Task-Specific Training and Job Design.

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Abstract

This paper provides a simple theoretical framework based on a new type of human capital introduced by Gibbons and Waldman (2004), called task-specific training, to understand job design. Mainly, in the presence of task-specific training, promotions might result ex-post in the underutilization of human capital and thus firms at the time of designing jobs should attempt to diversify this risk.

The results of the model are used to study the rationale for flexible work practices such as job rotation and ex-post task relocations, and to understand the college and experience premium.

Keywords: Task-Specific Training, Job Assignments, Job Design and Job Rotation

JEL-Classification: J41, J24, D21

1 Introduction

Since Becker's seminal paper, the literature on human capital has focused mainly on general-purpose and firm-specific human capital, and since Adam Smith in the Wealth of Nations emphasized the benefits of the division of labor, the literature on work organization has focused mainly on the benefits of specialization. Becker's human capital theory predicts that firms should not pay for general human capital since workers capture the full return on it, while firms should pay part of the cost of specific human capital since both parties share the return on it. Smith's division of labor principle predicts, due to learning-by-doing at the task level, that productivity is maximized by single-task jobs. These predictions, however, are at odds with a wide variety of innovations reagrding work organization in many modern firms[1]. Traditional forms of organization such as vertical job ladders and narrowly defined jobs that take advantage of extensive specialization are being replaced by what are known as flexible work practices characterized by multi-tasking, and work environments that favor continuing learning and the development of complementary skills be those general or specific[2].

This paper's contribution is to provide a simple theoretical framework based on a new type of human capital, called task-specific human capital[3], to understanding the main benefits and costs of designing multi-task jobs vis-a-vis designing single-task jobs. Following Gibbons and Waldman's (2004), task-specific training is understood as the part of on-the-job training that is specific to the tasks being performed on the job, as opposed to being specific to the firm. Task-specific and general training have a common feature which is that multiple firms value the training, but as opposed to general training, the contribution of task-specific training differs across jobs (within and outside of the firm).

In particular this paper extends the model in Balmaceda (2005) to multiple jobs and multiple tasks. The paper considers a two-period-four-job model between a firm and worker, both of whom are risk neutral and do not discount future income. There are two entry-level jobs, called training jobs, and two core jobs, labeled easy and difficult. Furthermore, there are two tasks, called core tasks, that have to be allocated between the core jobs, and two tasks, called training tasks, that have to be allocated between training jobs. At the beginning of period 1, before firms compete for workers, firms choose a task allocation, and right after that, firms compete for workers of unknown ability in a Bertrand-like fashion offering a job assignment given by one of the training jobs and a one period wage contract. Upon acceptance of a job, and during the first period, a worker acquires on-the-job firm- and task-specific training. A worker who is assigned to a

[1]For evidence on new work practices see, for instance, Hammer and Champy (1993), Osterman (1994, 2000), Black and Lynch (2000), Autor et al. (2003), and Lindbeck and Snower (2000, 2001).

[2]Osterman (1994) reports that in the US over half of all private establishments with 50 or more employer used some sort of team work and job rotation during 1992. Furthermore, he found that the higher the skill level required in an establishment, the more likely is it that multi-tasking and job rotation will be adopted.

[3]In what follows I use human capital and training as synonyms.

multi-task training job acquires $s \leq 1$ units of task-specific training in each of the two core tasks, while a worker who is assigned to a single-task training job acquires one unit of training in one of the core tasks. Thus, $1-s$ is a measure of the dis-economies of scope on training acquisition. At the beginning of period 2, a worker's ability is revealed to the incumbent employer, the market and the worker himself, and a productivity shock is realized that determines the output with the first-period employer in period 2. That is, symmetric learning is assumed. After the productivity shock is realized and the ability revealed, the market makes an offer consisting of a fixed wage and a job-assignment, and right after that, the incumbent firm offers a worker a job assignment and they either negotiate a one period contract for the period, or alternatively, they may either refuse to trade, or agree to trade with a third party instead. The wage determination procedure within the relationship is based on the outside option principle found, for example, in Sutton (1986)[4].

In this framework three different job configurations may emerge[5]: (i) training and core jobs are single-task (hereinafter, single-task firms); (ii) one of the training jobs is multi-task and core jobs are single-task (henceforth mixed firms); and (iii) one of the training jobs and one of the core jobs are multi-tasking (multi-tasking firms from here onwards). Choosing a job design that maximizes total career wages entails a trade-off between the benefits of specialization and the benefits of task complementarities plus the costs due to the dis-economies of scope on training accumulation. In the absence of uncertainty about a worker's innate ability, this trade-off is easily solved. When the increased productivity from specialization exceeds the increased productivity due to task complementarities plus the cost due to dis-economies of scope on task-specific training accumulation, then single-task firms maximize total career wages while when the opposite occurs, multi-tasking firms do so. However, when there is uncertainty about a worker's innate ability, this trade-off is complicated by the fact that ex-post it may be optimal to promote a worker to a job where his task-specific training is of no use. Under uncertainty about a worker's ability, single-task core jobs coupled with multi-tasking training jobs (mixed firms) have the advantage that a worker trained in both tasks will reap the benefit of one type of task-specific training regardless of the job to which he is promoted. In other

[4]The outside option principle assumes that bargaining and employment on the spot market are mutually exclusive. In this case, taking a job outside the firm or hiring a replacement worker terminates the bargaining process. Therefore, the no-trade payoff would be an *outside*, rather than an *inside* option in bargaining terminology (see, Sutton, 1986). The outcome of the game with an outside option depends on which party can take the outside option if an offer is rejected in the previous round of bargaining. In the case where only the responder can take the outside option, which is the case focused on this paper, there are three possible outcomes. The first possibility is one in which neither outside option is binding and thereby each party gets a share of the surplus created within the relationship (the surplus sharing outcome), which in this case is the total output. The second possibility is that only party i's outside option is binding, in which case he receives his outside option, and the other party's payoff is the total surplus minus party i's outside option. The third possibility occurs when both parties' outside options are binding, in which case the relationship terminates and each party gets the corresponding outside option.

[5]In fact, there are four possible configurations, but the one in which the training jobs are single-task jobs and one of the core jobs is a multi-tasking job never occurs in equilibrium.

words, mixed firms diversify the risk assocciated with the fact that at the time firms choose the job design they are unaware of the productivity (ability) of a worker. The downside is that ex-post productivity is lower due to the dis-economies of scope. In contrast, in single-task firms there are no dis-economies of scope, but upon a promotion task-specific training may be completely under-utilized. Lastly, multi-tasking firms face both problems–the dis-economies of scope and the possibility that upon a promotion, training is fully under-utilized, yet the upside is the enhanced productivity due to task complementarities.

Based on this intutition two observed flexible-work practices known as job rotation and task relocations are analyzed. In particular, it is shown that these practices are optimal when the benefits of task complementarities are large and the diseconomies of scope on training acquisition are small. Job-rotation trades-off the benefits of specialization to the benefits of multi-tasking, while flexible-work practices like ex-post task relocations trade-off the fixed cost F of setting up a flexible job design to the gain from avoiding the under-utilization of task-specific training upon a promotion.

The results of the paper are used to study the relationship between job design and wage inequality. In particular, it is shown that the college premium and the experience premium are higher in multi-tasking firms and that the tenure effect is smaller in multi-tasking firms. This together with the massive introduction of computerized information and communication systems, the introduction of flexible machine tools and programmable equipment, and the increase in human capital that occurred in the 1990s help us to explain the widening wage inequality during the 1990s[6].

This is the first paper that formally studies job design, promotions and task-specific training within the same framework. There is a small but growing literature that studies the issue of job design following Holmstrom and Milgrom's (1991) multitasking principal-agent model. Much of that literature focuses on how incentive contracts and measurability problems affect the allocation of tasks across jobs. Following Adam Smith, on the determinants of specialization, there is also some literature that looks at the issue of job design by focusing mainly on the returns to specialization vis-a-vis the costs of coordinating the activities of different workers (Yang and Borland, 1992; Becker and Murphy, 1992; Bolton and Dewatripont, 1994). These papers predicts that as the coordination costs decrease (due to lower communications costs), the degree of specialization among workers within a firm raises. Lastly, there is a literature begun by Lindbeck and Snower in which work organization is modelled through the time allocation of workers among several tasks. Specialization arises when workers perform one task while multi-tasking does when they allocate their work time among several tasks. In deciding time allocation, firms trade-off the return on intra-task learning–in other words, the more time a worker spends on a task, the higher his productivity in that task, and the return from inter-task learning–in other words, the more time spent in one task, the higher the productivity in the other task. None of these papers look at the issue of job design from the standpoint of

[6] See, for example, Autor et al. (2002, 2003).

task-specific training, promotions and task allocations across jobs and thus they cannot speak to the issues in this paper. Gibbons and Waldman (2005) is the only paper I am aware off that discusses job-design, task specific training, and promotions. In particular, they argue that jobs should be designed to minimize the underutilization of task-specific training. In terms of the model used here this means that all firms should adopt a mixed design. However, this holds only when neither dis-economies of scope nor benefits from task complementarities are too significant. Thus, their prediction holds only under certain parameters.

The rest of the paper is structured as follows. The next section, Section 2, presents the model. Section 3 presents the wage determination procedure, and derives the optimal job-assignment policy and optimal job design. In Section 4, flexible-work practices like job-rotation and task relocations are discussed. Section 5 discusses the link between job design and wage inequality. And finally, Section 6 offers some concluding remarks.

2 The Model

The paper considers a two-period–four-jobs model between workers (l) and identical firms (f), both of whom are risk neutral and do not discount future income. In particular, there are two training jobs, labeled 1 and 2, and two core jobs, the easy job, labeled e, and the difficult job, labeled d. There are two training tasks, labeled A and B, that have to be allocated between training jobs 1 and 2, and two core tasks, labeled α and β, that have to be allocated between core jobs in a manner discussed in more detail below.

At the beginning of each period, the firm and worker negotiate a one period contract for the supply of one unit of labor. During the first period, a worker is placed in a training job and acquires, through a learning-by-doing process, task-specific training (s) and firm-specific training (δ). That is, part of on-the-job training is specific to the tasks being performed in the job, as opposed to being specific to the firm, and as such, multiple firms value that, while the rest is specific to the firm and thus valuable only with the current employer. The accumulation of task-specific training depends on the task allocation as follows. When tasks A and B are allocated to training job 1, a worker who starts his career in that job acquires $s_{\alpha}^{1} \leq 1$ units of α-task specific training and $s_{\beta}^{1} \leq 1$ units of β-task specific training while a worker who starts his career in training job 2 acquires no task-specific training. The opposite occurs when both training tasks are allocated to training job 2. When the A-task is allocated to training job 1 and the B-task to training job 2, a worker who starts his career in training job 1 acquires one unit of α-task specific training ($s_{\alpha}^{1} = 1$) and no units of β-task specific training ($s_{\beta}^{1} = 0$), as opposed to a worker who starts his career in training job 2, who acquires one unit of β-task specific training ($s_{\beta}^{2} = 1$) and no units of α-task specific training ($s_{\alpha}^{2} = 0$). The opposite occurs when the A-task is allocated to training job 2 and the B-task to training job 1[7].

[7]Note that there is no direct cost of providing training, yet there is an opportunity cost when a worker is placed in a single task

In what follows $\mathbf{s}^j = \left(s^j_\alpha, s^j_\beta\right)$ denotes the level of task-specific training acquired by a worker who starts his career in training job j, $j \in \{1, 2\}$, and $\mathbf{t} = \left(t_A, t_\alpha, t_B, t_\beta\right)$ denotes the job design or task allocation, where the parameter t_A (t_B) takes the value 1 when task A (B) is allocated to training job 1 and takes the value 0 otherwise, while the parameter t_α (t_β) takes the value 1 when task α (β) is allocated to an easy job and takes the value 0 otherwise. In addition, $\left(t_\alpha, t_\beta\right)$ will be denoted by t_c, where the subscript c stands for core jobs. For example, $\mathbf{t} = (1, 1, 1, 1)$ is a job design in which training job 1 and an easy job are multi-task jobs while training job 2 and a difficult job have no tasks allocated to them[8], and $\mathbf{t} = (1, 1, 0, 0)$ is a job design in which training job 1 is specialized in task A and an easy job in task α while training job 2 is specialized in task B and an difficult job in task β.

A young worker's output (first period output) is normalized to 0, while an old worker's output (second period ouput) depends on his job-assignment, level of task- and firm-specific training, and innate ability $a \in [0, \overline{a}]$. This is drawn from a distribution function $\Phi(\bullet)$, with symmetric and unimodal density function $\phi(\bullet)$. The output of a worker whose ability is a, has acquired $\mathbf{s}^j$ units of task-specific training, and δ units of firm-specific training is $y_e\left(\mathbf{s}^j, t_c, a\right) + \delta + \eta$ when placed in an easy job and stays with the first-period employer and $y_e\left(\mathbf{s}^j, t_c, a\right)$ if he switches employer, while that is $y_d\left(\mathbf{s}^j, t_c, a\right) + \delta + \eta$ when placed in a difficult job and stays with the first-period employer and $y_d\left(\mathbf{s}^j, t_c, a\right)$ if he switches employer, where

$$y_e\left(\mathbf{s}^j, t_c, a\right) \equiv m_e + n_e a + \mu\left(t_c, \mathbf{s}^j\right)\left(s^j_\alpha t_\alpha + s^j_\beta t_\beta\right) + \sigma\sqrt{s^j_\alpha s^j_\beta t_\alpha t_\beta},$$
$$y_d\left(\mathbf{s}^j, t_c, a\right) \equiv m_d + n_d a + \mu\left(t_c, \mathbf{s}^j\right)\left(s^j_\alpha\left(1 - t_\alpha\right) + s^j_\beta\left(1 - t_\beta\right)\right) + \sigma\sqrt{s^j_\alpha s^j_\beta\left(1 - t_\alpha\right)\left(1 - t_\beta\right)},$$

and η is a firm-worker specific productivity shock drawn from a distribution function $F(\cdot)$, with density $f(\cdot)$ and support $\left[\underline{\eta}, \overline{\eta}\right]$.

- (A1): (i) If $t_A = t_B = 1$, then $s^1_\alpha = s^1_\beta = s \leq 1$ and $s^2_\alpha = s^2_\beta = 0$ and if $t_A = t_B = 0$, then $s^1_\alpha = s^1_\beta = 0$ and $s^2_\alpha = s^2_\beta = s \leq 1$; (ii) $\mu\left(t_c, \mathbf{s}^j\right) = \mu \in \left[\frac{1}{2}, 1\right]$ if $t_c = (1, 1)$ and $s^j_\alpha = s^j_\beta = s$ and $\mu\left(t_c, \mathbf{s}^j\right) = 1$ otherwise; and (iii) $\sigma \in [-2\mu, \bar{\sigma}]$ where $\bar{\sigma} > 0$.

Part (i) ensures that the accumulated amount of task-specific training depends only on the task allocation. This implies that on-the-job training neither depends on other characteristics of training jobs nor on workers' personal characteristics. The fact that the level of task-specific training is the same for each type of task

training job since he cannot be trained in another task.

[8] Because at each hierarchical level there are two tasks only, when in a given level one of the jobs is multi-tasking the other have no tasks attached to it. This however does not mean that a worker allocated to that job produces nothing since the productivity of innate ability is positive despite the fact that there is no task allocated to that job. One can think of this as the case in which core jobs are defined by a basic task; the difficult task that gives rise to the productivity of innate ability in that job, and the easy task that gives rise to the productivity of innate ability in that job. In addition, the easy and difficult tasks cannot be combined in one job, and the sensitivity of each task to a worker's innate ability is different from one another.

implies that $1 - s$ is a measure of the dis-economies of scope in training acquisition. In particular, when $s = 1$, there are no dis-economies of scope in training accumulation. Part (ii) tells us that there is benefit to specialization which is given by $1 - \mu$. Observe that the increased productivity from specialization is due to the fact that a worker performs one task only and not that he is trained in one task only. Part (iii) allows core tasks to be either complements or substitutes, and ensures that the productivity of task-specific-training is always positive. The parameter σ measures the degree of complementarity (substitutability) between tasks.

It is worth noting that firm- and task-specific training are separable in the production technology and that the latter has the same marginal product inside as well as outside of the firm when a worker is placed in the same job and the task allocation is the same across firms. This guarantees that the results of the paper do not depend on either the complementarity or substitutability between task- and firm-specific training nor on that between ability and different types of training.

- (A2): (i) $m_e \geq m_d$ and $n_d \geq n_e$; (ii) $m_e + n_e Ea \lesseqqgtr m_d + n_d Ea$; and (iii) $a^* - \frac{2\mu+\bar{\sigma}}{\triangle n} > 0$ and $a^* + \frac{2\mu+\bar{\sigma}}{\triangle n} < \overline{a}$ for all μ, where $a^* = \frac{\triangle m}{\triangle n}$, $\triangle n \equiv n_d - n_e$ and $\triangle m \equiv m_e - m_d$.

Part (i) assures that the marginal product of ability is higher for a difficult job than for an easy job and that an old worker with no ability and no task-specific training is more productive in an easy job than in a difficult job. Part (ii), which differs from the standard assumption in the literature, allows a worker of average ability to be more productive in a difficult job than in an easy job; and part (iii) says that regardless of the level of task-specific training and task complementarities, there is a positive probability that a worker whose ability is sufficiently high will be more productive in a difficult job, and one whose ability is sufficiently low will be more productive in an easy job.

The precise timing is as follows. At the beginning of period 1, before firms compete for workers, firms choose a job design $\mathbf{t}$. Then, firms compete for workers in a Bertrand-like fashion offering a job assignment consisting of a training job and a one period wage contract to workers of unknown ability. At the beginning of period 2, the worker's ability is revealed to the incumbent employer, the market and to the worker himself, and a productivity shock is realized that determines the output with the first-period employer in period 2 is realized–that is, symmetric learning is assumed. After the productivity shock is realized and the ability is revealed, the market makes a wage and a job-assignment offer and then the incumbent firm offers a worker a job assignment and negotiates the wage for the period. The wage determination procedure within the relationship, which I discuss in more detail below, is based on the outside option principle found, for example, in Sutton (1986).

3 The Analysis

3.1 Preliminaries

The bargaining game between the incumbent firm and worker adopted here is Rubinstein's alternating-offer game with the addition of outside options for both the firm and the worker. Bargaining takes place over a number of periods. At the beginning of the second period, the worker is chosen to be the proposing party with probability θ–the worker's bargaining power–and the firm with probability $1-\theta$–the firm's bargaining power. If the proposing party is the worker, he proposes a wage w_2. The firm can either accept or reject this offer, if it accepts, then the firm gets $y_{j(a)}\left(\mathbf{s}^j, t_c, a\right)+\delta+\eta-w_2$, where $j\left(a\right)$ denotes the job in the which a worker of ability a is placed by the incumbent in period 2 and t_c the task allocation chosen by the firm, while if it rejects then either the firm and worker gets zero and bargaining goes to the next round where the firm makes a proposal or the firm chooses to terminate the bargaining process taking its outside option. If bargaining is terminated the worker also gets his outside option which is equal to $y_{j'(a)}\left(\mathbf{s}^j, t'_c, a\right)$, where $j'\left(a\right)$ denotes the job in the which a worker of ability a is placed by the outside firm and t'_c the task allocation chosen by the firm offering the worker's outside option. Note that only the respondeing party is allowed to terminate bargaining. This ensures a unique solution for the bargaining game. Furthermore, because complete information is assumed, the bargaining process ensures that trade is ex-post efficient; that is, the firm-worker relationship continues whenever continuing the relationship generates more benefit than separating; *i.e.,* $y_{j(a)}\left(\mathbf{s}^j, t_c, a\right)+\delta+\eta \geq y_{j'(a)}\left(\mathbf{s}^j, t'_c, a\right)$. It follows from this and the outside option principle that when neither outside option is binding, the surplus from continuing the relationship is divided according to each party's bargaining power (hereinafter, the surplus sharing outcome); that is, the worker gets $\theta\left[y_{j(a)}\left(\mathbf{s}^j, t_c, a\right)+\delta+\eta\right]$ and the firm gets $(1-\theta)\left[y_{j(a)}\left(\mathbf{s}^j, t_c, a\right)+\delta+\eta\right]$; when only the worker's outside option is binding, the worker gets his outside option and the firm gets the total surplus minus the worker's outside option; that is, $y_{j(a)}\left(\mathbf{s}^j, t_c, a\right)+\delta+\eta-y_{j'(a)}\left(\mathbf{s}^j, t'_c, a\right)$; and when only the firm's outside option is binding, the worker gets the total surplus from continuing the relationship and the firm gets its outside option. Finally, when the worker and the firm's outside options are both binding, they are better-off terminating the relationship and each getting the corresponding outside option because what is generated by continuing the relationship is less than what can be generated if the firm and worker terminate their relationship.

Because the firm's outside option is 0 and the worker's outside option is $y_{j'(a)}\left(\mathbf{s}^j, t'_c, a\right) \geq 0$ for all a, whenever the firm's outside option is binding it is optimal to terminate the relationship[9]. Let us define

[9]To visualize this, notice that the firm's outside option is binding when $(1-\theta)\left[y_{j(a)}\left(\mathbf{s}^j, t_k, a\right)+\delta+\eta\right]<0$. This implies that the surplus within the relationship $y_{j(a)}\left(\mathbf{s}^j, t_k, a\right)+\delta+\eta$ is negative, which means that $y_{j(a)}\left(\mathbf{s}^j, t_k, a\right)+\delta+\eta<y_{j'(a)}\left(\mathbf{s}^j, t'_k, a\right)$. That is, the surplus inside the relationship is lower than the surplus when the relationship is terminated.

$\eta_{j(a)}(\theta, a) \equiv \frac{y_{j'(a)}(\mathbf{s}^j, t_c', a)}{\theta} - y_{j(a)}(\mathbf{s}^j, t_c, a) - \delta$ as the minimum productivity shock ensuring that the firm and worker share the surplus from the relationship, and $\underline{\eta}_{j(a)}(a) \equiv y_{j'(a)}(\mathbf{s}^j, t_c', a) - y_{j(a)}(\mathbf{s}^j, t_c, a) - \delta$ as the minimum productivity shock below which separation is efficient. Thus, the firm and worker's payoffs are as follows:

$$\pi_2 \equiv \begin{cases} (1-\theta)\left[y_{j(a)}(\mathbf{s}^j, t_c, a) + \delta + \eta\right] & \text{if } \eta \geq \eta_{j(a)}(\theta, a), \\ y_{j(a)}(\mathbf{s}^j, t_c, a) + \delta + \eta - y_{j'(a)}(\mathbf{s}^j, t_c', a) & \text{if } \underline{\eta}_{j(a)}(a) \leq \eta < \eta_{j(a)}(\theta, a), \\ 0 & \text{if } \eta < \underline{\eta}_{j(a)}(a), \end{cases} \tag{1}$$

and

$$w_2 \equiv \begin{cases} \theta\left[y_{j(a)}(\mathbf{s}^j, t_c, a) + \delta + \eta\right] & \text{if } \eta \geq \eta_{j(a)}(\theta, a), \\ y_{j'(a)}(\mathbf{s}^j, t_c', a) & \text{if } \underline{\eta}_{j(a)}(a) \leq \eta < \eta_{j(a)}(\theta, a), \\ y_{j'(a)}(\mathbf{s}^j, t_c', a) & \text{if } \eta < \underline{\eta}_{j(a)}(a), \end{cases} \tag{2}$$

where π_2 denotes the firm's second-period profit and w_2 denotes a worker's second period wage.

The second period job-assignment policy adopted by outside firms is to assign a worker to the job where his productivity is highest. That is, to allocate a worker of ability a with task-specific training $\mathbf{s}^j$ in a difficult job whenever $y_d(\mathbf{s}^j, t_c', a) \geq y_e(\mathbf{s}^j, t_c', a)$ and in an easy job otherwise. This condition boils down to

$$a \geq a(\mathbf{s}^j, t_c') \equiv a^* + \frac{\mu(t_c, \mathbf{s}^j)\left(s_\alpha^j(2t_\alpha' - 1) + s_\beta^j(2t_\beta' - 1)\right) + \sigma\sqrt{s_\alpha^j s_\beta^j}(t_\alpha' + t_\beta' - 1)}{\triangle n}.$$

This is summarized in the next lemma.

Lemma 1 *The outside firm assigns a worker of ability a with task-specific training* $\mathbf{s}^j$ *to a difficult job when* $a \geq a(\mathbf{s}^j, t_c')$ *and to an easy job otherwise; and (ii)* $a(\mathbf{s}^j, t_c')$ *is non-decreasing in* t_c'*, increasing in* σ *if* $t_c' = \mathbf{1}$*, and decreasing in* σ *if* $t_c' = \mathbf{0}$.

This result says that the outside firm's second-period job assignment policy depends not only on a worker's innate ability, but also on the task allocation and the level of task-specific training. In particular, when in the outside firm both core tasks are allocated to an easy job, the outside firm is less likely to assign a trained worker to a difficult job than an untrained worker, while when both core tasks are allocated to a difficult job, the opposite occurs. In contrast, when one of the core tasks is allocated to an easy job and the other to a difficult job, a trained worker who has acquired task-specific training in both tasks, i.e., $\mathbf{s}^j = (s, s)$, is equally likely to be assigned to a difficult job as an untrained worker. In fact, the outside firm places a worker whose task-specific training is (s, s) in an easy job with probability $\Phi(a^*)$ and in a difficult job with probability $1 - \Phi(a^*)$.

In principle, the incumbent firm's job-assignment policy might not be the same as the one adopted by outside firms since a promotion may result in a binding outside option while a non-promotion may not and vice-versa or the outside option may bind in either case or in none. In the next lemma it is shown that the incumbent employer follows the same job-assignment policy as the outside firm. Formal proofs of all results are placed in the appendix unless otherwise stated.

Lemma 2 *The incumbent employer adopts the same promotion policy as the outside firm. That is, a worker of ability a with task-specific training* $\mathbf{s}^j$ *is assigned to a difficult job if and only if* $a \geq a\left(\mathbf{s}^j, t_c\right)$ *and to an easy job otherwise.*

This lemma is due to symmetric learning since this results in a worker's outside option unaffected by the incumbent's decision to promote. This implies that the incumbent's promotion decision is based entirely on whether a worker is more productive in an easy or difficult job and not on the outside option. Thus, the incumbent assigns a worker to a difficult job when $y_d\left(\mathbf{s}^j, t_c, a\right) > y_e\left(\mathbf{s}^j, t_c, a\right)$ and to an easy job otherwise.

From these two lemmas one concludes that under spot contracting an old worker is assigned to a job where the output, conditional on his training level, task allocation and ability, is maximized regardless of which firm employs the worker in the second period. In short, job assignments are ex-post efficient.

A worker's period-2 expected wage is given by:

$$\begin{aligned} U^l\left(t_c, a\right) \equiv\ & E_a \int_{\eta_{j(a)}(\theta,a)}^{\overline{\eta}} \theta\left[y_{j(a)}\left(\mathbf{s}^j, t_c, a\right) + \delta + \eta\right] dF(\eta) + \\ & E_a \int_{\underline{\eta}}^{\eta_{j(a)}(\theta,a)} y_{j'(a)}\left(\mathbf{s}^j, t_c', a\right) dF(\eta), \end{aligned} \tag{3}$$

and the firm's period-2 expected profit is given by:

$$\begin{aligned} U^f\left(t_c, a\right) \equiv\ & E_a \int_{\eta_{j(a)}(\theta,a)}^{\overline{\eta}} (1-\theta)\left[y_{j(a)}\left(\mathbf{s}^j, t_c, a\right) + \delta + \eta\right] dF(\eta) + \\ & E_a \int_{\underline{\eta}_{j(a)}(a)}^{\eta_{j(a)}(\theta,a)} \left[y_{j(a)}\left(\mathbf{s}^j, t_c, a\right) + \delta + \eta - y_{j'(a)}\left(\mathbf{s}^j, t_c', a\right)\right] dF(\eta). \end{aligned} \tag{4}$$

These payoffs are easily understood when one observes that when the productivity shock is higher than $\eta_{j(a)}(\theta, a)$, the outside option is non-binding and thus a worker stays with the incumbent and shares the surplus from the relationship according to their bargaining power. When the productivity shock is below $\eta_{j(a)}(\theta, a)$, but above $\underline{\eta}_{j(a)}(a)$, a worker's outside option binds, but what is generated inside of the relationship is higher than what is generated outside of it. Thus, the relationship continues and a worker is paid his outside option. Lastly, when the productivity shock is lower than $\underline{\eta}_{j(a)}(a)$, what is generated inside of the relationship is lower than what is generated outside of it, and thus a worker leaves the firm and each gets the outside option.

Let us define $W_j(\mathbf{t})$, $j \in \{1, 2\}$, as the total career's wages when a worker starts his career in training job j and chooses a firm with job design $\mathbf{t}$. In the first period, firms compete for workers in a Bertrand-like fashion with the well-known result that in equilibrium, the present value of firm profits must be zero. In the absence of discounting, this implies that the wage of a young worker must be equal to a worker's output of an untrained worker, which in this case was normalized to zero, plus the firm's second period expected profit $U^f(t_c, a)$[10].

Because firms are ex-ante symmetrical in every dimension, from here onwards I will assume that the equilibrium job design is symmetric across firms, i.e., $\mathbf{t} = \mathbf{t}'$. Thus, the first-period wage is $w_1^j = U^f(t_c, a)$ and the total career wages are $W_j(\mathbf{t}) = U^f(t_c, a) + U^l(t_c, a)$, where

$$W_j(\mathbf{t}) \equiv E_{a>a(\mathbf{s}^j,t_c)} y_d\left(\mathbf{s}^j, t_c, a\right) + E_{a\leq a(\mathbf{s}^j,t_c)} y_e\left(\mathbf{s}^j, t_c, a\right) + \int_{-\delta}^{\overline{\eta}} (\delta + \eta)\, dF. \tag{5}$$

The next lemma finds conditions under which a worker's total career wages are maximized by starting his career in training job 1 for each possible job design. It is useful to have in mind that there are sixteen possible job designs. In four of them, training job 1 is a multi-task job and training job 2 has no task allocated to it. There are four in which the opposite occurs, while in the remaining eight cases, training jobs are single-task jobs.

Proposition 1 *(i) If training job* 1 *is multi-task, then a worker's career wages are maximized by staring in job* 1 *(*$W_1(\mathbf{t}) > W_2(\mathbf{t})$*), while the opposite occurs if training job* 2 *is multi-task; (ii) if training jobs are single-task while one of the core jobs is multi-task, then a worker's career wages are the same regardless of the starting job (*$W_1(\mathbf{t}) = W_2(\mathbf{t})$*); (iii) if training and core jobs are single-task and the task learned in job* 1 *is productive in an easy job, then a worker's career wages are maximized by starting in job* 1 *when* $a^* \leq Ea$ *and in job* 2 *when* $a^* > Ea$*; and (iv) if training and core jobs are single-task and the task learned in job* 1 *is productive in a difficult job, then a worker's career wages are maximized by starting in job* 1 *when* $a^* > Ea$ *and in job* 2 *when* $a^* \leq Ea$.

When either training job is multi-task, total career wages are higher when a worker starts his career in a multi-task job. The reason is that regardless of a worker's innate ability, he will be assigned with positive probability to a job where at least one of the two types of task-specific training increases second period productivity and thus second period wage. In fact, when core jobs are single-task, a worker will reap the benefits of either α-task or β-task specific training with probability one.

In contrast when training jobs are single-task, the job in which it is optimal to start depends on whether core jobs are either single- or multi-tasking. If one of the core jobs is multi-task, total career wages are the same when a worker starts his career in either training job since a worker's productivity in either core job is the same whether he or she comes from training job 1 or 2. If core jobs are single-task, $a^* \leq Ea$ –that is, an untrained worker of unknown ability is more productive in the difficult job– and task-specific training

[10]Notice that this is non-negative since the firm can always ensure a profit of zero in the second period by hiring an untrained worker or closing down.

acquired in training job 1 increases his productivity in an easy job only, while that acquired in training job 2 do so in a difficult job only, total career wages are maximized by assigning a worker to job 2. This maximizes a worker's expected productivity since it minimizes the probability that a worker is promoted to a job where his task-specific training is useless. The opposite occurs when an untrained worker of unknown ability is more productive in an easy job ($a^* > Ea$).

3.2 Optimal Job Design

Because all firms compete for workers in a Bertrand-like fashion with symmetric information and they are ex-ante identical, the optimal job design will be the one that maximizes the expected life-time wages for a worker. In the absence of discounting, this is exactly a worker's equilibrium expected output in period 2, which is $W_j(\mathbf{t})$. Hence, the equilibrium job design will be the one that maximizes expected period 2 output.

Formally, the optimal task allocation, denoted by $\mathbf{t}^*$, solves the following problem:

$$\max_{\mathbf{t}\in[0,1]^4} W_j(\mathbf{t}).$$

When designing jobs, firms must trade-off the benefits of specialization against the costs due to the dis-economies of scope on training accumulation and the benefits of task complementarities. When there is no uncertainty about a worker's innate ability in the sense that firms know ex-ante whether an untrained worker will be more productive in an easy or difficult job, the trade-off is trivially solved. Suppose that firms know with certainty that a worker's innate ability will be below a^*–that is, a worker with no training will be always more productive in the easy job. Then, a worker specialized in the A task who is placed in the easy job produces $m_e + n_e a + 1$ while a worker who has been trained in both tasks produces $m_e + n_e a + s(2\mu + \sigma)$. Thus, if $1 > s(2\mu + \sigma)$, it is better to allocate one training task to each training job and one core task to the easy job. In other words, when the benefits of specialization $1 - \mu$ exceed the benefit from task complementarity σ minus the loss productivity due to the dis-economies of scope on training acquisition $\mu(1-2s) + \sigma(1-s)$, the optimal is to design both core and training jobs as single-task jobs, while when the opposite occurs one of the training jobs and an easy job are designed as multi-task jobs. When there is uncertainty about a worker's innate ability in the sense that firms do not know ex-ante whether an untrained worker will be more productive in an easy or difficult job, the trade-off is not easily solved since ex-post it may be optimal to assign a worker to a job where his task-specific training is useless.

Before deriving the optimal job design under uncertainty, several useful cutoffs must be defined. Let $s_E \equiv \triangle n \int_{a^*}^{a^*+\frac{1}{\triangle n}} \Phi(a)\, da$ and $s_D \equiv 1 - \triangle n \int_{a^*-\frac{1}{\triangle n}}^{a^*} \Phi(a)\, da$[11]. The cutoff s_E (s_D) is the minimum training level for which total career wages from a multi-task training job and single-task core jobs are equal to those from single-tasking at both levels.

[11] Notice that $s_E > \frac{1}{2}$ and $s_D > \frac{1}{2}$.

Next let $s(\sigma, \mu)$ be the lowest dis-economies of scope on training acquisition above which multi-tasking at both levels yields higher output than single-tasking at both levels. That is, the lowest s such that $(2\mu + \sigma) s \geq 1$. Lastly, let define $s_E(\sigma, \mu)$ $(s_D(\sigma, \mu))$ as the minimum dis-economies of scope above which multi-tasking at the training level and in an easy (difficult) job yields higher total career wages than multi-tasking at the training level but single-tasking at the core level.

Proposition 2 *(1) Suppose that $a^* > Ea$. (i) If $s \leq \min\{s_E, s(\sigma, \mu)\}$, then both training and core jobs are single-task; (ii) if $s_E(\sigma, \mu) \geq s > s_E$, then one of the training jobs is multi-task and core jobs are single-task; and (iii) if $s > \max\{s_E(\sigma, \mu), s(\sigma, \mu)\}$, then one of the training jobs is mult-task and an easy job is multi-task; and*
(2) Suppose that $a^* \leq Ea$. (i) If $s \leq \min\{s_D, s(\sigma, \mu)\}$, then both training and core jobs are single-task; (ii) if $s_D(\sigma, \mu) \geq s > s_D$, then one of the training jobs is multi-task and core jobs are single-task; and (iii) if $s > \max\{s_D(\sigma, \mu), s(\sigma, \mu)\}$, then one of the training jobs is mult-task and a diffcult job is multi-task;

Proof. See the Appendix. ■

The results in this proposition are depicted in figure 1 for the case in which $a^* > Ea$. The figure looks exactly the same for the case in which $a^* \leq Ea$. The intuion can be easily grasped from the figure.

When the dis-economies of scope are sufficiently severe ($s \leq s_j$, $j = E, D$) and the degree of task complementarity is low neither training nor core jobs are designed as multi-task jobs. The reason is that neither the benefits from complementarities nor the benefits from diversifying the risk associated with whether it might be optimal ex-post to allocate a worker to a job where his task-specific training is unproductive can outweigh the benefits from specialization plus the cost due to the dis-economies of scope. When s continues to be small ($s \leq s_j$), but the benefits from task complementarities are such that $s > s(\sigma, \mu)$, multi-tasking at both levels is optimal since the productivity gain due to complementarities outweighs the costs due to multi-tasking (dis-economies of scope and forgone specialization benefits).

When the dis-economies of scope are less severe ($s > s_j$) but still important ($s \leq s_j(\sigma, \mu)$), one of the training jobs is multi-tasking and core jobs are single-tasking. The reason is that the productivity gain due to task complementarities cannot outweigh the benefit from diversifying the risk (through training the worker in both tasks) associated with the fact that ex-post it might be optimal to allocate a worker to a job where his task-specific training goes completely under-utilized. In other words, the benefits from diversifying the risk that task-specific training may add no value ex-post is higher than the benefit due to task complementarities. Finally, when the dis-economies of scope are small ($s > \max\{s_E(\sigma, \mu), s(\sigma, \mu)\}$) and the benefits from task complementarities are sufficiently large ($\sigma > 0$), multi-tasking at both levels maximizes total career wages. In this case, the productivity gain due to task complementarities more than outweighs the productivity gain due to specialization, the cost due to the dis-economies of scope, and the

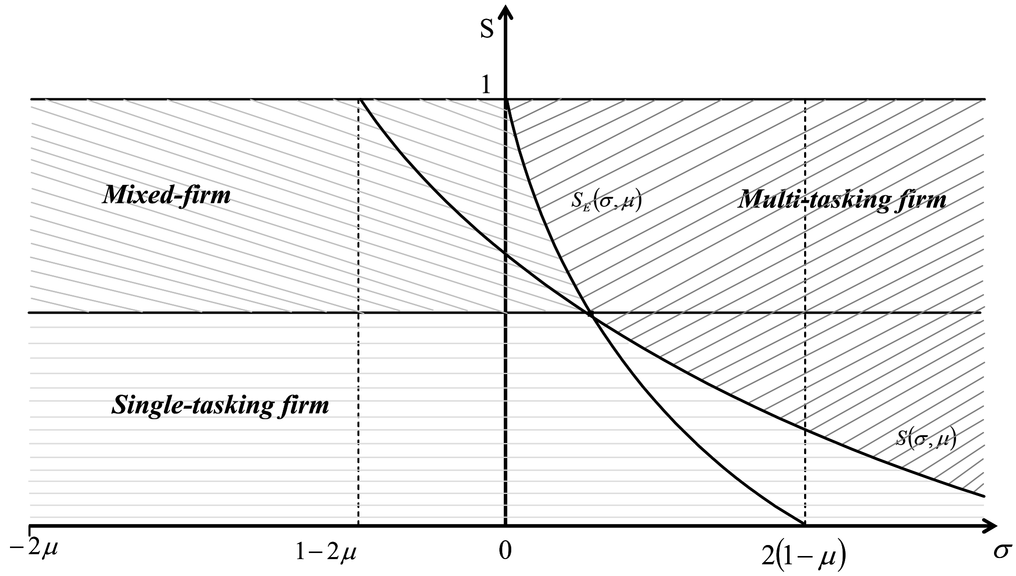

Figure 1:

benefit from diversifying the risk that training goes unutilized ex-post. In particular, when an untrained worker of average ability is more productive in a difficult job, this job is designed as a multi-task job while the opposite occurs when an untrained worker of average ability is more productive in a easy job since this minimizes the probability that a worker trained in both tasks is ex-post assigned to a job where his training has no value. Thus, the firm, by allocating the tasks where an untrained worker of average ability is more productive, ensures that a trained worker will be assigned more often to a job in which his ability and task-specific training are more productive, which in turn reduces the ex-post under-utilization of task-specific training.

In the next proposition I derive the comparative statics with respect to the main parameters μ, s and σ.

Proposition 3 *Multi-tasking is more likely as s, μ and σ increase.*

This result implies that as the benefits from specialization fall, the gains due to complementarities increases and the dis-economies of scope decrease, the parameter space under which single-tasking at the training and core level maximizes career wages is enlarged. It also implies that the parameter space under which multi-tasking at the training level only and that under which specialization are optimal shrinks. Thus, the factors favoring mult-tasking are: lower productivity gains due to specialization, lower dis-economies of scope on training acquisition and higher task complementarities.

3.3 Discussion

In this sub-section, I briefly discuss the relationship between multi-tasking and wages, and I argue that firms have an incentive to invest in task-specific training.

First, the results from proposition (3) suggest that any technological change that increases task complementarities, decreases the dis-economies of scope and reduces the benefits due to specialization thus creates conditions that favor the adoption multi-tasking jobs at some level. For instance, Autor et al. (2002, 2003) finds that the adoption of flexible work practices such as multi-tasking is positively related to the massive introduction of computerized information and communication systems –spurred by precipitous real price declines, the introduction of flexible machine tools and programmable equipment, and the increase in human capital that occurred during the 1990s. Several other papers have documented the existence of a positive correlation between the adoption of flexible work practices and the trends mentioned above for the US and other OECD countries during the 1990s (see, for instance, Black and Lynch 2005, Bresnahan et al. 2002; Caroli and VanReenen 2001; and Autor et al. 2001). Thus, to the extent that the new technologies developed during the 1980s and 1990s affect the parameters (s, σ, μ) in the proper direction, the results in this paper provide a rationale for the adoption of multi-tasking and other flexible-work practices.

Second, the model makes concrete predictions concerning the working of internal labor markets and job design. The model predicts that a worker's wage is determined by the market with probability $E_a F\left(\eta_{j(a)}(\theta, a)\right)$ and inside the firm (by bargaining between the current employer and the worker) with probability $1 - E_a F\left(\eta_{j(a)}(\theta, a)\right)$. When a wage is determined by the market it is given by: $y_{j(a)}\left(\mathbf{s}^j, t_c, a\right)$ while when it is internally determined is given by: $\theta\left[y_{j(a)}\left(\mathbf{s}^j, t_c, a\right) + \delta + \eta\right]$. In addition, the outside option principle ensures that an internally determined wage is higher than market determined wage.

Because a necessary condition for multi-tasking jobs to be optimal is that $(2\mu + \sigma) s \geq 1$, wages are less often determined internally by multi-tasking firms. In fact, the higher s, μ and σ, the less likely are internally determined wages. This means that in specialized firms, wages are more often determined by bargaining.

A worker whose average ability satisfies $Ea < a^*$ is promoted to the easy job with probability $\Phi\left(a^* + \frac{s(2\mu+\sigma)}{\triangle\beta}\right)$ in a multi-tasking firm and with probability $\Phi\left(a^* + \frac{1}{\triangle\beta}\right)$ in a specialized firm. It follows from this and proposition (2) that a worker is more likely to be promoted to easy job in a multi-tasking firm. In contrast, a worker whose average ability is given by $Ea > a^*$ is promoted to an easy job with probability $\Phi\left(a^* - \frac{s(2\mu+\sigma)}{\triangle\beta}\right)$ in a multi-tasking firm and with probability $\Phi\left(a^* - \frac{1}{\triangle\beta}\right)$ in a specialized firm. It follows from this and proposition (2) that a worker is more likely to be promoted to a difficult job in a multi-tasking firm.

A worker's probability of being promoted to an easy job in a mixed firm is $\Phi(a^*)$ regardless of his average ability. This implies that the promotion decision in this case is independent of the level of task-specific training and that a worker with $Ea < a^*$ is promoted less frequently to an easy job in a mixed firm than in any other type of firm while the opposite occurs when $Ea > a^*$.

Third, the model also provides a rationale for firm sponsored task-specific training. This is so in spite of the fact that Becker's human capital theory predicts that firms will not provide task-specific training because that increases a worker's productivity by the same amount inside as well as outside of the firm. However, Balmaceda (2005) shows that under the wage bargaining procedure used here, firms do have an incentive to invest in training that is equally productive inside as well as outside of the firm. The reason is simple. When a worker's outside option is non-binding, which occurs with probability $1 - F\left(\eta_{j(a)}(\theta, a)\right)$, the firm and worker share the return of task-specific training. In fact, one can see from equation (4), that the firm's payoff when the surplus sharing outcome occurs is $(1-\theta)\left[y_{j(a)}\left(\mathbf{s}^j, t_c, a\right) + \delta + \eta\right]$ and thus the firm grabs a share $(1-\theta)$ of the marginal return on task-specific training. In addition, Bertrand-like competition during the first period impedes firms from recovering their investment in task-specific training by lowering the first-period wage. Thus, firms do provide and pay for task-specific training as long as the surplus sharing outcome ocurrs with positive probability.

4 Flexible-work Practices

4.1 Job Rotation

Job rotation involves the movement of employees through a range of jobs in order to either increase interest and motivation or to improve multi-skilling or both[12]. This approach widens the activities of a worker by switching him or her through a range of tasks. For example, at General Electric Co. –a veteran of rotational training– recent college graduates and MBAs are hired directly into one of the company's seven rotational programs, not into a specific position. Over the course of the two-year program, GE trainees are transferred across locations and businesses, sometimes even out of the country[13]. They earn a professional salary and can garner raises based on frequent evaluations by their direct manager or local program leader.

More (1980) documents that during the industrial revolution, one of the methods of developing skills was a practice called migration, which involved rotating trainees among tasks within and among firms[14]. Similarly, casual observation shows that training of medical students around the world includes a system of period rotation among all specialities.

[12]Osterman (1994) reports that in the US over half of all private establishments with 50 or more employer used some sort of team work and job rotation during 1992. Furthermore, he found that the higher the skill level required in an establishment, the more likely is it that multi-tasking and job rotation will be adopted.

[13]Four rotations of six to eight months each, plus classroom instruction, provide trainees with intensive, accelerated experiences that it may take years to gather in a standard corporate career.

[14]The Shakers in the US and the *Kibbutzim* in Israel have used job rotation as a practice to develop skills for many years.

The results of the model can be re-interpreted to provide a rationale for job rotation as a justification for the enhancement of multi-skilling. In order to do so, let us assume that for technological reasons training jobs cannot be designed as multi-task jobs and thus the only feasible way in which a worker can acquire task-specific training in more than one task is by being moved from one job to another during the training period.

In what follows, it is assumed that a worker who starts his career in training job 1 (job 2) acquires one unit of α-task (β-task) specific training if he stays the whole period in that job– that is there is no job rotation– and $s \leq 1$ units of α-task (β-task) specific training and $s \leq 1$ units of β-task (α-task) specific training if he is moved from job 1 (2) to job 2 (1) half-way through the first period.

Because of Bertrand-like competition, job-rotation will be observed only when total career wages from job rotation are higher than those from specialization. Adopting a job-rotation policy yields the same payoff as when one of the training jobs is designed as a multi-task job and specialization corresponds to the case in which both training jobs are single-task jobs. Thus, the next proposition readily follows from proposition (2) in Section 3.2.

Proposition 4 *(i) Job rotation maximizes total career wages as long as* $s \geq s_R(\sigma, \mu)$, *where* $s_R(\sigma, \mu) \equiv \min\{s_E, s_E(\sigma, \mu)\}$ *if* $a^* > Ea$ *and* $s_R(\sigma, \mu) \equiv \min\{s_D, s_D(\sigma, \mu)\}$ *otherwise; and (ii)* $s_R(\sigma, \mu)$ *is non-increasing with* σ *and* μ.

This proposition establishes that a job-rotation policy maximizes a worker's total careers wages when the dis-economies of scope in task-specific training acquisition are not too severe. In addition, it says that the lower the return from specialization and the larger the return from task complementarities, the more likely a firm is to adopt a job rotation policy.

It is worthy of note that there are two different rationales for job rotation. The most obvious one is that it allows reaping the benefits of task complementarities between two types of task-specific training. The less obvious rationale is that it allows diversifying the risk that task-specific training goes fully unused ex-post. The former arises when the benefits from task complementarities are large while the latter when they are small.

4.2 Ex-post Task Re-allocations

The evidence shows that flexible work practices are more common today in the US economy and OECD countries than a decade ago. For instance, Osterman (1994, 2000) reports that there is an increasing use of some forms of flexible work in the US economy. In particular, multi-tasking in its different forms is becoming a key requirement in many successful organizations. Here a particular kind of flexible work practice is studied, which is the possibility of relocating tasks ex-post.

In order to make the problem interesting, I will assume that ex-post task relocations are feasible if and only if a particular kind of technology such as flexible machine tools and programmable equipment is adopted. The cost of this technology is F per-worker[15].

It is easy to show that the optimal task allocation ex-post is to re-allocate tasks to the job where an untrained worker is more productive because that maximizes a worker's output ex-post.

Lemma 3 *The optimal task allocation ex-post, denoted by* t_c^r*, is* $t_c^r = (1,1)$ *if* $a \leq a^*$ *and* $t_c^r = (0,0)$ *if* $a > a^*$*.*

Given this result and the fact that for any task allocation, the market as well as the incumbent firm follows the same second-period job assignment policy, it is easy to see that total career wages, denoted by $W_j^r(\mathbf{t})$, are given by:

$$W_j^r(\mathbf{t}) \equiv E_{a>a^*}\left\{\int_{-\delta}^{\overline{\eta}} y_d\left(\mathbf{s}^j, t_c^r, a\right) dF + \int_{\underline{\eta}}^{-\delta} y_d\left(\mathbf{s}^j, t_c^r, a\right) dF\right\} +$$
$$E_{a\leq a^*}\left\{\int_{-\delta}^{\overline{\eta}} y_e\left(\mathbf{s}^j, t_c^r, a\right) dF + \int_{\underline{\eta}}^{-\delta} y_e\left(\mathbf{s}^j, t_c^r, a\right) dF\right\} + \int_{-\delta}^{\overline{\eta}} (\delta+\eta)\, dF.$$

The next result derives the optimal initial task-allocation given that tasks can be relocated ex-post at no cost.

Lemma 4 *If* $s(2\mu+\sigma) \leq 1$*, then both training and core jobs are designed as single-task while if* $s(2\mu+\sigma) > 1$*, then one of the training jobs and one of the core jobs are designed as multi-task, and an easy job is ex-post multi-task when* $a \leq a^*$ *while a difficult job is so when* $a > a^*$*.*

The driving force behind this result is that ex-post task relocations allow the firm to minimize the under-utilization of task-specific training since tasks are re-assigned to a job where a worker's innate ability is more productive. This implies that when the total productivity of task-specific training is higher under multi-tasking than under single-tasking–that is $s(2\mu+\sigma) > 1$– training jobs are multi-task, otherwise they are single-task.

Because total career wages when a non-flexible technology is adopted are $W_j(\mathbf{t}^*)$, while those when a flexible technology is adopted are $W_j^r(\mathbf{t}^r)$, a flexible technology is adopted when the benefits of avoiding under-utilization of task-specific training upon a promotion ($W_j^r(\mathbf{t}^r) - W_j(\mathbf{t}^*)$) exceed the fixed cost of adopting a flexible technology (F). Simple comparisons of total career wages lead to the following result.

Let $j = D$ if $a^* \leq Ea$ and $j = E$ otherwise.

Proposition 5 *(i) If* $s \leq \min\left\{s_j, s(\sigma,\mu)\right\}$*, then a flexible technology is adopted if* $F < 1 - \int_{a^*}^{a^*+\frac{1}{\triangle n}} \Phi(a)\, da$*; (ii) if* $s(\sigma,\mu) \geq s > s_E$*, then a flexible technology is adopted if* $F < 1-s$ *while if* $s_j(\sigma,\mu) \geq s > s(\sigma,\mu)$*,*

[15] An commonly used argument to explain why some companies fail to adopt flexible-work practices is that they require considerable up-front costs, but their returns are long-run in nature (Porter, 1992).

a flexible technology is adopted if $F < 1 - s(2\mu + \sigma)$*; and (iii) if* $s > \max\{s_E(\sigma, \mu), s(\sigma, \mu)\}$*, then a flexible technology is adopted if* $F < s(2\mu + \sigma) - \int_{a^*}^{a^* + \frac{s(2\mu+\sigma)}{\triangle n}} \Phi(a)\, da$.

As expected when the up-front cost per worker is small, a flexible technology is adopted. The most interesting part of this result is that the higher the return of task complementarities and the lower the benefit of specialization, the more likely it is to observe task relocation practices. This is so because task relocations ensure ex-post that it is possible to exploit the benefits of task complementarities regardless of a worker's realized ability.

5 Wage Inequality and Job Design

The empirical evidence for the US (see, for instance, Gottschalk 1997) shows that since the late 1970s the college premium (the difference between the average weekly earnings of collegate graduates and those of high-school graduates) has increased significantly and the experience premium (the difference between the average weekly earnings of people who have been in the labor force for a long time and those who have been there for a short period of time) for males grew rapidly during the 1970s, continued growing during the 1980s but then leveled off at a very high level during the 1990s. The experience premium for females also started growing in the early 1970s, but the largest increases came in the 1980s and 1990s. Thus, it is plausible that changes in overall wage inequality reflect to certain extent the fact that less educated workers lost relative to the more educated, and more experienced workers gained relative to younger workers. However, inequality increased not only among different groups such as whites and blacks, older and younger workers, but also within groups of workers having the same trait. That is, within a group the residuals of a wage regression for a worker of the 90th percentile compared to the one in the 10th percentile grew 38 percent between 1973 and 1994.

Within the same period of time, the nature of jobs has been changing and firms have been moving from specialization toward multi-tasking. Hammer and Champy (1994) provides several examples of how large US companies like Kodak, IBM, Hallmark and Bell Atlantic moved in several areas from the old-fashioned division of labor principle to the most modern principle of mult-tasking. Osterman (1994, 2000) reports that there is an increasing use of some forms of flexible work, many of which involve some kind of multi-tasking, in the US economy. In fact, he finds that in 1992, 24.6 percent of establishments involved a substantial share of their workforce in two or more practices, one of which was job rotation; by 1997 it had grown to 38.3 percent.

Research using US data shows that the adoption of newer more flexible work practices in the production process have raised the demand for skilled labour, i.e., that new technologies and new work practices are skill biased (see e.g., Autor and Katz 1999 and Caroli and Van Reenen 2001). Furthermore, the evidence points to a complementarity between the adoption of these newer more flexible work practices and the use of new production and telecommunications technologies. New work organization practices were strongly associated with problem-solving and interpersonal skill increases, suggesting that these practices are broadening the set of skills sought by manufacturers. Traditional academic skills (e.g., math and reading) also were linked to the use of flexible technologies and new work organization practices, but increases in general skill requirements were reported less frequently than were requirements for computer, interpersonal, and problem-solving skills (see, Gale et. al 2002, Autor et. al. 2002 and 2003).

The experience premium in the present setting, denoted by Exp, is given by $E(w_2) - E(w_1)$. After some simple algebra and integration by parts one can show that the experience premium is:

$$Exp = E_a \left\{ (2\theta - 1) \left[y_{j(a)} \left(\mathbf{s}^j, t_c, a \right) + \delta + \overline{\eta} \right] + \int_{-\delta}^{\overline{\eta}} F(\eta)\, d\eta - 2\theta \int_{\eta_{j(a)}(\theta, a)}^{\overline{\eta}} F(\eta)\, d\eta \right\}. \qquad (6)$$

For the sake of simplicity, in what follows it is assumed that:

- (A3): $a^* > Ea$, $\theta = \frac{1}{2}$ and $F(\eta)$ is uniform with support $[-\overline{\eta}, \overline{\eta}]$.

Proposition 6 *(i) If* $s \leq \min\{s_E, s(\sigma, \mu)\}$*, then the experience premium is higher in a single-task firm; (ii) if* $s_E(\sigma, \mu) \geq s > s_E$*, then the experience premium is higher in a mixed firm; and (iii) if* $s >$ $\max\{s_E(\sigma, \mu), s(\sigma, \mu)\}$*, then the experience premium is higher in a multi-tasking firm and increases with* μ *and* σ*.*

This proposition shows that in a cross-section of firms the experience premium is higher in multi-tasking firms. That is, workers hired in a multi-tasking firm have a higher experience premium than those in single-task and mixed firms. Furthermore, those in mixed firms have a higher experience premium than those in single-task firms. This implies that workers who engage in mult-tasking training have a higher experience premium that those who specialize in one task only. It is worth noting that this explanation for the experience premium is driven by technological changes and not changes in the demand for skills. Nonetheless, the model can also help us understand the change in demand for skills if one believes that s is positively correlated with the schooling level. This is true as long as a worker's ability to learn on the job is positively correlated with his schooling level (either because schooling is useful to learning on the job or because there is a positive relationship between schooling and innate ability to learn on the job). The reason is simple. An increase in schooling implies a decrease in the dis-economies of scope on training accumulation and thus it results in a larger s. For a given return on specialization μ and a given return on complementarities σ, this means that multi-tasking is more likely to be the optimal job design and thus a schooling premium arises since wages are higher for workers with a higher schooling level.

While there is no specific evidence that I am aware of concerning the relationship between return on tenure and multi-tasking, the model predicts that there is a link between tenure effect and job design that is worth highlighting.

Recall that the tenure coefficient is the part of the wage growth which is not common to both stayers and leavers. In this setting, the tenure coefficient is

$$T = \max\left\{\theta\left[y_{j(a)}\left(\mathbf{s}^{j}, t_c, a\right) + \delta + \eta\right], y_{j(a)}\left(\mathbf{s}^{j}, t_c, a\right)\right\} - y_{j(a)}\left(\mathbf{s}^{j}, t_c, a\right),$$

and thus the the tenure effect, denoted by$E(T)$, is (after some simple algebra and integration by parts)

$$E(T) = E_a\left\{(\theta - 1)\, y_{j(a)}\left(\mathbf{s}^{j}, t_c, a\right) + \theta\left(\delta + \overline{\eta}\right) - \theta\int_{\eta_{j(a)}(\theta,a)}^{\overline{\eta}} F(\eta)\, d\eta\right\}. \tag{7}$$

The next result presents the link between the tenure effect and job design.

Proposition 7 *(i) If $s \leq \min\{s_E, s(\sigma,\mu)\}$, then the tenure effect is lower in a single-task firm; (ii) if $s_E(\sigma,\mu) \geq s > s_E$, then the tenure effect is lower in a mixed firm; and (iii) if $s > \max\{s_E(\sigma,\mu), s(\sigma,\mu)\}$, then the tenure effect is lower in a multi-tasking firm and the tenure effect decreases with μ and σ.*

This proposition shows that in a cross-section of firms the tenure effect is lower in multi-tasking firms. That is, workers hired in a multi-tasking firm have a lower tenure effect than those in single-task and mixed firms. Furthermore, those in mixed firms have a lower tenure effect that those in single-task firms.

In short, the model predicts that in a cross-section of firms, workers in multi-tasking firms have more experience and college premium but a lower tenure effect than more specialized workers. This suggests that the massive introduction of computerized information and communication systems, the introduction of flexible machine tools and programmable equipment, and the increase in human capital that occurred in the 1990s can aid at explaining the college and experience premium.

6 Final Comments

In this paper I have studied how a new type of training affects the design of the workplace. In particular, I have shown that an optimal job design is determined by a particular interaction between three elements: (i) the dis-economies of scope on task-specific training acquisition; (ii) the degree of complementarity between core tasks; and (iii) the increased productivity due to specialization. The novelty of this paper is that the interaction between these three dimension of job design is driven by the fact that at the time of designing jobs, firms are uncertain about workers' ability. Mainly, firms have to combine these elements to maximize total career wages having in mind that ex-post it might be optimal to allocate a worker to a job where his task-specific training is useless. This plays against specialization and multi-tasking at the core level, and thus an organizational form in which multi-tasking at the training level and single-tasking at the core level takes place is optimal under certain parameters.

The paper also contributes to understanding when and why flexible-work practices such as job-rotation and task re-allocations should be adopted and how multi-tasking can help to explain the college and experience premium.

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A Appendix

Proof. Proposition 2. Let the outside option be $y_{j'(a)}\left(\mathbf{s}^j, t_c', a\right)$, then there are two cases to consider: (1) when the outside option is non-binding; and (ii) when it is binding. I will take each in turn.

Let the outside option be non-binding regardless of whether a worker is promoted or not. Then, the incumbent chooses to promote a worker to a difficult job if and only if $(1-\theta)\left[y_d\left(\mathbf{s}^j, t_c, a\right)+\delta+\eta\right] \geq (1-\theta)\left[y_e\left(\mathbf{s}^j, t_c, a\right)+\delta+\eta\right]$, which boils down to the efficient promotion policy. That is, promote if and only if $y_d\left(\mathbf{s}^j, t_c, a\right) \geq y_e\left(\mathbf{s}^j, t_c, a\right)$.

Suppose next that the outside option is binding irrespective of whether a worker is either promoted to a difficult job or to an easy job. Then, the incumbent choose to allocate a worker to a difficult job if and only if $y_d\left(\mathbf{s}^j, t_c, a\right)+\delta+\eta-y_{j'(a)}\left(\mathbf{s}^j, t_c', a\right) \geq y_e\left(\mathbf{s}^j, t_c, a\right)+\delta+\eta-y_{j'(a)}\left(\mathbf{s}^j, t_c', a\right)$, which again boils down to the efficient promotion policy.

Let the outside option be binding when a worker is assigned to a difficult job, but be non-binding when he is assigned to an easy job. This requires $\frac{y_{j'(a)}\left(\mathbf{s}^j, t_c', a\right)}{\theta}-y_e\left(\mathbf{s}^j, t_c, a\right)-\delta \leq \eta < \frac{y_{j'(a)}\left(\mathbf{s}^j, t_c', a\right)}{\theta}-y_d\left(\mathbf{s}^j, t_c, a\right)-\delta$, which holds if and only if $y_d\left(\mathbf{s}^j, t_c, a\right) \leq y_e\left(\mathbf{s}^j, t_c, a\right)$. Thus, in this case a worker is not promoted whenever $y_d\left(\mathbf{s}^j, t_c, a\right)+\delta+\eta-y_{j'(a)}\left(\mathbf{s}^j, t_c', a\right) < (1-\theta)\left[y_e\left(\mathbf{s}^j, t_c, a\right)+\delta+\eta\right]$. This inequality can be written as $y_d\left(\mathbf{s}^j, t_c, a\right)+\delta+\theta\eta-y_{j'(a)}\left(\mathbf{s}^j, t_c', a\right) < (1-\theta)\left[y_e\left(\mathbf{s}^j, t_c, a\right)+\delta\right]$. The left-hand side is then smaller than $(1-\theta)\left[y_d\left(\mathbf{s}^j, t_c, a\right)+\delta\right]$, which is lower than $(1-\theta)\left[y_e\left(\mathbf{s}^j, t_c, a\right)+\delta\right]$ since $y_d\left(\mathbf{s}^j, t_c, a\right) \leq y_e\left(\mathbf{s}^j, t_c, a\right)$. Thus, a worker is never promoted when $y_d\left(\mathbf{s}^j, t_c, a\right) \leq y_e\left(\mathbf{s}^j, t_c, a\right)$.

Finally, let the outside option be binding when a worker is assigned to an easy job, but be non-binding when he is assigned to a difficult job. This requires $\frac{y_{j'(a)}\left(\mathbf{s}^j, t_c', a\right)}{\theta}-y_d\left(\mathbf{s}^j, t_c, a\right)-\delta \leq \eta < \frac{y_{j'(a)}\left(\mathbf{s}^j, t_c', a\right)}{\theta}-y_e\left(\mathbf{s}^j, t_c, a\right)-\delta$, which is possible only when $y_d\left(\mathbf{s}^j, t_c, a\right) > y_e\left(\mathbf{s}^j, t_c, a\right)$. Thus, in this case a worker is promoted whenever $(1-\theta)\left[y_d\left(\mathbf{s}^j, t_c, a\right)+\delta+\eta\right] > y_e\left(\mathbf{s}^j, t_c, a\right)+\delta+\eta-y_{j'(a)}\left(\mathbf{s}^j, t_c', a\right)$. This inequality can be written as $(1-\theta)\, y_d\left(\mathbf{s}^j, t_c, a\right) > y_e\left(\mathbf{s}^j, t_c, a\right)+\theta(\delta+\eta)-y_{j'(a)}\left(\mathbf{s}^j, t_c', a\right)$. The right-hand side is then smaller than $(1-\theta)\left[y_e\left(\mathbf{s}^j, t_c, a\right)+\delta\right]$, which is lower than $(1-\theta)\left[y_d\left(\mathbf{s}^j, t_c, a\right)+\delta\right]$ since $y_d\left(\mathbf{s}^j, t_c, a\right) > y_e\left(\mathbf{s}^j, t_c, a\right)$. Thus, a worker is always promoted when $y_d\left(\mathbf{s}^j, t_c, a\right) > y_e\left(\mathbf{s}^j, t_c, a\right)$. ■

Proof. of Prooposition 1.

Let us define $\epsilon\left(\mathbf{s}^j, 1-t_c\right) \equiv \mu\left(1-t_c, \mathbf{s}^j\right)\left(s_1^j\left(1-t_\alpha\right)+s_2^j\left(1-t_\beta\right)\right)+\sqrt{s_1^j s_2^j}\sigma^d\left(1-t_\alpha\right)\left(1-t_\beta\right)$ and $\epsilon\left(\mathbf{s}^j, t_c\right) \equiv \mu\left(t_c, \mathbf{s}^j\right)\left(s_1^j t_\alpha+s_2^j t_\beta\right)+\sqrt{s_1^j s_2^j}\sigma t_\alpha t_\beta$. Then $a\left(\mathbf{s}^j, t_c\right) \equiv a^*+\frac{\epsilon\left(\mathbf{s}^j, t_c\right)-\epsilon\left(\mathbf{s}^j, 1-t_c\right)}{\triangle n}$.

Notice first that

t_A	t_α	t_B	t_β	$\epsilon\left(\mathbf{s}^1, t_c\right)$	$\epsilon\left(\mathbf{s}^1, 1-t_c\right)$	$\epsilon\left(\mathbf{s}^2, t_c\right)$	$\epsilon\left(\mathbf{s}^2, 1-t_c\right)$	$\triangle\epsilon_1$	$\triangle\epsilon_2$	$a\left(\mathbf{s}^1, t_c\right)-a\left(\mathbf{s}^2, t_c\right)$
1	1	1	1	$s(2\mu+\sigma)$	0	0	0	$s(2\mu+\sigma)$	0	$s(2\mu+\sigma)$
1	1	1	0	s	s	0	0	0	0	0
1	1	0	1	1	0	1	0	1	1	0
1	1	0	0	1	0	0	1	1	-1	1
1	0	1	1	s	s	0	0	0	0	0
1	0	1	0	0	$s(2\mu+\sigma)$	0	0	$-s(2\mu+\sigma)$	0	$-(s(2\mu+\sigma))$
1	0	0	1	0	1	1	0	-1	1	-1
1	0	0	0	0	1	0	1	-1	-1	0
0	1	1	1	1	0	1	0	1	1	0
0	1	1	0	0	1	1	0	-1	1	-1
0	1	0	1	0	0	$s(2\mu+\sigma)$	0	0	$s(2\mu+\sigma)$	$-(s(2\mu+\sigma))$
0	1	0	0	0	0	s	s	0	0	0
0	0	1	1	1	0	0	1	1	-1	1
0	0	1	0	0	1	0	1	-1	-1	0
0	0	0	1	0	0	s	s	0	0	0
0	0	0	0	0	0	0	$s(2\mu+\sigma)$	0	$-s(2\mu+\sigma)$	$s(2\mu+\sigma)$

Notice next that $W_1(\mathbf{t})-W_2(\mathbf{t})$ is given by:

$$E_{a>a\left(\mathbf{s}^1, t_c\right)}\left(m_d+n_d a+\epsilon\left(\mathbf{s}^1, 1-t_c\right)\right)+E_{a\leq a\left(\mathbf{s}^1, t_c\right)}\left[m_e+n_e a+\epsilon\left(\mathbf{s}^1, t_c\right)\right]-$$
$$E_{a>a\left(\mathbf{s}^2, t_c\right)}\left(m_d+n_d a+\epsilon\left(\mathbf{s}^2, 1-t_c\right)\right)-E_{a\leq a\left(\mathbf{s}^2, t_c\right)}\left[m_e+n_e a+\epsilon\left(\mathbf{s}^2, t_c\right)\right].$$

Let us suppose first that $a\left(\mathbf{s}^1, t_c\right)=a\left(\mathbf{s}^2, t_c\right)$. This implies that $\epsilon\left(\mathbf{s}^1, t_c\right)-\epsilon\left(\mathbf{s}^1, 1-t_c\right)=\epsilon\left(\mathbf{s}^2, t_c\right)-\epsilon\left(\mathbf{s}^2, 1-t_c\right)$ and therefore $W_1(\mathbf{t})-W_2(\mathbf{t})$ is given by:

$$\epsilon\left(\mathbf{s}^1, 1-t_c\right)-\epsilon\left(\mathbf{s}^2, 1-t_c\right).$$

It readily follows from this that $W_1(\mathbf{t})-W_2(\mathbf{t})>0$ if and only if $\mathbf{t}\in\{(1,1,1,0),(1,0,1,1)\}$, $W_1(\mathbf{t})-W_2(\mathbf{t})<0$ if and only if $\mathbf{t}\in\{(0,0,0,1),(0,1,0,0)\}$, and $W_1(\mathbf{t})-W_2(\mathbf{t})=0$ if and only if $\mathbf{t}\in\{(1,1,0,1),(1,0,0,0),(0,$

Next let us assume that $a\left(\mathbf{s}^1, t_c\right) > a\left(\mathbf{s}^2, t_c\right)$. Then, $W_1(\mathbf{t}) - W_2(\mathbf{t})$ is given by:

$$-\tfrac{1}{\triangle n}E_{a\left(\mathbf{s}^1,t_c\right)\geq a>a\left(\mathbf{s}^2,t_c\right)}(a^* - a) + E_{a>a_1(\mathbf{t})}\epsilon\left(\mathbf{s}^1, 1-t_c\right) + E_{a\leq a\left(\mathbf{s}^1,t_c\right)}\epsilon\left(\mathbf{s}^1, t_c\right) -$$
$$E_{a>a\left(\mathbf{s}^2,t_c\right)}\epsilon\left(\mathbf{s}^2, 1-t_c\right) - E_{a\leq a\left(\mathbf{s}^2,t_c\right)}\epsilon\left(\mathbf{s}^2, t_c\right),$$

The cases under which $a\left(\mathbf{s}^1, t_c\right) > a\left(\mathbf{s}^2, t_c\right)$ are the following: $\mathbf{t} \in \{(1,1,1,1), (1,1,0,0), (0,0,1,1), (0,0,0,0)\}$. Suppose that $\mathbf{t} = (1,1,1,1)$, then $W_1(\mathbf{t}) - W_2(\mathbf{t})$ is given by:

$$-\frac{1}{\triangle n}E_{a^*+\frac{s(2\mu+\sigma)}{\triangle n}\geq a>a^*}\left(a^* - a\right) + E_{a\leq a^*+\frac{s(2\mu+\sigma)}{\triangle n}}s\left(2\mu+\sigma\right) > 0.$$

Suppose next that either $\mathbf{t} = (0,0,1,1)$ or $\mathbf{t} = (1,1,0,0)$, then:

$$W_1(1,1,0,0) = E_{a>a^*+\frac{1}{\triangle n}}(m_d + n_d a) + E_{a\leq a^*+\frac{1}{\triangle n}}[m_e + n_e a + 1] + \int_{-\delta}^{\overline{\eta}}(\delta + \overline{\eta})\,dF,$$
$$W_2(1,1,0,0) = E_{a>a^*-\frac{1}{\triangle n}}[m_d + n_d a + 1] + E_{a\leq a^*-\frac{1}{\triangle n}}[m_e + n_e a] + \int_{-\delta}^{\overline{\eta}}(\delta + \overline{\eta})\,dF.$$

It readily follows from this that $W_1(1,1,0,0) \geq W_2(1,1,0,0)$ if and only if:

$$E_{a>a^*+\frac{1}{\triangle n}}(m_d + n_d a) + E_{a\leq a^*+\frac{1}{\triangle n}}[m_e + n_e a + 1] \geq$$
$$E_{a>a^*-\frac{1}{\triangle n}}[m_d + n_d a + 1] + E_{a\leq a^*-\frac{1}{\triangle n}}[m_e + n_e a]$$

After a few steps of simple algebra this can be written as:

$$-\triangle n\int_{a^*-x}^{a^*+x}\left(a - a^*\right)\phi(a)\,da + \left[\Phi\left(a^* + \frac{1}{\triangle n}\right) + \Phi\left(a^* - \frac{1}{\triangle n}\right) - 1\right] \geq 0,$$

where $x = \frac{1}{\triangle n}$.

Let us write $\int_{a^*-x}^{a^*+x}(a - a^*)\phi(a)\,da$ as $\int_{-x}^{x}a\phi(a + a^*)$ and then as $\int_{-x}^{0}a\phi(a + a^*)\,da + \int_{0}^{x}a\phi(a + a^*)\,da$. Use a change of variables in the first integral of $u = -a$, and in the second, allow $u = a$. Because of symmetry, $\phi(u) = \phi(-u)$, the two integrals can be written as:

$$\int_0^x u\left[\phi\left(a^* + u\right) - \phi\left(a^* - u\right)\right]du.$$

If $a^* = Ea$, because of symmetry of the density function, half of the population would have $a > a^*$ and half would have $a \leq a^*$. This implies that if $a^* > Ea$, $\phi(a^* + u) < \phi(a^* - u)$, while if $a^* \leq Ea$, $\phi(a^* + u) \geq \phi(a^* - u)$. Furthermore, since $\phi(a)$ is unimodal and symmetric around the mean, then $\Phi\left(a^* + \frac{1}{\triangle n}\right) + \Phi\left(a^* - \frac{1}{\triangle n}\right) > 1$ if $a^* > Ea$ while $\Phi\left(a^* + \frac{1}{\triangle n}\right) + \Phi\left(a^* - \frac{1}{\triangle n}\right) \leq 1$ if $a^* \leq Ea$. Thus, if $a^* \leq Ea$, $W_1(1,1,0,0) \leq W_2(1,1,0,0)$ while if $a^* > Ea$, $W_1(1,1,0,0) > W_2(1,1,0,0)$.

Suppose next that $\mathbf{t} = (0,0,0,0)$, then $W_1(\mathbf{t}) - W_2(\mathbf{t})$ is given by:

$$-\frac{1}{\triangle n}E_{a^*\geq a>a^*-\frac{s(2\mu+\sigma)}{\triangle n}}\left(a^* - a\right) - E_{a>a^*-\frac{s(2\mu+\sigma)}{\triangle n}}\left(s\left(2\mu+\sigma\right)\right) < 0.$$

Next let us assume that $a\left(\mathbf{s}^1, t_c\right) < a\left(\mathbf{s}^2, t_c\right)$, then $W_1(\mathbf{t}) - W_2(\mathbf{t})$ is given by:

$$\frac{1}{\triangle n} E_{a\left(\mathbf{s}^2,t_c\right)\geq a > a\left(\mathbf{s}^1,t_c\right)}\left(a^* - a\right) + E_{a > a\left(\mathbf{s}^1,t_c\right)}\epsilon\left(\mathbf{s}^1, 1-t_c\right) + E_{a\leq a\left(\mathbf{s}^1,t_c\right)}\epsilon\left(\mathbf{s}^1, t_c\right) - E_{a > a\left(\mathbf{s}^2,t_c\right)}\epsilon\left(\mathbf{s}^2, 1-t_c\right) - E_{a\leq a\left(\mathbf{s}^2,t_c\right)}\epsilon\left(\mathbf{s}^2, t_c\right),$$

The cases under which $a\left(\mathbf{s}^1, t_c\right) < a\left(\mathbf{s}^2, t_c\right)$ are the following: $\mathbf{t} \in \{(1,0,1,0), (1,0,0,1), (0,1,1,0), (0,1,0,1)\}$. Suppose that $\mathbf{t} = (1,0,1,0)$, then $W_1(\mathbf{t}) - W_2(\mathbf{t})$ is given by:

$$\frac{1}{\triangle n} E_{a^*\geq a > a^* - \frac{s(2\mu+\sigma)}{\triangle n}}\left(a^* - a\right) + E_{a > a^* - \frac{s(2\mu+\sigma)}{\triangle n}} s\left(2\mu+\sigma\right) > 0.$$

Suppose next that either $\mathbf{t} = (1,0,0,1)$ or $\mathbf{t} = (0,1,1,0)$, then $W_1(\mathbf{t}) - W_2(\mathbf{t})$ is given by:

$$\frac{1}{\triangle n} E_{a^* + \frac{1}{\triangle n}\geq a > a^* - \frac{1}{\triangle n}}\left(a^* - a\right) + 1 - \Phi\left(a^* - \frac{1}{\triangle n}\right) - \Phi\left(a^* - \frac{1}{\triangle n}\right) > 0.$$

This is positive if and only if $a^* > Ea$ and the opposite occurs otherwise.

Suppose next that $\mathbf{t} = (0,1,0,1)$, then $W_1(\mathbf{t}) - W_2(\mathbf{t})$ is given by:

$$\frac{1}{\triangle n} E_{a^* + \frac{s(2\mu+\sigma)}{\triangle n}\geq a > a^*}\left(a^* - a\right) - E_{a\leq a^* - \frac{\epsilon(2\mu+\sigma)}{\triangle n}} s\left(2\mu+\sigma\right) < 0.$$

■

Proof. of Proposition 2.

To save on notation, let us define $y_d(a)$ as $m_d + n_d a$ and $y_e(a)$ as $m_e + n_e a$, then

t_A	t_α	t_B	t_β	$\max\{W_1(\mathbf{t}), W_2(\mathbf{t})\}$
1	1	1	1	$E_{a>a^*+\frac{s(2\mu+\sigma)}{\triangle n}} y_d(a) + E_{a\leq a^*+\frac{s(2\mu+\sigma)}{\triangle n}} [y_e(a) + s(2\mu+\sigma)]$
1	1	1	0	$E_{a>a^*} y_d(a) + E_{a\leq a^*} y_e(a) + s$
1	1	0	1	$E_{a>a^*+\frac{1}{\triangle n}} y_d(a) + E_{a\leq a^*+\frac{1}{\triangle n}} [y_e(a) + 1]$
1	1	0	0	$\begin{matrix} E_{a>a^*+\frac{1}{\triangle n}} y_d(a) + E_{a\leq a^*+\frac{1}{\triangle n}} [y_e(a) + 1] & \text{if } a^* > Ea \\ E_{a>a^*-\frac{1}{\triangle n}} [y_d(a) + 1] + E_{a\leq a^*-\frac{1}{\triangle n}} y_e(a) & \text{if } a^* < Ea \end{matrix}$
1	0	1	1	$E_{a>a^*} y_d(a) + E_{a\leq a^*} y_e(a) + s$
1	0	1	0	$E_{a>a^*-\frac{s(2\mu+\sigma)}{\triangle n}} [y_d(a) + s(2\mu+\sigma)] + E_{a\leq a^*-\frac{s(2\mu+\sigma)}{\triangle n}} y_e(a)$
1	0	0	1	$\begin{matrix} E_{a>a^*-\frac{1}{\triangle n}} [y_d(a) + 1] + E_{a\leq a^*-\frac{1}{\triangle n}} y_e(a) & \text{if } a^* < Ea \\ E_{a>a^*+\frac{1}{\triangle n}} y_d(a) + E_{a\leq a^*+\frac{1}{\triangle n}} [y_e(a) + 1] & \text{if } a^* > Ea \end{matrix}$
1	0	0	0	$E_{a>a^*-\frac{1}{\triangle n}} [y_d(a) + 1] + E_{a\leq a^*-\frac{1}{\triangle n}} y_e(a)$
0	1	1	1	$E_{a>a^*+\frac{1}{\triangle n}} y_d(a) + E_{a\leq a^*+\frac{1}{\triangle n}} [y_e(a) + 1]$
0	1	1	0	$\begin{matrix} E_{a>a^*-\frac{1}{\triangle n}} [y_d(a) + 1] + E_{a\leq a^*-\frac{1}{\triangle n}} y_e(a) & \text{if } a^* < Ea \\ E_{a>a^*+\frac{\epsilon(1\sigma)}{\triangle n}} y_d(a) + E_{a\leq a^*+\frac{1}{\triangle n}} [y_e(a) + 1] & \text{if } a^* > Ea \end{matrix}$
0	1	0	1	$E_{a>a^*+\frac{s(2\mu+\sigma)}{\triangle n}} y_d(a) + E_{a\leq a^*+\frac{s(2\mu+\sigma)}{\triangle n}} [y_e(a) + s(2\mu+\sigma)]$
0	1	0	0	$E_{a>a^*} y_d(a) + E_{a\leq a^*} y_e(a) + s$
0	0	1	1	$\begin{matrix} E_{a>a^*+\frac{1}{\triangle n}} y_d(a) + E_{a\leq a^*+\frac{1}{\triangle n}} [y_e(a) + 1] & \text{if } a^* > Ea \\ E_{a>a^*-\frac{1}{\triangle n}} [y_d(a) + 1] + E_{a\leq a^*-\frac{1}{\triangle n}} y_e(a) & \text{if } a^* < Ea \end{matrix}$
0	0	1	0	$E_{a>a^*-\frac{1}{\triangle n}} [y_d(a) + 1] + E_{a\leq a^*-\frac{1}{\triangle n}} y_e(a)$
0	0	0	1	$E_{a>a^*} y_d(a) + E_{a\leq a^*} y_e(a) + 1$
0	0	0	0	$E_{a>a^*-\frac{s(2\mu+\sigma)}{\triangle n}} [y_d(a) + s(2\mu+\sigma)] + E_{a\leq a^*-\frac{s(2\mu+\sigma)}{\triangle n}} y_e(a)$

It readily follows from this that a worker's career wages can take 6 possible values which are the following:

$$W_1 = E_{a>a^*+\frac{s(2\mu+\sigma)}{\triangle n}} y_d(a) + E_{a\leq a^*+\frac{s(2\mu+\sigma)}{\triangle n}} [y_e(a) + s(2\mu+\sigma)]$$
$$W_2 = E_{a>a^*} y_d(a) + E_{a\leq a^*} y_e(a) + s$$
$$W_3 = E_{a>a^*+\frac{1}{\triangle n}} y_d(a) + E_{a\leq a^*+\frac{1}{\triangle n}} [y_e(a) + 1]$$
$$W_4 = E_{a>a^*-\frac{s(2\mu+\sigma)}{\triangle n}} [y_d(a) + s(2\mu+\sigma)] + E_{a\leq a^*-\frac{s(2\mu+\sigma)}{\triangle n}} y_e(a)$$
$$W_5 = E_{a>a^*-\frac{1}{\triangle n}} [y_d(a) + 1] + E_{a\leq a^*-\frac{1}{\triangle n}} y_e(a)$$
$$W_6 = \begin{cases} E_{a>a^*-\frac{1}{\triangle n}} [y_d(a) + 1] + E_{a\leq a^*-\frac{1}{\triangle n}} y_e(a) & \text{if } a^* < Ea \\ E_{a>a^*+\frac{1}{\triangle n}} y_d(a) + E_{a\leq a^*+\frac{1}{\triangle n}} [y_e(a) + 1] & \text{if } a^* > Ea \end{cases}$$

First notice that $W_1 > W_4$ if and only if $a^* > Ea$ while $W_1 \leq W_4$, otherwise. Second, notice that $W_3 > W_5$ if and only if $a^* \leq Ea$ while $W_3 \leq W_5$, otherwise. Thus, the relevant carrer's wages are W_1, W_2 and W_3 if $a^* > Ea$ and W_2, W_4 and W_5 otherwise.

Let us suppose first that $a^* > Ea$; that is, $W_1 > W_4$. Then,

$$W_2 - W_1 = \triangle n \int_{a^*}^{a^*+\frac{s(2\mu+\sigma)}{\triangle n}} \left(a - a^*\right) \phi\left(a\right) da + s - s\left(2\mu + \sigma\right) \Phi\left(a^* + \frac{s\left(2\mu+\sigma\right)}{\triangle n}\right).$$

Integrating by parts, this becomes:

$$W_2 - W_1 = s - \triangle n \int_{a^*}^{a^*+\frac{s(2\mu+\sigma)}{\triangle n}} \Phi\left(a\right) da.$$

Notice that $\frac{\partial(W_2-W_1)}{\partial s} = 1 - \Phi\left(a^* + \frac{s(2\mu+\sigma)}{\triangle n}\right)(2\mu+\sigma)$, $\frac{\partial^2(W_2-W_1)}{\partial s^2} = -\phi\left(a^* + \frac{s(2\mu+\sigma)}{\triangle n}\right)\frac{(2\mu+\sigma)^2}{\triangle n} < 0$, and $\frac{\partial(W_2-W_1)}{\partial s \partial \sigma} = -\Phi\left(a^* + \frac{s(2\mu+\sigma)}{\triangle n}\right) - \phi\left(a^* + \frac{s(2\mu+\sigma)}{\triangle n}\right)\frac{(2\mu+\sigma)s}{\triangle n} < 0$. Because $W_2 - W_1 = 0$ at $s = 0$ and $W_2 - W_1$ is strictly concave in s, there exists an $s \geq 0$, denoted by $s_E(\sigma, \mu)$, such that $W_2 - W_1 < 0$ for all $s \geq s_E(\sigma, \mu)$. Notice also that $s_E(\sigma, \mu) = 1$ for all $\sigma < 1 - 2\mu$ and thus $W_2 - W_1 > 0$ for all $s \in [0, 1]$ and that $s_E(\sigma, \mu) = 0$ for all $\sigma > 2(1-\mu)$ and thus $W_2 - W_1 < 0$ for all $s \in [0, 1]$. In addition, notice that $\frac{\partial s_E(\sigma,\mu)}{\partial \sigma} > 0$ for all $\sigma < 1 - 2\mu$ and that for σ sufficiently large the opposite occurs.

Next notice that $W_2 - W_3$ is given by:

$$\triangle n \int_{a^*}^{a^*+\frac{1}{\triangle n}} \left(a - a^*\right) d\Phi\left(a\right) + s - \Phi\left(a^* + \frac{1}{\triangle n}\right).$$

Integrating by parts this becomes

$$s - \triangle n \int_{a^*}^{a^*+\frac{1}{\triangle n}} \Phi\left(a\right) da.$$

Because $W_2 - W_3$ increases with s, and at $s = 1$, $W_2 - W_3 > 0$ while at $s = 0$, $W_2 - W_3 > 0$ there exists an $s_E \in (0, 1)$ such that $W_2 - W_3 > 0$ for all $s > s_E$. Furthermore, $s_E > \frac{1}{2}$. This follows from the fact that,

$$\frac{1}{2} - \triangle n \int_{a^*}^{a^*+\frac{1}{\triangle n}} \Phi\left(a\right) da \leq \frac{1}{2} - \triangle n \int_{a^*}^{a^*+\frac{1}{\triangle n}} \frac{1}{2} da = 0,$$

where the inequality follows from the fact that $\Phi\left(a^*\right) > \frac{1}{2}$ and $\Phi\left(a\right)$ is increasing in a.

Finally, note that if $s\left(2\mu + \sigma\right) > 1$, then $W_3 - W_1$ is given by:

$$\triangle n \int_{a^*+\frac{1}{\triangle n}}^{a^*+\frac{s(2\mu+\sigma)}{\triangle n}} \left(a - a^*\right) d\Phi\left(a\right) + \Phi\left(a^* + \frac{1}{\triangle n}\right) - s\left(2\mu+\sigma\right) \Phi\left(a^* + \frac{s\left(2\mu+\sigma\right)}{\triangle n}\right).$$

Integrating by parts, $W_3 - W_1 < 0$ if and only if:

$$-\triangle n \int_{a^*+\frac{1}{\triangle n}}^{a^*+\frac{s(2\mu+\sigma)}{\triangle n}} \Phi\left(a\right) da < 0.$$

Whereas if $s(2\mu+\sigma) \le 1$, $W_3 - W_1 > 0$ if and only if:

$$\triangle n \int_{a^*+\frac{s(2\mu+\sigma)}{\triangle n}}^{a^*+\frac{1}{\triangle n}} \Phi(a)\, da > 0.$$

This implies that if $s(2\mu+\sigma) > 1$, then $W_3 < W_1$ and $W_2 < W_1$ if and only if $s > s_E(\sigma,\mu)$, otherwise $W_2 \ge W_1$. Whereas if $s(2\mu+\sigma) \le 1$, the relevant payoffs are W_3 and W_2 and $W_2 - W_3 > 0$ if and only $s > s_E$.

Let us suppose next that $a^* \le Ea$; that is, $W_1 \le W_4$, then

$$W_2 - W_4 = \triangle n \int_{a^*-\frac{s(2\mu+\sigma)}{\triangle n}}^{a^*} (a^* - a)\,\phi(a)\, da + s - s(2\mu+\sigma)\left[1 - \Phi\left(a^* - \frac{s(2\mu+\sigma)}{\triangle n}\right)\right].$$

Integrating by parts, this becomes:

$$W_2 - W_4 = s - s(2\mu+\sigma) + \triangle n \int_{a^*-\frac{s(2\mu+\sigma)}{\triangle n}}^{a^*} \Phi(a)\, da.$$

Notice that $\frac{\partial(W_2-W_4)}{\partial s} = -(1+\sigma) + \Phi\left(a^* - \frac{s(2\mu+\sigma)}{\triangle n}\right)(2\mu+\sigma)$, $\frac{\partial^2(W_2-W_1)}{\partial s^2} = -\phi\left(a^* - \frac{s(2\mu+\sigma)}{\triangle n}\right)\frac{(2\mu+\sigma)^2}{\triangle n} < 0$, and $\frac{\partial(W_2-W_1)}{\partial s \partial \sigma} = -1 + \Phi\left(a^* - \frac{s(2\mu+\sigma)}{\triangle n}\right) - \phi\left(a^* + \frac{s(2\mu+\sigma)}{\triangle n}\right)\frac{(2\mu+\sigma)s}{\triangle n} < 0$. Because $W_2 - W_4 = 0$ at $s = 0$ and $W_2 - W_4$ is strictly concave, there exists an $s \ge 0$, denoted by $s_D(\sigma,\mu)$, such that $W_2 - W_4 < 0$ for all $s \ge s_D(\sigma,\mu)$. Notice also that $s_D(\sigma,\mu) = 1$ for all $\sigma < 1 - 2\mu$ and thus $W_2 - W_4 > 0$ for all $s \in [0,1]$ and that $s_D(\sigma,\mu) = 0$ for all $\sigma > 2(1-\mu)$ and thus $W_2 - W_4 < 0$ for all $s \in [0,1]$. In addition, notice that $\frac{\partial s_D(\sigma,\mu)}{\partial \sigma} > 0$ for all $\sigma < 1 - 2\mu$ and that for σ sufficiently large the opposite occurs.

Next notice that $W_2 - W_5$ is given by:

$$\triangle n \int_{a^*-\frac{1}{\triangle n}}^{a^*} (a^* - a)\, d\Phi(a) + s - \Phi\left(a^* - \frac{1}{\triangle n}\right).$$

Integrating by parts this becomes

$$\triangle n \int_{a^*-\frac{1}{\triangle n}}^{a^*} \Phi(a)\, da + s - 1.$$

Because $W_2 - W_5$ increases with s and at $s = 1$, $W_2 - W_5 > 0$, then there exists an $s_D \in (0,1)$ such that $W_2 - W_5 > 0$ for all $s > s_D$. Furthermore, $s_D > \frac{1}{2}$. This follows from the fact that,

$$\triangle n \int_{a^*-\frac{1}{\triangle n}}^{a^*} \Phi(a)\, da + \frac{1}{2} - 1 \le -\frac{1}{2} + \triangle n \int_{a^*-\frac{1}{\triangle n}}^{a^*} \frac{1}{2} da = 0,$$

where the inequality follows from the fact that $\Phi(a^*) \le \frac{1}{2}$ and $\Phi(a)$ is increasing in a.

Finally, note that if $s(2\mu+\sigma) \le 1$, then $W_5 - W_4$ is given by:

$$\triangle n \int_{a^*-\frac{1}{\triangle n}}^{a^*-\frac{s(2\mu+\sigma)}{\triangle n}} (a - a^*)\, d\Phi(a) + 1\left[1 - \Phi\left(a^* - \frac{1}{\triangle n}\right)\right] - s(2\mu+\sigma)\left[1 - \Phi\left(a^* - \frac{s(2\mu+\sigma)}{\triangle n}\right)\right].$$

Integrating by parts, $W_5 - W_4 > 0$ if and only if:

$$-\triangle n \int_{a^* - \frac{1}{\triangle n}}^{a^* - \frac{s(2\mu+\sigma)}{\triangle n}} \Phi(a)\, da + 1 - s(2\mu + \sigma) > 0.$$

Notice that $\frac{\partial (W_5 - W_4)}{\partial s} < 0$, and at $s = \frac{1}{(2\mu+\sigma)}$, $W_5 - W_4 = 0$. Thus, $W_5 - W_4 > 0$ for all $s(2\mu + \sigma) \leq 1$.

If $s(2\mu + \sigma) > 1$, then $W_5 - W_4 < 0$ if and only if:

$$\triangle n \int_{a^* - \frac{s(2\mu+\sigma)}{\triangle n}}^{a^* - \frac{1}{\triangle n}} \Phi(a)\, da + 1 - s(2\mu + \sigma) < 0.$$

Notice that $\frac{\partial (W_5 - W_4)}{\partial s} < 0$, and at $s = \frac{1}{(2\mu+\sigma)}$, $W_5 - W_4 = 0$. Thus, $W_5 - W_4 < 0$ for all $s(2\mu + \sigma) > 1$. ■

Proof. of Lemma 3.Let us take a worker whose training level is $\mathbf{s}^j = (s, s)$**.** Then his output when placed in a difficult job and the two core tasks are allocated to adifficult job is $m_d + n_d a + s(2\mu + \sigma)$ while when placed in an easy job and the two core tasks are allocated to an easy job is $m_e + n_e a + s(2\mu + \sigma)$. It readily follows from this that the former is higher when $a > a^*$ and lower otherwise. Consider now the case in which each core job is allocated one core task, then a worker's productivity when placed in a difficult job is $m_d + n_d a + s$ while when placed in an easy job is $m_e + n_e a + s$. It readily follows from this that the former is higher when $a > a^*$ and lower otherwise. Thus, the output ex-post is maximized by allocating the two core task to an easy (difficult) job when $2\mu + \sigma > 1$ and $a \leq a^*$ ($a > a^*$) and by allocating one core task to each core job when $2\mu + \sigma \leq 1$.

Let us take a worker whose training level is $\mathbf{s}^j = (1, 0)$. Then his output when placed in core job j, $j \in \{E, D\}$, and the two core tasks are allocated to a difficult job is $m_j + n_j a + 1$ while when each core job is allocated one core task his output is $m_j + n_j a + 1$ if the task in which the worker is trained is allocated to job j and is $m_j + n_j a$ otherwise. Thus, the output ex-post is maximized by either allocating both tasks to the job where the worker's ability yields the highest output or by allocating the right task to the job where the worker's ability yields the highest output. ■

Proof. of Lemma 4.The proof is trivial. It is enough to note that the possible payoffs under each possible job design are as follows.

$$W_1 = E_{a>a^*} y_d(a) + E_{a \leq a^*} y_e(a) + s(2\mu + \sigma)$$
$$W_2 = E_{a>a^*} y_d(a) + E_{a \leq a^*} y_e(a) + s$$
$$W_3 = E_{a>a^*} y_d(a) + E_{a \leq a^*} y_e(a) + 1$$

Notice that $W_3 \geq W_2$ and thus multi-tasking at the training level and ex-post single tasking at the core level is never optimal when $2\mu + \sigma \leq 1$. Notice also that $W_1 > W_2$ when $2\mu + \sigma > 1$ and that $W_1 > W_3$ when $s(2\mu + \sigma) > 1$. Thus, multi-tasking at the training level and ex-post single tasking at the core level is never optimal and multi-tasking at both levels is optimal whenever $s(2\mu + \sigma) > 1$ while single-tasking at both levels is optimal otherwise. ■

Proof. of Proposition 6.Suppose that $\theta = \frac{1}{2}$, then $E(Exp) = E_a \int_{-\delta}^{y_{j(a)}(\mathbf{s}^j, t_c, a)-\delta} F(\eta)\, d\eta > 0$. The experience premium in a mult-tasking firm is

$$E_{a>a^*+\frac{s(2\mu+\sigma)}{\triangle n}} \int_{-\delta}^{y_d(a)-\delta} F(\eta)\, d\eta + E_{a\le a^*+\frac{s(2\mu+\sigma)}{\triangle n}} \int_{-\delta}^{y_e(a)+s(2\mu+\sigma)-\delta} F(\eta)\, d\eta,$$

the experience premium in single-task firm is

$$E_{a>a^*+\frac{1}{\triangle n}} \int_{-\delta}^{y_d(a)-\delta} F(\eta)\, d\eta + E_{a\le a^*+\frac{1}{\triangle n}} \int_{-\delta}^{y_e(a)+1-\delta} F(\eta)\, d\eta,$$

and that in mixed firm is

$$E_{a>a^*} \int_{-\delta}^{y_d(a)+s-\delta} F(\eta)\, d\eta + E_{a\le a^*} \int_{-\delta}^{y_e(a)+s-\delta} F(\eta)\, d\eta.$$

Assuming that $F(\eta)$ is uniform with support $[-\overline{\eta}, \overline{\eta}]$, one gets the result from proposition 2.

$$\begin{array}{c} E(Exp) = E_a \int_{\eta_{j(a)}(\theta,a)}^{\overline{\eta}} (2\theta-1)\left[y_{j(a)}\left(\mathbf{s}^j, t_c, a\right)+\delta+\eta\right] dF(\eta) + \\ \int_{\underline{\eta}}^{\eta_{j(a)}(\theta,a)} y_{j(a)}\left(\mathbf{s}^j, t_c, a\right) dF(\eta) - \int_{\eta_{j(a)}(a)}^{\eta_{j(a)}(\theta,a)} (\delta+\eta)\, dF(\eta). \end{array} \tag{8}$$

■

Proof. of Proposition 7.

The tenure effect in a multi-tasking firm is

$$\begin{array}{c} -\frac{1}{2} E_{a>a^*+\frac{s(2\mu+\sigma)}{\triangle n}} \left\{ y_d(a) + \int_{y_d(a)-\delta}^{\overline{\eta}} F(\eta)\, d\eta \right\} + \\ -\frac{1}{2} E_{a\le a^*+\frac{s(2\mu+\sigma)}{\triangle n}} \left\{ y_e(a) + s(2\mu+\sigma) + \int_{y_e(a)+s(2\mu+\sigma)-\delta}^{\overline{\eta}} F(\eta)\, d\eta \right\} + \frac{1}{2}(\delta+\overline{\eta}), \end{array}$$

the tenure effect in a single-task firm is

$$\begin{array}{c} -\frac{1}{2} E_{a>a^*+\frac{1}{\triangle n}} \left\{ y_d(a) + \int_{y_d(a)-\delta}^{\overline{\eta}} F(\eta)\, d\eta \right\} + \\ -\frac{1}{2} E_{a\le a^*+\frac{1}{\triangle n}} \left\{ y_e(a) + 1 + \int_{y_e(a)+1-\delta}^{\overline{\eta}} F(\eta)\, d\eta \right\} + \frac{1}{2}(\delta+\overline{\eta}), \end{array}$$

and that in a mixed firm is

$$\begin{array}{c} -\frac{1}{2} E_{a>a^*} \left\{ y_d(a) + \int_{y_d(a)+s-\delta}^{\overline{\eta}} F(\eta)\, d\eta \right\} + \\ -\frac{1}{2} E_{a\le a^*} \left\{ y_e(a) + \int_{y_e(a)+s-\delta}^{\overline{\eta}} F(\eta)\, d\eta \right\} + \frac{1}{2}(\delta+\overline{\eta}-s). \end{array}$$

Assuming that $F(\eta)$ is uniform with support $[-\overline{\eta}, \overline{\eta}]$, the result readily follows from proposition 2.

$$E(T) = E_a \int_{\eta_{j(a)}(\theta,a)}^{\overline{\eta}} \left\{ \theta\left[y_{j(a)}\left(\mathbf{s}^j, t_c, a\right)+\delta+\eta\right] - y_{j(a)}\left(\mathbf{s}^j, t_c, a\right) \right\} dF(\eta) > 0.$$

■