.688	0.000

Filtered data

	Left Tail					Right Tail				
	k*	$\bar{\chi}$	s.e	t-test	p-value	k*	$\bar{\chi}$	s.e	t-test	p-value
Chilean $/ Real	149	0.147	0.094	–9.082	0.000	52	0.001	0.139	–7.203	0.000
Chilean $/Sol	199	0.346	0.095	–6.849	0.000	190	0.211	0.088	–8.978	0.000
Mexican $/Real	172	0.160	0.088	–9.503	0.000	185	0.208	0.089	–8.916	0.000
Mexican $ /Can $	191	0.262	0.091	–8.080	0.000	199	0.204	0.085	–9.331	0.000

(b) Asia and Oceania

Raw data

	Left Tail						Right Tail				
	ρ	k*	$\bar{\chi}$	s.e	t-test	p-value	k*	$\bar{\chi}$	s.e	t-test	p-value
Won/Thai Baht	0.296	181	0.572	0.177	–3.666	0.000	183	0.538	0.114	–4.062	0.000
Yen/Thai Baht	0.330	183	0.324	0.098	–6.906	0.000	131	0.313	0.115	–5.983	0.000
Yen/Australian $	0.283	193	0.424	0.103	–5.622	0.000	198	0.547	0.110	–4.115	0.000
N. Zealand$/Australia$	0.737	184	0.775	0.131	–1.720	0.042	126	0.740	0.155	–1.676	0.047

Filtered data

	Left Tail					Right Tail				
	k*	$\bar{\chi}$	s.e	t-test	p-value	k*	$\bar{\chi}$	s.e	t-test	p-value
Won/Thai Baht	153	0.431	0.116	–4.921	0.000	65	0.362	0.169	–3.777	0.000
Yen/Thai Baht	181	0.265	0.094	–7.822	0.000	187	0.182	0.086	–9.454	0.000
Yen/Australian $	199	0.121	0.079	–11.066	0.000	168	0.313	0.101	–6.786	0.000
N. Zealand$/Australia$	198	0.239	0.088	–8.640	0.000	197	0.399	0.099	–6.031	0.000

(c) Selected Large Economies and the Euro

Raw data											
	Left Tail						Right Tail				
	ρ	k*	$\bar{\chi}$	s.e	t-test	p-value	k*	$\bar{\chi}$	s.e	t-test	p-value
Yen/Euro	0.344	170	0.392	0.101	−6.014	0.000	167	0.534	0.119	−3.924	0.000
U.S.$/Euro	0.344	168	0.484	0.114	−4.507	0.000	183	0.468	0.109	−4.902	0.000
Australian $/Euro	0.356	147	0.395	0.115	−5.256	0.000	199	0.586	0.112	−3.679	0.000
Won/Euro	0.231	190	0.409	0.102	−5.785	0.000	199	0.448	0.103	−5.384	0.000

Filtered data										
	Left Tail					Right Tail				
	k*	$\bar{\chi}$	s.e	t-test	p-value	k*	$\bar{\chi}$	s.e	t-test	p-value
Yen/Euro	184	0.181	0.087	−9.405	0.000	171	0.233	0.094	−8.139	0.000
U.S.$/Euro	155	0.428	0.115	−4.981	0.000	153	0.248	0.101	−7.445	0.000
Australian $/Euro	93	0.130	0.117	−7.429	0.000	107	0.169	0.113	−7.348	0.000
Won/Euro	186	0.258	0.092	−8.048	0.000	152	0.066	0.086	−10.795	0.000

Notes: (1) The sample period is 1998-2002 for panels (a) and (b), and 1999-2002 for panel (c). The data is on a daily frequency, and was obtained from the Bank of Canada. (2) ρ is the Pearson correlation coefficient over the whole sample period. (3) k* represents the optimal threshold obtained by exponential regression procedure. (4) $\bar{\chi}$ is computed based on tail index estimation of Fréchet transformed margins of daily co-exceedances of return pairs, Z=min(S,T). Asymptotic dependence cannot be rejected if $\bar{\chi} = 1$.

Figure 1 Evolution of the Nominal Exchange Rate for some Selected Countries:1998-2002

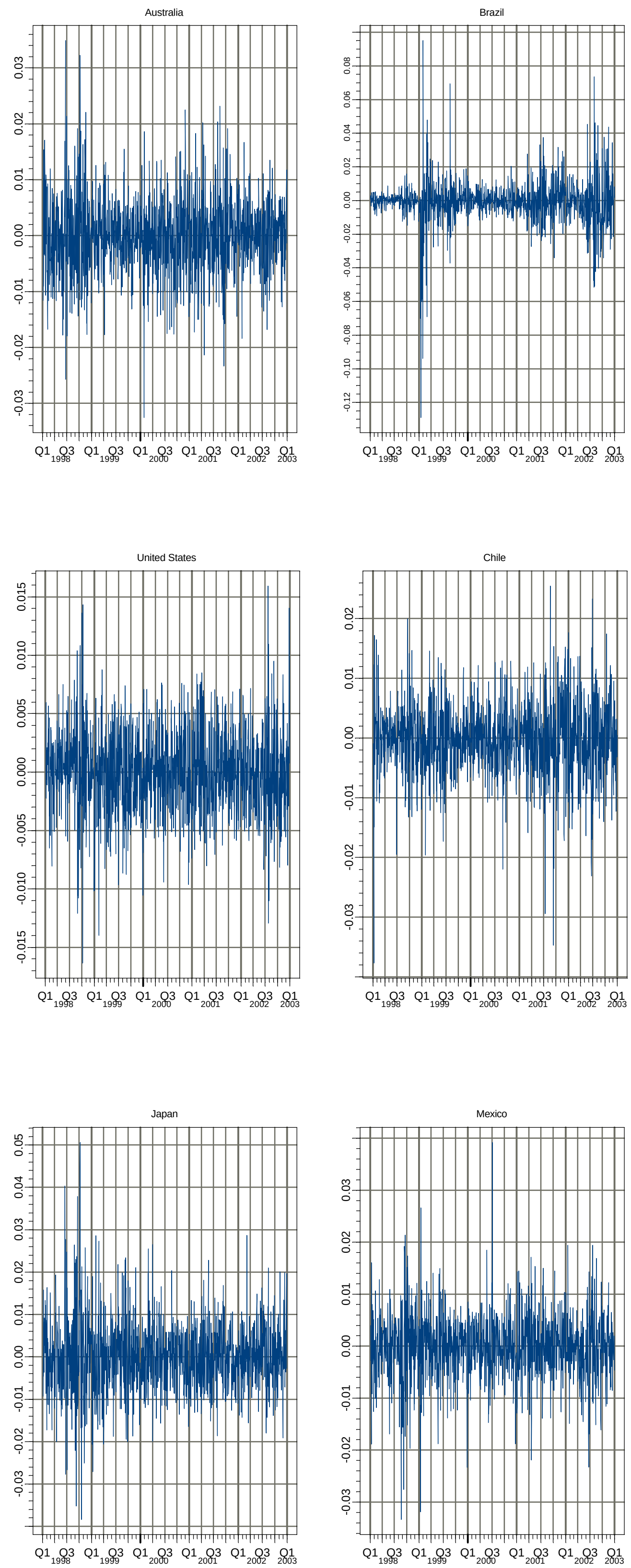
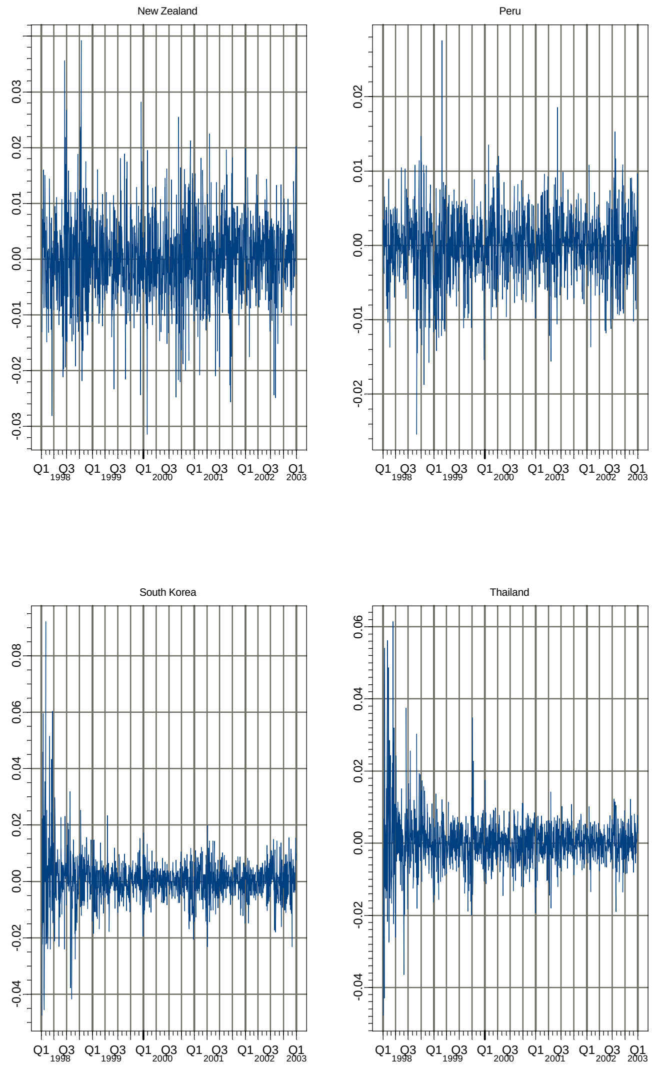
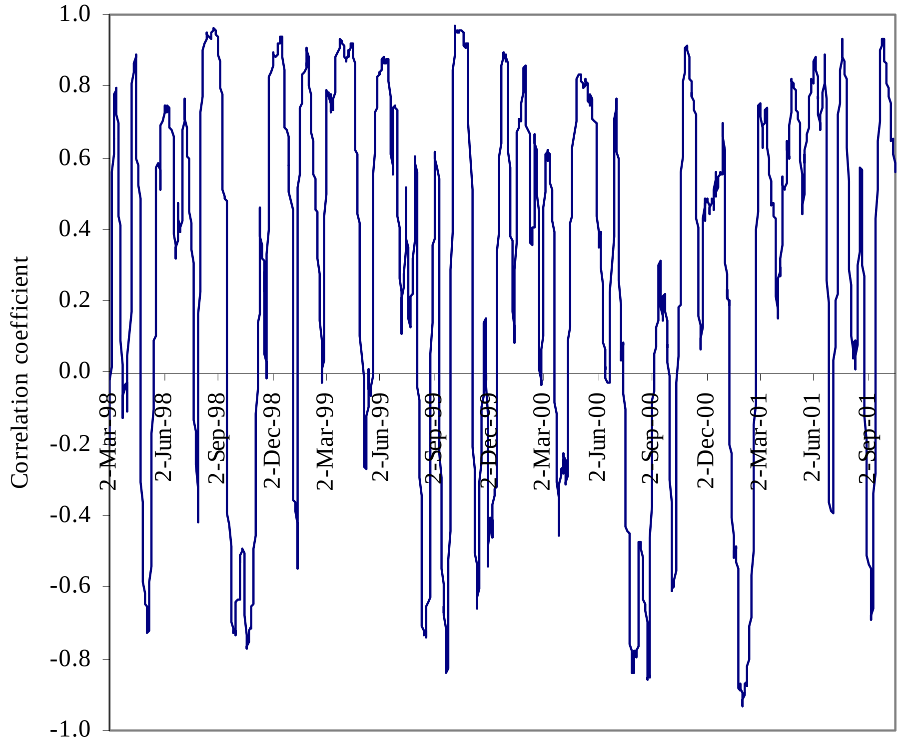


Figure 1 Continued



Notes: (1) Data source: Bank of Canada. (2) The figures are daily.

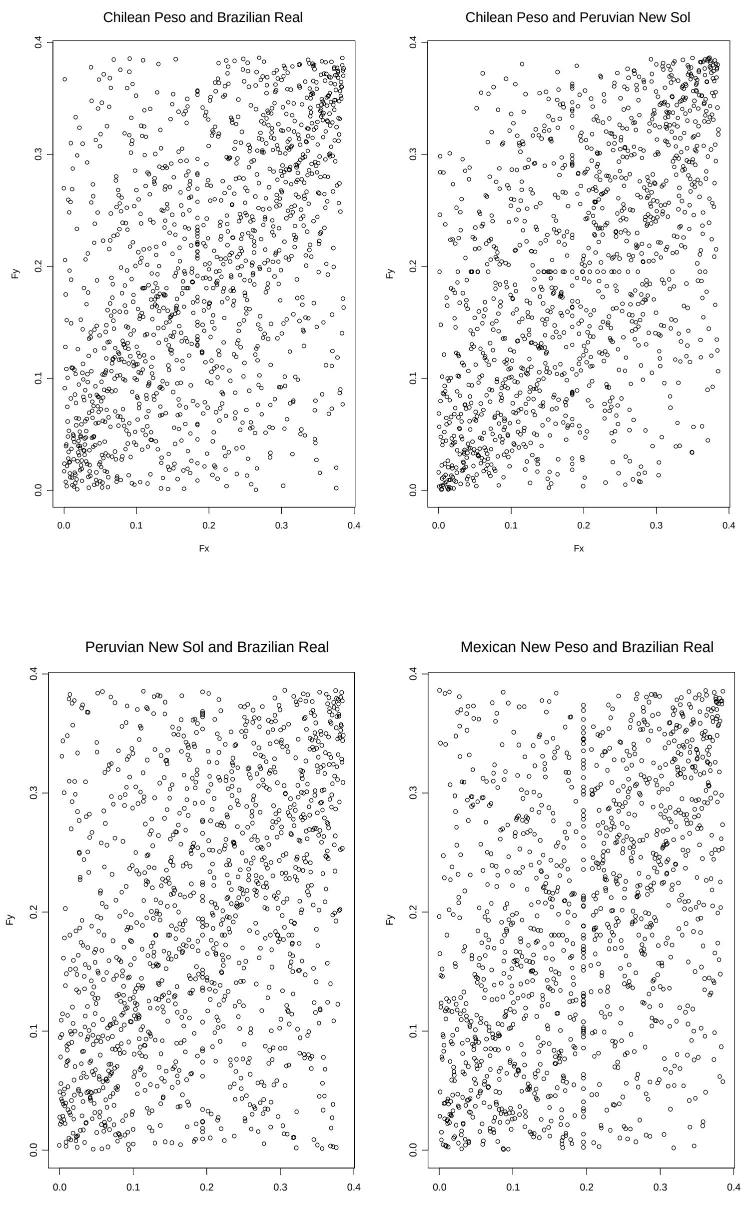
Figure 2 Moving Average Correlation Coefficient between the Returns on the Brazilian Real and the Chilean Peso



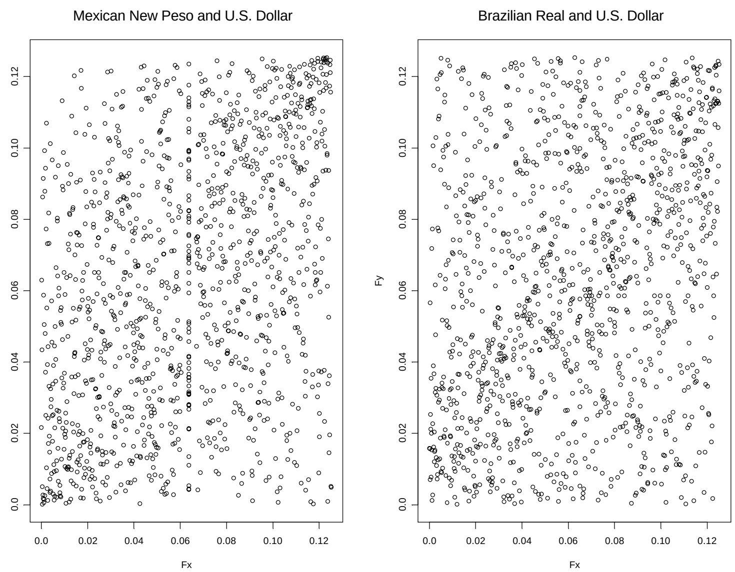
Note: The correlation coefficient corresponds with an equally weighted moving average Pearson correlation coefficient between the daily percent changes of Chile and Brazil's exchange rates. The coefficient is calculated by taking moving blocks of 20 observations (i.e., the average number of business days in a month).

Figure 3 LeftTail Dependence of Returns Pairs

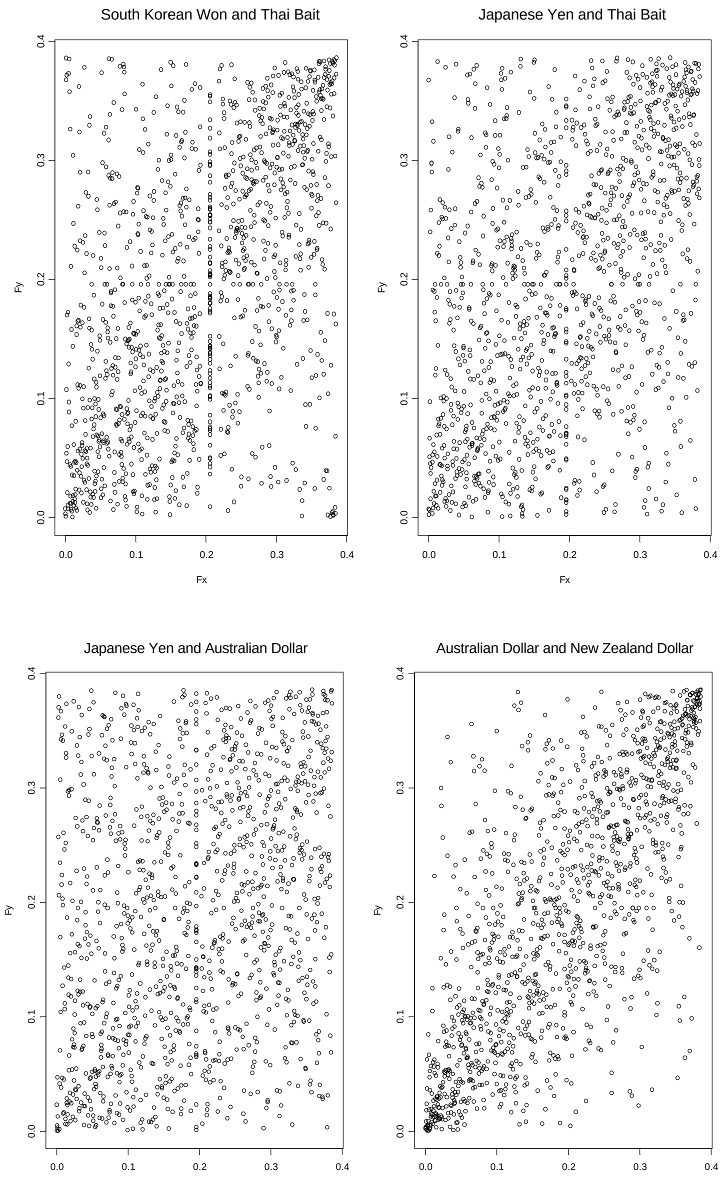
(a) South and North America



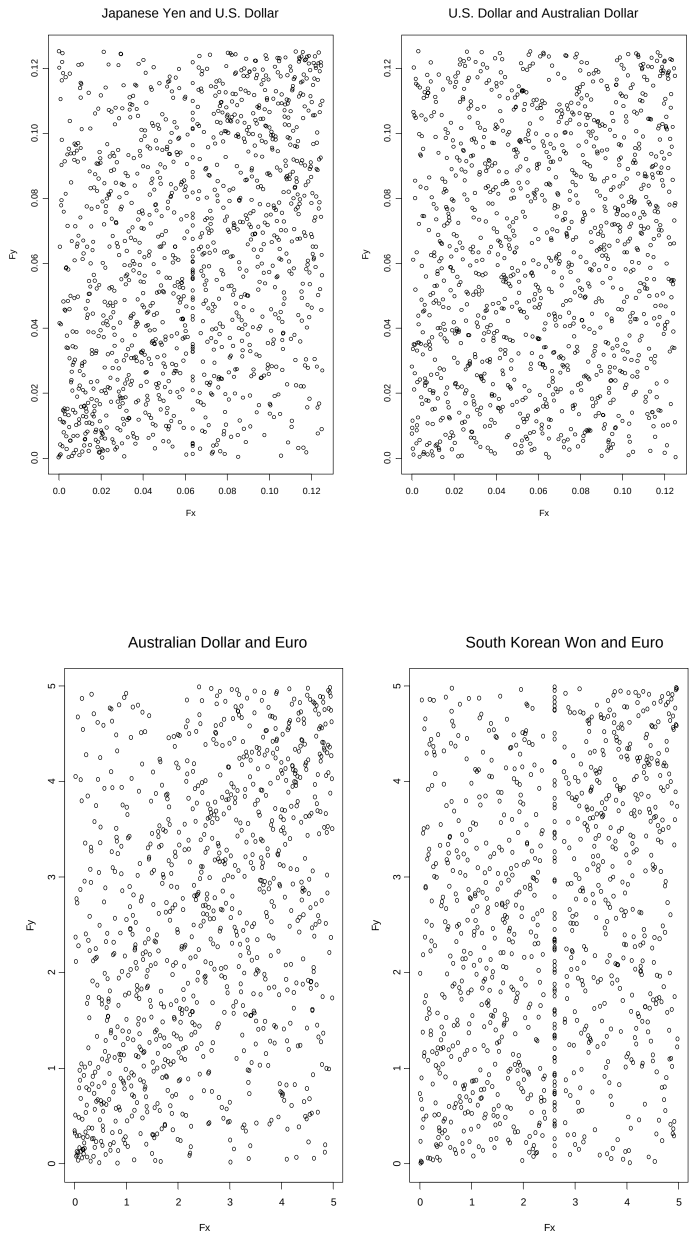
(a) South and North America (continued)



(b) Asia and Oceania



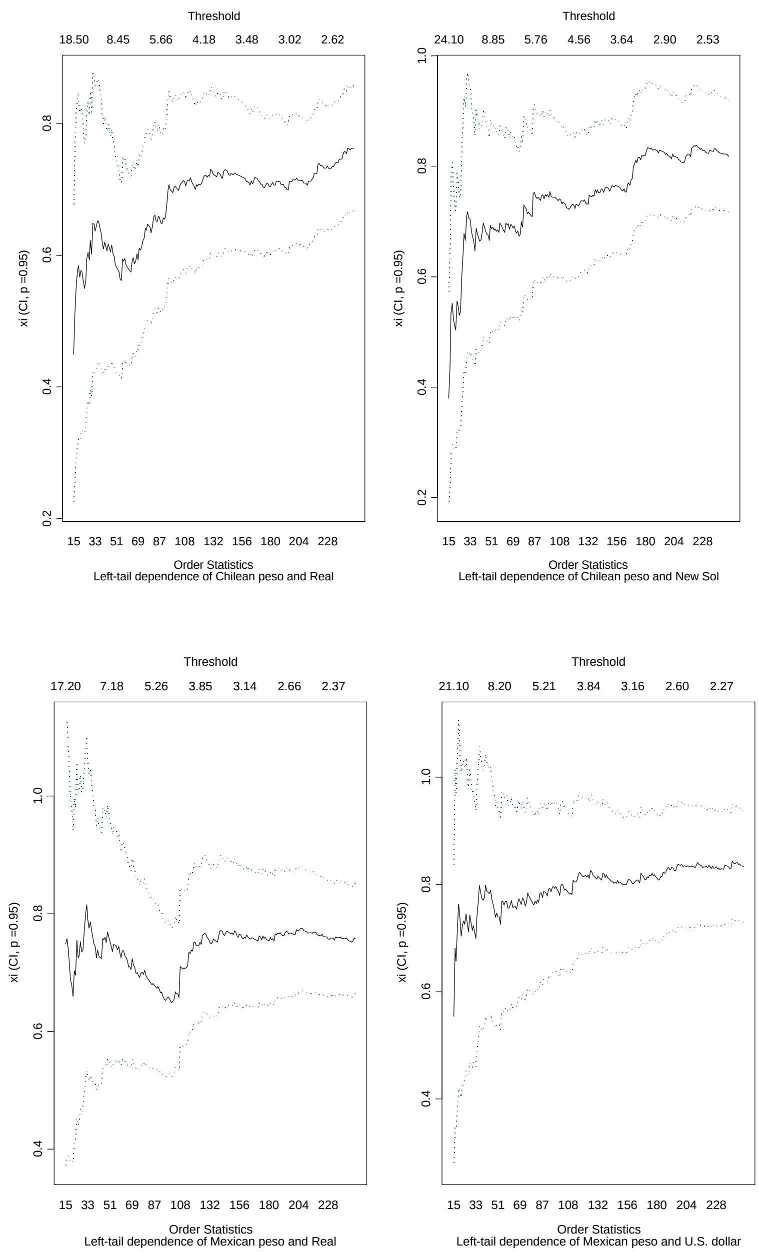
(c) Selected Large Economies and the Euro



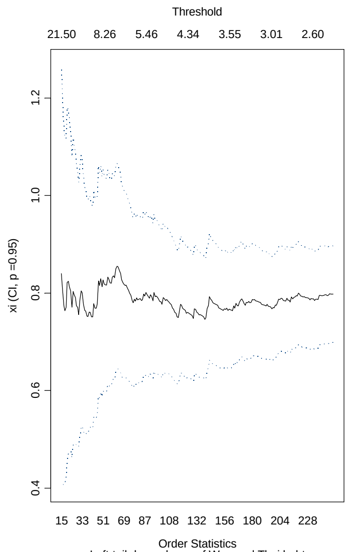
Notes: (X_t,Y_t) represents a pair of negative returns, t=2,…,T. The random variables $u_t = F_X(X_t)$ and $v_t = F_Y(Y_t)$ are both distributed as uniform, where F_X and F_Y are the marginal distribution functions. An informal procedure to detect extremal dependence consists of examining the large values of u_t and v_t.

Figure 4 Hill estimator of Left Tail of Returns Pairs

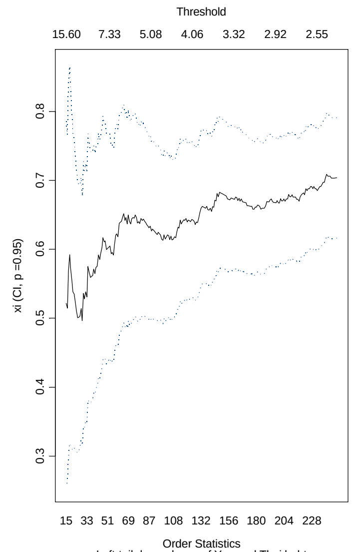
(i) South and North America



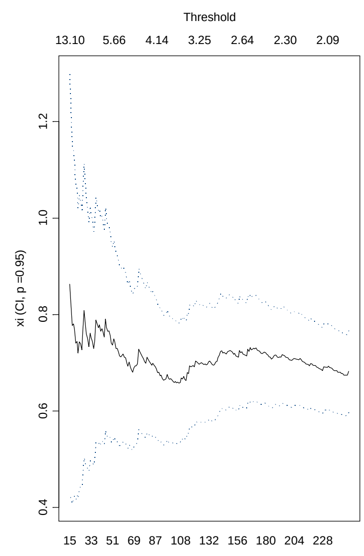
(ii) Asia and Oceania



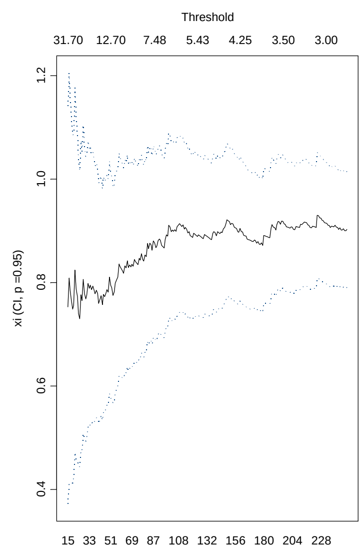
Left-tail dependence of Won and Thai baht



Left-tail dependence of Yen and Thai baht

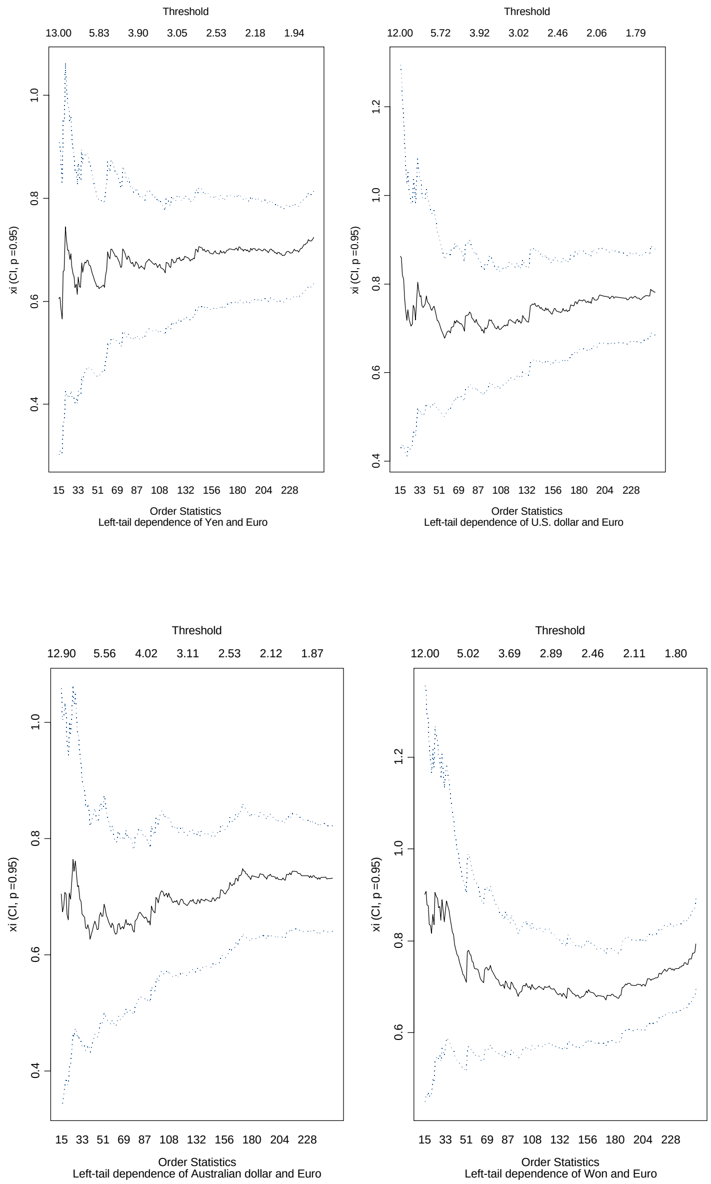


Left-tail dependence of Yen and Australian dollar



Left-tail dependence of N.Zealand dollar and Australian dollar

(iii) Selected Large Economies and the Euro



Notes: (1) $\hat{\zeta}$ is the Hill estimator of the tail index of Fréchet transformed margins of daily co-exceedances of return pairs, Z=min(S,T). (2) The dotted lines are 95-percent confidence bands.