Nº 230

SEQUENTIAL PROCUREMENT AUCTIONS AND
THEIR EFFECT ON INVESTMENT DECISIONS

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DOCUMENTOS DE TRABAJO

Serie Economía

Sequential Procurement Auctions and Their Effect on Investment Decisions

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March 2007

Abstract

In this paper we characterize the optimal procurement mechanism and the investment level for an environment where two projects must be adjudicated sequentially, and the winner of the first project has the opportunity to invest in a distributional upgrade for its costs in the second project. We study 4 cases, based on the commitment level of the seller and the observability of the investment decision. We find that with commitment, the second period mechanism gives an advantage to the first period winner, and induces an investment level that is greater than the efficient one. With non-commitment, the second period mechanism gives a disadvantage to the first period winner, and induces an investment level that is smaller than the efficient one. Observability is irrelevant in the commitment case, but makes the effects more pronounced in the non-commitment case.

1. Introduction

During the last decades procurement auctions have been widely used as mechanisms to assign high cost projects and tasks concerning goods and services. By the year 1998, the sum of all governments expenditures in procurements (excluding defense and labor compensation) was estimated as 7.1% of the worldwide GDP[1] . The repeated utilization of procurement auctions by specific institutions (governments or private sector), which contract with the same pool of firms over time, has made the study of cost reduction investment by these firms specially relevant.

The objective of this paper is twofold. First, to characterize the cost-minimizing procurement mechanisms in an environment with repeated interaction between a buyer and multiple sellers, where investment can be undertaken as a cost-reduction device, and also as an strategic action devised to obtain advantages in future procurement auctions. Second, to analyze the effects these cost-minimizing mechanisms have on supplier's investment decisions. This is particularly relevant if the

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[1]Source: OECD, *The Size of Government Procurement Markets*, Journal of Budgeting Vol.1, No.4, March 2002, ISBN 9264196307.

relative size of expected expenditures is big compared with the value of just one particular purchase, since in that case the use of tools that may reduce future costs, even at the cost of raising today's expenditures, can be particularly profitable.

We consider a buyer who wants to procure two consecutive projects and faces $n \geq 2$ potential suppliers. Costs for performing both tasks are distributed independently across time and competitors, and they are private information to each firm. We assume, though, that the first procurement winner (whose identity is public information for the second procurement) acquires an intrinsic advantage characterized by a *distributional upgrade*. Such phenomenon reflects a knowledge acquired concerning the task performed, to which the rest of the firms have no access. Moreover, this upgrade can be influenced by a costly cost-reducing investment that the winner may carry out between the two procurements, increasing his chances of obtaining lower costs to perform the second period task. In such a context, second period rules affect the investment decisions of first period winners, and therefore, the degree of complementarity between both tasks, making it endogenous and chosen strategically by the first-period winner.

We use the mechanism design approach to characterize the cost-minimizing mechanism, which we determine under the assumptions of full commitment and non-commitment of the buyer. In the case of full-commitment, we assume that the rules for both mechanisms are decided at t=0, before any procurement mechanism or investment occurs. In the non-commitment case, the second-period rules are determined after the winner of the first period is announced. In each case, we study the case of observable and non-observable investment. If investment is observable and there is non-commitment, we assume that the second-period rules are determined after the investment level is decided.

We compare these cost-minimizing mechanisms, and the investment level they induce, with the the ex-post efficient benchmark: each period the project is assigned to the least-cost supplier, and the investment level induced is such that the marginal cost of investment equals the marginal benefit (in expected terms) of cost reduction. We also show that, independent of investment observability, such an incentive compatible mechanism exists and consists of two second price sealed bid procurements.

We show that, under full commitment of the buyer, the investment level induced by a cost-minimizing mechanism does not depend on whether investment is observable or not, and it is higher than the efficient level , so *over-investment* occurs. This is so because the cost-minimizing mechanism gives an *advantage gap* to the first period winner in the second period. That is, he can get the second contract even if his cost is higher than the minimum of the other competitors. Moreover, in the mentioned environment and without observability, a higher advantage gap in the second period mechanism induces a higher level of investment. This result can seem counterintuitive, since an advantage gap can make the first period winner "relax", knowing he owns a big advantage over competitors and likely to win anyway. However, there is a second effect that dominates: investment, by decreasing costs, increases the expected profits in case the firm gets the contract. Since with a bigger advantage gap this happens more often, the expected payoff of a cost reduction is higher, thus investment is more profitable.

In the non-commitment case, it is optimal for the buyer to give a *disadvantage* to the first period winner, who holds a distributional advantage over its competitors. This fact is anticipated

by the winner, leading to investment levels below efficiency. Observability then makes a difference: by observing the investment level, the buyer can react optimally with even more disadvantageous mechanisms and, as a consequence, the first period winner invests below the level chosen when investment is not observable.

Finally, we derive some comparative statics with respect to the number of players. As it increases, all the investment levels mentioned above decrease, but weakly preserving the order among them. For a number of firms sufficiently large, the investment level in all environments collapses to zero: if the number of competitors is large enough, inducing investment is too expensive for the buyer relative to the marginal benefit in cost reduction. This is so because the marginal gain for the buyer introduced by investing, that is the probability of getting a cost lower than the minimum of *all other competitors*, decreases with the number of competitors.

These results have important normative implications: (i)cost-minimizing buyers, with the ability to commit, and working in a dynamic[2] environment, induce investment levels above the efficient level, and (ii) a buyer's lack of commitment induces under-investment in the same context. So, in a dynamic context, mechanisms are not only optimal rules to assign tasks under asymmetric information, they are also tools to induce more complementarity between projects, and therefore, they are a way to generate incentives in cost reduction,

Our work is related to the literature in various ways. With the methodology of mechanism design, Pesendorfer and Jofre-Bonnet in [**?**], derive the optimal mechanism for the case where the complementarity is exogenous, there is full commitment and only two players. In that paper, the *advantage-gap* that the first-period winner acquires for the second procurement, is independent of the complementarity of the projects (in fact they could be substitutes), which is an exogenous feature. This is not the case in our model, since the advantage-gap and the degree of complementarity are related through the investment incentives.

With respect to the investment incentives in one-shot procurements, literature has focused mainly on the consequences of an *investment stage* before a procurement. In this context, Piccione and Tan [**?**] analyze the implementation of the efficient solution when firms are ex ante symmetric and they can all invest (non observably) in R&D cost-reduction. They found that the answers is affirmative if this technology presents diminishing returns to scale, and the mechanism can be a first or second price sealed bid procurement. Dasgupta [**?**], in a similar model (investment stage and non observability), shows that in a context of first and second price sealed bid procurements, the buyer's lack of commitment induces investment levels below efficiency. Moreover, he proves that commitment rises the level of investment, but always below the efficient one.

Finally, Arozamena and Cantillon in [**?**], analyze the effects of allowing only one firm to invest before the procurement auction when this action observable by competitors. Their main result is the *under-investment* in first price sealed bid auctions: as a response to the cost-reduction investment, rival firms will bid more aggressively, therefore reducing the investment incentives. We also get under-investment, but for a different reason: under non-commitment, is the mechanism designer (the buyer) that changes behavior, giving an advantage to worse firms, and therefore decreasing the incentives to invest.

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[2]By dynamic we understand more than one period

2. THE MODEL

2.1 The Environment

Consider a risk-neutral buyer who wants to procure two projects, one at $t = 1$ and the other at $t = 2$. The set of competing firms is $N = \{1, ..., n\}$, $n \geq 2$, all of which are risk-neutral and live for the two periods. The buyer is compelled to procure the two goods or services[3]. In each period, the cost of undertaking the project for a firm is drawn from the interval $C = [\underline{c}, \bar{c}]$, and they become private information of the firm. At $t = 1$, these costs are independently distributed according to a distribution $F(\cdot)$, differentiable, that satisfies $f(c) \equiv F'(c) > 0$ if $c \in C$ (so firms are *ex-ante* symmetric).

At $t = 2$, the competitors costs are drawn independently from those drawn in period 1, and independently across firms as well. The costs of the first-period losers are taken from the same distribution $F(\cdot)$. Instead, the winner of the first procurement (from now on *the winner*) has the option of investing an amount I between auctions, and changes his distribution to $G(\cdot, I)$, with the same support as before. This investment has a monetary cost $\Psi(I)$ for the winner. Assumption **??** below implies a distribution improvement for the winner as a function of investment: an increase in investment implies an increase in the chance of obtaining lower costs relative to higher ones. As a consequence, higher investment induces a "better" cost distribution in the usual sense of first order stochastic dominance. The formal result is in the lemma **??** in the next section.

Assumption 1 $G \in C^2(C \times \mathbb{R}_+)$. *For all* $0 \leq I' < I \in \mathbb{R}$ *and* $c' < c \in C$,

$$\frac{f(c')}{f(c)} < \frac{\frac{\partial G}{\partial c}(c', I')}{\frac{\partial G}{\partial c}(c, I')} < \frac{\frac{\partial G}{\partial c}(c', I)}{\frac{\partial G}{\partial c}(c, I)}$$

Also, in setting $I' = 0$, this assumption implies an *ex-ante* degree of complementarity between projects: the winner acquires a knowledge concerning the task performed (for instance the so called *know-how*), which is not available to the losers, and enables him to improve his initial distribution in the last period. Of course, the final degree of complementarity will depend on the amount the firm invests in developing this *know-how*.

We also impose that the marginal benefit of investment is decreasing:

Assumption 2 *For all* $I \in \mathbb{R}$, $\frac{\partial^2 G}{\partial I^2}(\cdot, I) < 0$ *in* $(\underline{c}, \bar{c})$.

We now state two technical assumption, the first one is the monotone likelihood ratio, and the second is a technical condition needed for integrability.

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[3]In auction theory is usual to take into account the seller's valuation, let's say $t_0 > -\infty$, for the good he is selling. In the case procurements, such valuation corresponds to the cost, call it C_0, for which the buyer would carry out the project. In what follows, we suppose that $C_0 = +\infty$

Assumption 3 $\frac{F(c)}{f(c)}$, $\frac{G(c,I)}{\frac{\partial G}{\partial c}(c,I)}$ are increasing in c (regular). Also, $\frac{F(c)}{f(c)}$ is differentiable (in particular, F is twice differentiable).

Assumption 4 There exists $f \in L^1(I\!R)$ such that

$$\left| \frac{\partial G}{\partial I}(c,I) \right| = \frac{\partial G}{\partial I}(c,I) < f(c),\ \forall\, I \in I\!R$$

Finally, for the investment technology we assume

Assumption 5 $\Psi(\cdot)$ is twice differentiable and satisfies $\Psi'(\cdot) > 0$, $\Psi''(\cdot) \geq 0$.

For notation purposes, we denote the joint density of the first-period distribution as $f^n(c) = \prod_{j=1}^{n} f(c_j)$. As usual, we define $c_{-i} = (c_1, ..., c_{i-1}, c_{i+1}, ..., c_n)$ and $f^{n-1}(c_{-j}) = \prod_{i \neq j} f(c_i)$.

The previous assumptions are not hard to satisfy. For example, we the family of distributions introduced in Piccione and Tan [?], which they argue is a way of modeling investment in cost reduction of R&D technologies, satisfies them.

Example 6 Suppose that $F(\cdot)$ is a twice differentiable concave distribution. Then, it is straightforward that verifies the regularity assumption. The family of distributions given by

$$G(c,0) = F(c)^\eta\ \text{ with } 0 < \eta < 1$$

and

$$G(c,I) = 1 - (1 - G(c,0))^{\gamma I + 1}\ \text{ with } \gamma > 0$$

satisfies assumptions ?? and ??, ??, and ?? (see Appendix A for a detailed proof).

2.2 The Mechanisms

We consider second period procurement mechanisms that can depend on the identity of the first-period winner, but not on the cost-realization (revelation) of that first period. Moreover, we do not allow the buyer to exclude sellers in the second period if they do not participate in the first. The reason for these assumptions is that, otherwise, the buyer can extract the full second period surplus, by threatening the sellers (in the first period) with very biased rules in the second.

The fact that costs are drawn independently across time enables the buyer to pay attention only to incentive compatible mechanisms because the revelation principle applies. We will focus on two types of environments: *full commitment* and *non-commitment* of the buyer. In the first case, the buyer can commit to the first and second period mechanisms. In the second, he cannot, and he will adjust according to the information he gathers before the second period.

In each case, we analyze when investment is observable and when it is not. The difference is that in the observable case, the investment, and therefore the winner's cost distribution for the

second period is public information (as well as the losers' distributions). Thus, in this setting the mechanisms used by the buyer at $t = 2$ may depend on the level of investment chosen by the winner of the first procurement auction.

Definition 7 *A direct mechanism, when investment is not observable, is given by the tuple* $\Gamma_{no} = (t^1, q^1, t^2_w, q^2_w, t^2_l, q^2_l)$, *where* $t^1 : C^n \longrightarrow I\!\!R^n$, $q^1 : C^n \longrightarrow \Delta_n$, $t^2_w : C^n \longrightarrow I\!\!R$, $q^2_w : C^n \longrightarrow [0, 1]$, $t^2_l : C^n \longrightarrow I\!\!R^{n-1}$, $q^2_l : C^n \longrightarrow [0, 1]^{n-1}$, *such that* $q^2_w(c) + \sum_{i \neq w} q^2_{l,i}(c) = 1$ *for all* $c \in C^n$.

Definition 8 *A direct mechanism, when investment is observable, is given by the tuple*

$$\Gamma = (t^1, q^1, \{t^2_{w,I}\}_{I \geq 0}, \{q^2_{w,I}\}_{I \geq 0}, \{t^2_{l,I}\}_{I \geq 0}, \{q^2_{l,I}\}_{I \geq 0})$$

where $t^1 : C^n \longrightarrow I\!\!R^n$, $q^1 : C^n \longrightarrow \Delta_n$, $t^2_{w,I} : C^n \longrightarrow I\!\!R$, $q^2_{w,I} : C^n \longrightarrow [0, 1]$, $t^2_{l,I} : C^n \longrightarrow I\!\!R^{n-1}$, $q^2_{l,I} : C^n \longrightarrow [0, 1]^{n-1}$, *such that* $q^2_{w,I}(c) + \sum_{i \neq w} q^2_{l,I,i}(c) = 1$ *for all* $c \in C^n$ *and* $I \geq 0$.

When investment not observable, $t^s(c) = (t^s_1(c), ..., t^s_n(c))$, and $t^s_i(c)$ corresponds to the payment to firm $i \in N$ at time $s = 1, 2$, conditional on the vector report $c = (c_1, ..., c_n)$. Analogously, $q^s(c) = (q^s_1(c), ..., q^s_n(c))$, with $q^s_i(c)$ the probability that competitor $i \in N$ wins the procurement auction at time $s = 1, 2$ conditional on the same vector cost report. Finally, when investment can be monitored, the functions are essentially the same, but now the second period rules may depend on the investment level carried out by the first period winner prior to the last procurement.

A natural question that arises is whether the mechanism designer can improve by using mechanisms with additional features. For instance, the buyer can make use of second period rules which depend on the first period winner's identity. In Appendix B we show that there is no improvement, in terms of reducing buyer's expected expenditures, when such history-dependent mechanisms are taken into account. We can therefore restrict our analysis to mechanisms considered in definitions **??** and **??**.

3. Preliminary Results

We first state a lemma involving some distributional consequences of assumption **??**. In particular, that the monotone likelihood ratio property implies first order stochastic dominance as the investment level decreases.

Lemma 9 *Suppose that assumption* **??** *holds, then:*

(i)

$$\frac{\frac{\partial G}{\partial c}(c, I)}{1 - G(c, I)} < \frac{\frac{\partial G}{\partial c}(c, I')}{1 - G(c, I')}, \forall c \in C, \ 0 \leq I < I'.$$

(ii)

$$\frac{G(c, I)}{\frac{\partial G}{\partial c}(c, I)} < \frac{G(c, I')}{\frac{\partial G}{\partial c}(c, I')}, \forall c \in C, \ 0 \leq I < I'.$$

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(iii) *For every $c \in C$ fixed, the function $G(c, \cdot)$ is increasing. This is equivalent to* first order stochastic dominance *as I decreases in the family of distributions $\{G(\cdot, I)| \ I \geq 0\}$.*

Proof: Appendix A.

$\square$

Condition (i) in the previous lemma is used by Arozamena and Cantillon in [**?**] when they refer to a distributional upgrade. It states that, conditional on a certain cost level c, it is more likely to obtain lower costs as investment increases.

We now turn to the model. In both periods the buyer specifies probabilities of winning the project and payments (transfers) to each of the competitors depending on their costs reports. At $t = 1$ firms are denoted by subscripts $i = 1, 2, ..., n$. So, if $c' = (c'_1, c'_2, ..., c'_n)$ is the vector of reported costs, the probability that firm i wins the first project, conditional on that report vector, is $q^1_i(c')$, $i = 1, 2$. By $Q^1_i(c'_i)$ we denote the expected probability that player i wins the procurement auction conditional on his announcement c'_i, $i = 1, 2$. Then, it satisfies

$$Q^1_i(c'_i) = \int_C q^1_i(c'_i, c_{-i}) f^{n-1}(c_{-i}) dc_{-i}, \ i \in N. \tag{1}$$

The first period transfer for player i conditional on the report vector $c' = (c'_1, ..., c'_n)$ corresponds to the expression $t^1_i(c')$, $i \in N$. Then, the expected transfer for player i in this period, conditional on his announcement c'_i will be denoted by $T^1_i(c'_i)$ and verifies

$$T^1_i(c'_i) = \int_C t^1_i(c'_i, c_{-i}) f^{n-1}(c_{-i}) dc_{-i}, \ i \in N. \tag{2}$$

for $i \in N$.

For the observable case rename the first period winner by w, $w \in N$. The expression $q^2_{w,I}(c')$ denotes the probability that this last competitor wins the second project if he had carried out an investment I conditional on the reports vector $c' = (c'_1, ..., c'_n)$. Analogously, $q^2_{l,I,i}(c')$ corresponds to the probability that player $i \neq w$ wins the second procurement conditional on being a first-period loser, the reports vector c' and the amount of investment I chosen by the first period winner. The expected second period probabilities satisfy

$$Q^2_{w,I}(c'_i) = \int_{C^{n-1}} q^2_{w,I}(c'_i, c_{-i}) f^{n-1}(c_{-i}) dc_{-i} \tag{3}$$

$$Q^2_{l,I,i}(c'_j) = \int_{C^{n-1}} q^2_{l,I,i}(c'_j, c_{-j}) f^{n-2}(c_{-w,j}) \frac{\partial G}{\partial c}(c_w, I) dc_{-j} \tag{4}$$

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with $i \neq w$, $i \in N$.

Transfers $t_{w,I}(\cdot)$ and $t_{l,I,i}(\cdot)$, $i \neq w$, $i \in N$, are defined in the same way, that is, they depend on the investment decided by the first period winner, on player's reports and on being a first period winner or loser. The expression $T_{w,I}^2(c_w')$ corresponds to the expected transfer for the first period winner at $t = 2$, when reported c_w' as his cost and chosen an investment level I ($T_{l,I,i}(c_i')$ is defined in the same way with the obvious changes). Finally, in this setting, $\Pi_{w,I}^2(c_w, c_w')$ will be the expected utility at $t = 2$ for the same competitor when his real cost is c_w and reported c_w' (analogously for firm $i \neq w$, $\Pi_{l,I,i}^2(c_i, c_i')$, $i \in N$). Then we have,

$$\Pi_{w,I}^2(c_w, c_w') = T_{w,I}^2(c_w') - c_w Q_{w,I}^2(c_w') - \Psi(I), \ i \in N. \tag{5}$$

$$\Pi_{l,I,i}^2(c_i, c_i') = T_{l,I,i}^2(c_i') - c_i Q_{l,I,i}^2(c_i'), \ i \neq w, \ i \in N. \tag{6}$$

Assume from now on that the buyer wants the first period winner to invest an amount I. We denote by $\Pi_i^1(c_i, c_i')$ the discounted expected utility at $t = 1$ for firm i with cost c_i and reported cost c_i', conditional on revealing real costs at $t = 2$ and on the fact that every first period winner must invest I. It satisfies

$$\Pi_{i,I}^1(c_i, c_i') = T_i^1(c_i') - c_i Q_i^1(c_i') + \beta Q_i^1(c_i') \int_C \Pi_{w,I}^2(c, c) \frac{\partial G}{\partial c}(c, I) dc + \beta[1 - Q_i^1(c_i')] \int_C \Pi_{l,I,i}^2(c, c) f(c) dc. \tag{7}$$

This last expression consists in expected payments and costs of the first procurement (the first two terms), and the ones related to the second period expected utility (which depends on being the winner or a loser in the first procurement auction), all conditional on the first period report and cost c_i' and c_i, respectively.

As we said before, we can restrict the analysis to direct mechanisms. Truth-telling in the second period is condensed in:

$$IC_o^2 \begin{cases} \Pi_{w,I}^2(c_w, c_w) \geq \Pi_{w,I}^2(c_w, c_w'), \ \forall \ c_w, c_w' \in C, \ \forall I \geq 0. \\ \\ \Pi_{l,I,i}^2(c_i, c_i) \geq \Pi_{l,I,i}^2(c_i, c_i'), \ \forall \ c_i, c_i' \in C, \ \forall \ i \neq w, \ i \in N, \ \forall I \geq 0 \end{cases}$$

The corresponding for $t = 1$:

$$IC_o^1: \ \forall \ i \in N \text{ and } I \geq 0, \ \Pi_{i,I}^1(c_i, c_i) \geq \Pi_{i,I}^1(c_i', c_i), \ \forall \ c_i, c_i' \in C.$$

There is no problem in defining incentive compatibility for any possible investment level $I \geq 0$. As we will see later, participation constraints will be constructed so that only the levels that the buyer allows are realized. The following lemma is the common used characterization of incentive compatible mechanisms:

Lemma 10 (Incentive Compatibility): *In this context, Γ is incentive compatible if and only if*

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(i) *For all $i \in N$ and $I \geq 0$,*

- $Q_i^1(\cdot)$ *is not increasing*

- $\Pi_{i,I}^1(c_i, c_i) = \Pi_{i,I}^1(\bar{c}, \bar{c}) + \int\limits_c^{\bar{c}} Q_i^1(c_i) dc_i$ *for all $c_i \in C$*

(ii) *For all $I \geq 0$,*

- $Q_k^t(\cdot)$ *is not increasing, $k = (w, I), (l, I, i)$, $i \neq w$, $i \in N$.*

- $\Pi_k^2(c_k, c_k) = \Pi_k^2(\bar{c}, \bar{c}) + \int\limits_c^{\bar{c}} Q_k^2(c_k) dc_k$ *for all $c_k \in C$, $k = (w, I), (l, I, i)$, $\forall i \neq w$, $i \in N$.*

Proof: Appendix A.

$\square$

To conclude, when investment is not observable, mechanisms (probabilities and transfers) can't depend the level of investment chosen by the first period winner. For this reason, mechanisms in this setting omit the variable I and all the above expressions can be obtained with this notational change. Nevertheless, this feature (mechanisms' investment-independence) is quite different from saying that these rules can't depend on an "optimal"[4] level of investment, which can certainly happen. In other words, mechanisms cannot be functions of investment, but they can be designed to induce, for instance, a certain investment I, that will appear implicitly in those rules.

4. REVENUE MAXIMIZATION UNDER FULL COMMITMENT

In this environment we assume the existence of institutions that may enforce the contracts established by the buyer. We start with the *investment-observability* case.

4.1 Investment Observability and Full Commitment

In this context, since investment is observable, the buyer can induce any amount of investment he wants. This can be done by setting transfers so low enough (even payments to the buyer) so that any other level chosen by the first period winner is unprofitable for him. Under this schemes, histories associated to investment levels different from the ones the buyer wants to induce never happen.

Assume from now on that the buyer wants to induce the first period winner to invest a level $I \geq 0$. Participation in the second procurement auction is ensured by

$$PC_o^2(I) \begin{cases} \Pi_{w,I}^2(c_w, c_w) \geq 0, \ \forall c_w \in C \\ \\ \Pi_{l,I,i}^2(c_i, c_i) \geq 0, \ \forall c_i \in C, \ i \neq w, \ i \in N. \end{cases}$$

[4]Optimal depends on the problem being solved.

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For any other amount of investment $\tilde{I} \neq I$ carried out by the first period winner, the buyer can simply set transfers low enough so that the expected second period utility for the first period winner is negative for any possible cost. This can be done because investment is observable, hence, transfers depend on this variable and, as a consequence, the buyer can punish the first period winner if this agent deviates from the specified level.

At $t = 1$, we consider the participation constraint presented in Pesendorfer and Jofre-Bonet [?], that is, participating in both procurements auctions is more profitable (in expected terms) than doing this only in the last one. This notion is condensed in

$$PC_o^1(I): \; \Pi_{i,I}^1(c_i, c_i) \geq \beta \int_C \Pi_{I,I,i}^2(c, c) f(c) dc, \; \forall \, c_i \in S, \; \forall \, i \in N$$

This last constraint is valid since the buyer may optimally restrict to mechanisms independent of the first period winner's identity, so if player i, $i \in N$, decides not to participate in the first procurement auction, his expected utility at $t = 1$ will correspond to $\beta \int_C \Pi_{l,I,i}^2(c, c) f(c) dc$ regardless of previous winner's identity.

We are now ready to state the optimization problem that faces the mechanism designer. Denote by $\mathcal{C} = \mathcal{C}(\Gamma, I)$ the expected procurement cost when the buyer uses mechanism Γ and wants to induce an investment level $I \geq 0$. Since the buyer can restrict to mechanisms that do not depend on the first period winner's identity (see Appendix B) this expression corresponds to:

$$\mathcal{C} = \sum_{i=1}^n \int_C T_i^1(c) f(c) dc + \beta \left[\int_C T_{w,I}^2(c) \frac{\partial G}{\partial c}(c, I) dc + \sum_{j \neq w} \int_C T_{l,I,j}^2(c) f(c) dc \right] \tag{8}$$

Therefore, this agent solves:

$$\mathcal{P}_o \left\{ \begin{array}{c} \min_{\Gamma, I} \; \mathcal{C}(\Gamma, I) \\[2ex] s.t \qquad IC_o^1, \; IC_o^2 \\[2ex] PC_o^1(I), \; PC_o^2(I) \end{array} \right.$$

We will refer to a *feasible mechanism* $\Gamma(I)$ if it fulfills the restrictions of the problem $\mathcal{P}_o(I)$, that is, the problem solved by a buyer who wants to induce an investment level I.

Now we present the essential result concerning mechanisms. Considering the degree of generality stated in Section 2, the minimizing cost mechanism under investment observability satisfies that: second period rules do not depend on (i) first period winner's identity, (ii) identities between first period losers and (iii) investment level carried out by the first period winner.

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Theorem 11 Under full-commitment and investment observability, the cost-minimizing mechanism, call it Γ^*, does not depends on the first period winner's identity nor investment, and it is characterized by

$$q_i^{1*}(c_1, ..., c_n) = \begin{cases} 1 & c_i + \frac{F(c_i)}{f(c_i)} < c_j + \frac{F(c_j)}{f(c_j)} \quad \forall j \neq i \\ 0 & \sim \end{cases} \tag{9}$$

$$q_w^{2*}(c_w, c_{-w}) = \begin{cases} 1 & c_w < g(c_{i,l}) \quad \forall i \neq w \\ 0 & \sim \end{cases} \tag{10}$$

$$q_{l,i}^{2*}(c_i, c_{-i}) = \begin{cases} 1 & g(c_i) = min\{c_w, g(c_j); \quad \forall j \neq w\} \\ 0 & \sim \end{cases} \tag{11}$$

with $g(c) = c + \left(1 + \frac{1}{n-1}\right) \frac{F(c)}{f(c)}$, $c \in C$.

Proof: Appendix A.

$\square$

Observation 1: It is important to emphasize that this mechanism does not depend on investment and, therefore, it is optimal in when cost-reducing opportunities are not available. In this context, with $n = 2$ we obtain Pesendorfer's optimal mechanism exposed in [**?**].

At $t = 1$ the minimizing rule correspond to the one derived by Myerson in [**?**] and it is efficient because of the first period symmetry across competitors and assumption **??**. Note that the first period winner obtains an *advantage gap* for the second procurement, that is, this firm is able to win the second procurement even when some other rivals present lower costs. This gap decreases with the number of firms: as long as the number of competitors increase, giving the same advantage to the first period winner is more expensive for buyer, since it is likely that some other competitors are below this agent's cost. Nevertheless, this gap never disappear, reflecting that sequentiality introduces *memory* in the optimal contract, expressed in the mentioned advantage. As a consequence, the minimizing-cost rule in the second period **always** sacrifices efficiency in order to reduce expected costs. Finally, since this mechanism does not depend on the level of investment I, it is also feasible when this variable is not observable.

Observation 2: The rule defined in the previous theorem, and the procedure to obtain it, does not rely on the *ex-ante* degree of complementarity among both projects. In other words, it is optimal when there is no distributional change across time, and even when projects are substitutes, notion captured by the inequality

$$\frac{f(c')}{f(c)} > \frac{\frac{\partial G}{\partial c}(c', 0)}{\frac{\partial G}{\partial c}(c, 0)}, \ \forall \ c' < c \in C$$

In the first case, this occurs because the buyer is able to inter-temporally distribute incentives across time in a better way than performing two independent procurements. In other words, transfers are

modified, when compared to the independent-procurements contract transfers, in order to reduce costs. In the case of substitutes projects, there is an additional feature to take into account, which is the advantage given to the competitor with the worst distribution. Nevertheless, this mechanism is still cost minimizing: it is less likely that the first period winner obtains lower costs than every loser, so even when the worst firm has an advantage, it wins the second period procurement less number of times in average. This effect, in addition to the inter-temporal incentives distribution capability, out-weights the increase in expected costs due to the advantage gap.

To establish the optimal contract for the buyer in this setting, it remains to show the cost-minimizing investment level that the buyer wishes to be implemented, call it I^*. The next result characterizes it:

Theorem 12 Under full-commitment of the buyer and investment observability, the cost-minimizing investment level I^* is the solution to

$$\max_{I \geq 0} \int_C [1 - F(g^{-1}(c))]^{n-1} G(c, I) dc - \Psi(I) \tag{12}$$

Proof: Appendix A.

$\square$

4.2   Investment Non-Observability and Full-Commitment

As we said before, in this case rules can't be functions of the investment level chosen by the first period winner. In this setting, it is the first period winner who decides the amount of investment to be carried out, but the buyer can design mechanisms to induce specific levels he may wish to implement. Therefore, the buyer solves:

$$\mathcal{P}_{no} \begin{cases} \min_{\Gamma_{no}, I} \ \mathcal{C}(\Gamma_{no}, I) \\[2ex] s.t \quad I \in arg \max_{\hat{I} \geq 0} \int_C \Pi_w^2(c, c) \frac{\partial G}{\partial c}(c, \hat{I}) dc \\[2ex] \qquad IC_{no}^1, \ IC_{no}^2 \\[2ex] \qquad PC_{no}^1(I), \ PC_{no}^2(I) \end{cases}$$

Because in this setting investment is not observable, participation and incentive-compatibility constraints are accompanied by "(no)". They are essentially the same ones of the observable case, but involving investment-independent mechanisms (definition 6). The first restriction must be added because now the investment level is chosen by the first period winner. It specifies that the cost-minimizing investment level that the buyer wishes to be implemented must be optimal under mechanism Γ_{no} for the first period winner as well. In other words, winner's transfer and probability of winning the second procurement, which are functions depending only on costs reports, must induce

the exact investment level the buyer wants if the first period winner is to choose this variable.

Denote by I^* the cost minimizing investment level when this variable is observable and $(\Gamma^*_{no}, I^*_{no})$ the solution to $\mathcal{P}_{no}$. It is clear that

$$\mathcal{C}(\Gamma^*, I^*) \leq \mathcal{C}(\Gamma^*_{no}, I^*_{no})$$

that is, at the optimum, the expected procurement cost is lower when investment is observable. This is due to the additional restrictions in $\mathcal{P}_{no}$: I^*_{no} must be optimal for the first period winner, and, the buyer is constrained to look in a smaller set of mechanisms (not investment dependent). In this context, if we find a feasible mechanism (Γ_{no}, I_{no}) such that $\mathcal{C}(\Gamma^*, I^*) = \mathcal{C}(\Gamma_{no}, I_{no})$, then, $(\Gamma^*_{no}, I^*_{no}) = (\Gamma_{no}, I_{no})$ and the problem is solved.

The next result establishes that, when investment is not observable, the expected total cost of the investment-observability solution $\mathcal{C}(\Gamma^*, I^*)$ can be achieved by using the same mechanism Γ^* (it is feasible because it doesn't depend on investment). Moreover, the investment level induced in both settings is the same. Therefore, the optimal solution of the full commitment case, (Γ^*, I^*), does not depend on investment observability.

Proposition 13 Under full commitment of the buyer and investment non-observability, the solution to $\mathcal{P}_{no}$ is $\Gamma^*_{no} = \Gamma^*$, $I^*_{no} = I^*$, with (Γ^*, I^*) the full commitment investment observability solution. Therefore, as (Γ^*, I^*) is the solution in both settings, observable and non-observable investment, we call it the full commitment cost-minimizing solution.

Proof: Appendix A.

$\square$

As an interpretation, when the buyer set rules according to the optimal mechanism Γ^*, he provides the right incentives to induce the first period winner to invest I^*, the same level that the auctioneer would have chosen himself.

Observation 3: More generally, if investment can't be observed, when facing a mechanism with second period expected probability for the first period winner $Q^2_w(\cdot)$, this agent decides his cost-reduction level by solving

$$\max_{I \geq 0} \int_C \Pi^2_w(c, c) \frac{\partial G}{\partial c}(c, I) dc - \Psi(I)$$

As $\int_C \Pi^2_w(c) \frac{\partial G}{\partial c}(c, I) dc = T^2_w(\bar{c}) - \bar{c} Q^2_w(\bar{c}) + \int_C Q^2_w(c) G(c, I) dc - \Psi(I)$ the problem that solves the first period winner when investment is not observable is equivalent to

$$\max_{I \geq 0} \int_C Q^2_w(c) G(c, I) dc - \Psi(I)$$

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This last expression will be used constantly throughout the rest of the paper. In the particular case of Γ^*, we have that

$$Q_w^{2*}(c) = [1 - F(g^{-1}(c))]^{n-1}$$

Then (??) is exactly the problem that the first period winner solves when investment is not observable and the buyer chooses Γ^*. From now one we assume that $I^* > 0$ and satisfies the first order condition of this problem.

Observation 4: It is interesting that even when projects are *ex ante* substitutes, if investment in cost reduction is relatively large, complementarity between them can arise endogenously. That is, mechanisms play a dual role in a context of sequential procurements: they are optimal ways of assigning tasks, and also, they are tools available for the buyer to induce cost reduction. If this last agent establishes the correct incentives, *ex ante* substitutes projects may become complements endogenously. The following numerical example illustrates this issue.

Example 14 *(Endogenous Complementarity):* *Suppose $n = 2$, $C = [0, 1]$, $F(c) = c$ if $c \in C$. Assume that $G(c, 0) = c^\eta$ and $G(c, I) = 1 - (1 - G(c, 0))^{\gamma I + 1}$ if $c \in C$. Ex-ante substitutes tasks is condensed in*

$$\frac{f(c')}{f(c)} > \frac{\frac{\partial G}{\partial c}(c', 0)}{\frac{\partial G}{\partial c}(c, 0)}, \ \forall c' < c$$

We will use three values[5] for η: 1.5, 3, 6. Because in each case $\eta > 1$, it is easy to see that the projects studied here are substitutes. Consider $\gamma = 5$ and an investment cost function $\Psi(I) = \frac{0.01I^2}{2}$. Recall that, under full commitment, the minimizing cost investment level I^* solves (??). For each η, the first order condition satisfied by I_η^* corresponds to

$$H_\eta'(I_\eta^*) = 5 \int_0^1 \left[1 - \frac{c}{2}\right] [1 - c^\eta]^{(5I_\eta^*+1)} |log(1 - c^\eta)| dc - 0.01 I_\eta^* = 0$$

Numerically, we find that $2.7 \leq I_{1.5}^* \leq 2.8$, $2.9 \leq I_3^* \leq 3.0$, $2.4 \leq I_5^* \leq 2.5$ and call each lower bound $\underline{I}_\eta$. Because of assumption ??, in order to show that projects become complements, it suffices to show that

$$\frac{f(c')}{\frac{\partial G}{\partial c}(c', \underline{I}_\eta)} < \frac{f(c)}{\frac{\partial G}{\partial c}(c, \underline{I}_\eta)}, \ \forall c' < c$$

in other words, that

$$L_\eta(c) = \frac{f(c)}{\frac{\partial G}{\partial c}(c, \underline{I}_\eta)}$$

is increasing in $c \in C$. Under our assumptions, this is equivalent to the function $\frac{\partial G}{\partial c}(c, \underline{I}_\eta)$ to be decreasing in the same variable. Numerical results are presented in the figures below.

---

[5]No particular reason exists in choosing this numbers
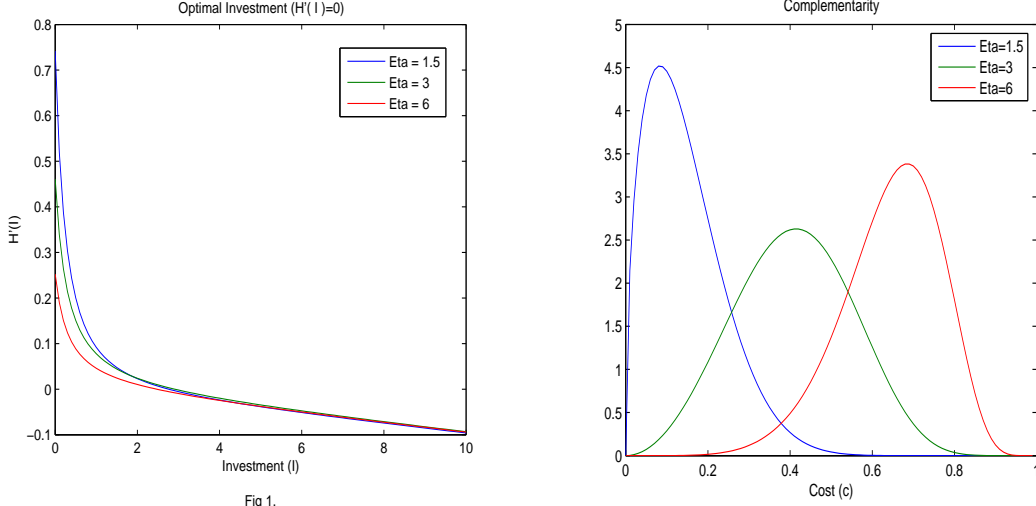
14

Fig 1.

Figure 1 shows $H'_\eta(\cdot)$ for each η considered. It illustrates that it is not necessarily true that higher ex-ante levels of substitution lead to lower investment levels (green and blue lines cross each other). On the other hand, in Figure 2 the functions $L_\eta(\cdot)$ are depicted. It is clear that as long the degree of substitution between projects decreases, the costs-zone in which each function increases becomes smaller. In other words, investment leads to bigger distributional improvements as both tasks are less substitutes. In the particular case of $\eta = 1.5$, it is interesting to note that projects almost become complements endogenously (numerically, only for costs levels under 0.084 the function increases).

5. Non-Commitment

In this environment we assume that the buyer cannot commit to a second period contract before the investment stage, and this fact is known by the competitors. This fact induces, through a second period mechanism that is disadvantageous to the first period winner, an investment level below the one induced when there is commitment. Also, in this setting, the observability of investment will do make a difference (unlike the case of commitment).

To begin with, suppose that the first period winner invested an amount I before the second procurement. Because the buyer is able to change the mechanism at any time before this procurement takes place, he has the incentive to impose new optimal rules considering the investment expenditures, $\Psi(I)$, as a sunk costs. Recall from the preliminary results section, that under an incentive compatible mechanism, the second period expected utility for the winner is expressed by

$$\Pi^2_{w,I}(c,c) = T^2_{w,I}(c) - cQ^2_{w,I}(c) - \Psi(I)$$

Therefore, as in a one-shot procurement, the auctioneer has the incentive to impose the following participation constraint:

$$T^2_{w,I}(c) - cQ^2_{w,I}(c) \geq 0, \ \forall c \in C$$

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which in our case can be written as

$$\Pi^2_{w,I}(c,c) \geq 0, \ \forall c \in C.$$

Finally, as in the previous sections, the second procurement expected cost corresponds to

$$\mathcal{C}^2(I) = \int\limits_C T^2_{w,I}(c)\frac{\partial G}{\partial c}(c,I)dc + \sum_{i \neq w}\int\limits_C T^2_{l,I,i}(c)f(c)dc$$

We start considering the case when investment is observable.

5.1 Investment Observability and Non-Commitment

Because investment is observable, the buyer can make us of mechanisms of the form

$$\Gamma^2 = (\{t^2_{w,I}\}_{I\geq 0}, \{q^2_{w,I}\}_{I\geq 0}, \{t^2_{l,I}\}_{I\geq 0}, \{q^2_{l,I}\}_{I\geq 0})$$

that is, second period rules considering that investment can be monitored. Therefore, when the first period winner has already chosen an investment level I (observable), the buyer solves:

$$\widehat{\mathcal{P}}_o(I) \begin{cases} \qquad \min_{\Gamma^2} \ \mathcal{C}^2(I) \\ \\ s.t \qquad \Pi^2_{w,I}(c,c) \geq -\Psi(I), \ \forall c \in C \\ \\ \qquad \Pi^2_{l,i}(c,c) \geq 0, \ \forall c \in C, \ \forall i \neq w, \ i \in N \\ \\ \qquad \qquad IC^2_o \end{cases}$$

Inequality $\Pi^2_{w,I}(c,c) \geq -\Psi(I)$ reflects the fact that the buyer consider the winner's investment expenditures as a sunk cost. It is worth to emphasize that because of the buyer's inability to pre-commit to contracts, he cannot decide the investment level even though it is observable. As a consequence, it is the first period winner, anticipating how the buyer will react when facing different investment levels, the one that determines how much investment will be carried out. The following result characterizes the mechanism and investment level induced under non-commitment of the buyer and investment observability. It also shows that the buyer gives *disadvantage* to the first period winner at $t = 2$: since the buyer cannot fully commit to contracts and the first period winner has improved his distribution, the buyer has no incentive to continue giving the mentioned *advantage gap* of the full commitment solution. Rather, it is optimal for him to give disadvantage to the winner due to the fact that it is more likely that this competitor report lower costs. As usual in auction theory, informational asymmetries harm the competitor with the best distribution in one-shot auctions.

Proposition 15 In the absence of buyer's commitment, the level of investment induced when this variable is observable, $\widehat{I}$, solves

$$\max_{I\geq 0} \int\limits_S [1 - F(g_1^{-1}(h(c,I)))]^{n-1}G(c,I)dc - \Psi(I) \tag{13}$$

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with $h(c, I) = c + \frac{G(c,I)}{\frac{\partial G}{\partial c}(c,I)}$ and $g_1(c) = c + \frac{F(c)}{f(c)}$. In this case, the second period cost-minimizing mechanism $\hat{\Gamma}^2(\hat{I})$ corresponds to

$$\hat{q}^2_{w,\hat{I}}(c_w, c_{-w}) = \begin{cases} 1 & c_w + \frac{G(c_w,\hat{I})}{\frac{\partial G}{\partial c}(c_w,\hat{I})}) < \min_{i \neq w} \left\{ c_i + \frac{F(c_i)}{f(c_i)} \,\middle|\, j \neq w \right\} \\ 0 & \sim \end{cases} \tag{14}$$

$$\hat{q}^2_{l,\hat{I},i}(c_i, c_{-i}) = \begin{cases} 1 & c_i + \frac{F(c_i)}{f(c_i)} < \min \left\{ c_w + \frac{G(c_w,\hat{I})}{\frac{\partial G}{\partial c}(c_w,\hat{I})}, c_j + \frac{F(c_j)}{f(c_j)} \,\middle|\, j \neq j, w \right\} \\ 0 & \sim \end{cases} \tag{15}$$

Proof : Appendix A.

$\square$

Since $h(\cdot, I) > g_1(\cdot)$ (Lemma **??**, (ii)), it can occur that the first period winner loses the second procurement even when having the lowest cost among all competitors. Therefore, this mechanism gives disadvantage to the first period winner.

5.2 Investment Non-Observability and Non-Commitment

We now assume that investment is not observable. If the buyer could monitor the investment level chosen by the first period winner, call it I, would react optimally imposing $\hat{\Gamma}^2(I)$ (see proposition **??**'s proof) defined by:

$$\hat{q}^2_{w,I}(c_w, c_{-w}) = \begin{cases} 1 & c_w + \frac{G(c_w,I)}{\frac{\partial G}{\partial c}(c_w,I)}) < \min_{i \neq w} \left\{ c_i + \frac{F(c_i)}{f(c_i)} \,\middle|\, j \neq w \right\} \\ 0 & \sim \end{cases} \tag{16}$$

$$\hat{q}^2_{l,I,i}(c_i, c_{-i}) = \begin{cases} 1 & c_i + \frac{F(c_i)}{f(c_i)} < \min \left\{ c_w + \frac{G(c_w,I)}{\frac{\partial G}{\partial c}(c_w,I)}, c_j + \frac{F(c_j)}{f(c_j)} \,\middle|\, j \neq j, w \right\} \\ 0 & \sim \end{cases} \tag{17}$$

Because now investment can't be observed, the buyer and first period winner enter into a simultaneous-move game. The action space for the first period winner is

$$A_w = [0, +\infty)$$

On the other hand, the buyer may choose any incentive compatible mechanism. Since the best-responses space for the buyer is $BR_b = \{\hat{\Gamma}(I) \mid I \geq 0\}$, we will only pay attention to this type of incentive compatible mechanisms. Recall that when the first period winner a mechanism with second period expected probability function $Q_w(\cdot)$, he chooses the optimal investment level solving

$$\max_{I \geq 0} \int_C Q^2_w(c)G(c, I)dc - \Psi(I)$$

Now we define a pure strategy equilibrium in this context:

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Definition 16 *A pure strategy equilibrium under non-commitment and investment non-observability is a tuple* $(\hat{\Gamma}^2(\hat{I}), \hat{I}) \in BR_b \times A_w$ *that solves*

$$
\begin{cases}
\min_{\hat{\Gamma}(I)} \mathcal{C}^2(I) \\[2mm]
s.t. \quad I \in \arg\max_{I \geq 0} \int_C \widehat{Q}^2_{w,I}(c) G(c, I) dc - \Psi(I) \\[2mm]
\Pi^2_{w,I}(c, c) \geq -\Psi(I), \ \forall c \in S \\[2mm]
\Pi^2_l(c, c) \geq 0, \ \forall c \in S \\[2mm]
IC^2_{no} \\[2mm]
\hat{\Gamma}^2(I) \in BR_b
\end{cases}
$$

with

$$
\widehat{Q}_{w,I}(c) = \begin{cases} (1 - F(g_1^{-1}(h(I,c))))^{n-1} & g_1^{-1}(h(I,c)) < \bar{c} \\ 0 & \sim \end{cases}
$$

and

$$
h(I, c) = c + \frac{G(c, I)}{\frac{\partial G}{\partial c}(c, I)}, \quad g_1(c) = c + \frac{F(c)}{f(c)}
$$

□

That is, in contrast with the observable case when the first period winner could move first choosing his best investment level (anticipating how the buyer would react), both players now move simultaneously. Consider the function

$$
U_w(I \mid \bar{I}) = \int_S [1 - F(g_1^{-1}(h(\bar{I}, c)))]^{n-1} G(c, I) dc - \Psi(I) \tag{18}
$$

which corresponds to the second period expected utility for the first period winner if invested an amount I when facing the mechanism $\hat{\Gamma}^2(\bar{I})$. Finally, define $V(I) = U_w(I \mid I)$ (this is exactly the function that appears in (**??**), Proposition **??**). It is important to stress that there are three forces that influence $V(\cdot)$'s behavior: $[1 - F(g_1^{-1}(h(I, c)))]^{n-1}$ which makes that $V(I)$ decrease as I grows, $G(c, I)$ that increases with I for each $c \in S$, and $-\Psi(I)$ with a negative effect. Because of assumption **??** and the double negative effect, we will assume that $V(\cdot)$ corresponds to:

(*type 1*) $V(0) \geq 0$, exists $\nu > 0$ such that $V(\cdot)$ is increasing in $[0, \nu]$ and then decreases for ever. Note that in this case $\nu = \hat{I}$, the investment level induced under *non-commitment* and investment observability (Proposition **??**).

(*type 2*) Strictly decreasing. In this case it is direct that $\hat{I} = 0$ by simply looking at **??**.

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In both cases we assume that $V(I)$ is negative for I sufficiently large. The following result establishes the existence of a pure equilibrium for each V-*type*, characterizes the equilibrium investment level induced and compares it with the one derived in the *non-commitment* and investment observability case.

Proposition 17 Assume that

$$\lim_{I \to \infty} \int_C [1 - F(g_1^{-1}(h(I,c)))]^{n-1} \frac{\partial G}{\partial I}(c,I)dc - \Psi'(I) < 0 \tag{19}$$

If $V(\cdot)$ is one of type 1, there is a unique pure strategy equilibrium $(\hat{\Gamma}^2(\hat{\hat{I}}), \hat{\hat{I}})$ and satisfies $\hat{\hat{I}} > \hat{I}$. It is characterized by

$$\left. \frac{\partial U_w}{\partial I}(I \mid \hat{\hat{I}}) \right|_{I = \hat{\hat{I}}} = 0 \tag{20}$$

If $V(\cdot)$ belongs to type 2, existence and uniqueness are also ensured, but it could be the case that $\hat{\hat{I}} = 0$ (in this case $\hat{I} = 0$ as well). If not, it is characterized by the above equation and satisfies the same relation with $\hat{I}$.

Proof: Appendix A.

$\square$

It is interesting to note that, under *non-commitment*, observability induces lower investment. If the buyer can't commit to the contracts, and investment is observable, when facing higher levels of this variable he answers with more disadvantageous mechanisms: it is optimal for him to impose $Q_w^2(c) = [1 - F(g_1^{-1}(h(I,c)))]^{n-1}$ that decreases with I. This happens because after cost reducing investment, the first period winner is more likely to obtain lower costs, and therefore, the buyer can give more disadvantage to this competitor in order to reduce expected expenditures. As a consequence, the investment level induced when this variable is observable falls in comparison with other case. Observe also that if $V(\cdot)$ corresponds to *type 2*, it does not means that there will be no investment: we only assert that if there is no positive one, then $(\hat{\Gamma}(0), 0)$ will be an equilibrium.

Observation 6: We first check that the distributional upgrade introduced in Example 1 satisfies the requirement (??) of Proposition ??. Note that $G(c,I) = 1 - (1 - F_w(c))^{\gamma I + 1}$ satisfies

$$\lim_{I \to \infty} \frac{\partial G}{\partial I}(c,I) = \lim_{I \to \infty} \left(-\gamma (1 - F_w(c))^{\gamma I + 1} log(1 - F_w(c)) \right) = 0$$

if $c \neq \bar{c}$, $\underline{c}$. Since

$$\lim_{I \to \infty} [1 - F(g_1^{-1}(h(I,c)))]^{n-1} \frac{\partial G}{\partial I}(c,I) = 0$$

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the Dominated Convergence Theorem allow us to conclude that

$$\lim_{I\to\infty}\int_S [1 - F(g_1^{-1}(h(I,c)))]^{n-1}\frac{\partial G}{\partial I}(I,c)dc - \Psi'(I) = -\lim_{I\to\infty}\Psi'(I) < 0.$$

Then, (**??**) is fulfilled.

To conclude this part, we compare the *full commitment* investment level I^* with the ones derived in this section. It states that commitment raises the investment levels chosen by the first period winner. Therefore, in a context of sequential procurement auctions, a buyer concerned about inducing cost reduction must commit to the contracts in order to achieve that goal.

Proposition 18 If I^* is the full commitment investment level, and $\hat{I}$ and $\hat{\hat{I}}$ the ones derived under no commitment when investment is observable and when it is not respectively, they satisfy $\hat{I} \leq \hat{\hat{I}} < I^*$. Equality between $\hat{I}$ and $\hat{\hat{I}}$ holds only if $V(\cdot)$ is one of type 2 (in this case, $\hat{I} = 0$).

Proof: Appendix A.

□

Basically, the intuition of the result is the following: disadvantageous mechanisms disincentive cost-reduction investment. In other words, the marginal cost of investing becomes stronger, when compared to the marginal benefit in reducing the expected second project cost realization, as long as rules become less attractive (in terms of reducing the chances to win the second procurement). Since the buyer favors the losers when there is no commitment, in contrast to the *advantage gap* given to the first period winner in the other environment, this pattern is observed.

6. EFFICIENCY

In this section we prove the existence and provide a characterization of an ex-post efficient mechanism. That is, one that in each period assigns the project to the lowest cost supplier and induces an investment level such that the marginal cost of investment equals the marginal benefit (in expected terms) of cost reduction. Also, we compare such an efficient investment level with the ones induced by the revenue maximizing mechanisms derived in the previous sections for different environments.

The efficient mechanism, that we will denote by Γ^e, must assign each project to the competitor with the lowest cost realization. Thus, the assignment rules in each period must be given by:

$$q_i^{t,e}(c) = \begin{cases} 1 & c_i < c_j \ \forall j \in N \\ 0 & \sim \end{cases} \tag{21}$$

20

for $t = 1, 2$. Given this rules, and if the first period winner invests a quantity I, the expected social cost is

$$\mathcal{C}(\Gamma^e, I) = \int_{C^n} \left[\sum_{i=1}^n c_i q_i^{1,e}(c) \right] f^n(c) dc + \beta \int_{C^n} \left[c_w q_w^{2,e}(c) + \sum_{i \neq w} c_i q_i^{2,e}(c) \right] f^{n-1}(c_{-w}) \frac{\partial G}{\partial c_w}(c_w, I) dc + \beta \Psi(I)$$

A simple calculation shows that

$$
\begin{aligned}
\mathcal{C}(\Gamma^e, I) &= n \int_C c[1 - F(c)]^{n-1} f(c) dc + \beta \int_C c[1 - F(c)]^{n-1} \frac{\partial G}{\partial c}(c, I) dc \\
&\quad + \beta(n-1) \int_C c[1 - F(c)]^{n-2}[1 - G(c, I)] f(c) dc + \beta \Psi(I)
\end{aligned}
\tag{22}
$$

The efficient investment level, I^e, is the solution to

$$\min_{I \geq 0} \mathcal{C}(\Gamma^e, I) \tag{23}$$

The next result characterizes I^e and states that the efficient mechanism exists regardless of investment observability.

Proposition 19 The efficient investment level I^e is the solution to

$$\max_{I \geq 0} \int_C [1 - F(c)]^{n-1} G(c, I) dc - \Psi(I) \tag{24}$$

Moreover, it can be induced regardless of investment observability using second price sealed bid procurement auctions each period.

Proof: Appendix A.

$\square$

Finally our main result:

Proposition 20 *(Over-investment):* The following relationship holds

$$\hat{I} \leq \hat{\hat{I}} < I^e < I^* \tag{25}$$

That is, in a context of sequential procurements and full commitment of the buyer, the cost minimizing investment level is bigger than the efficient one. Also, buyer's lack of commitment induces investment levels below efficiency.

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Proof: Appendix A.

$\square$

Proposition 6 is the reflect of an assertion previously stated: the more advantage given to the first period winner at the last procurement, the more investment induced. In this context, the efficient mechanism is a *fair rule* in terms of assigning no advantage to any competitor. Therefore, induces less investment than the *full commitment* mechanism and more investment relative to the non-commitment rules. As an example, consider two incentive compatible mechanisms $\widetilde{\Gamma}$ and $\overline{\Gamma}$, not depending on investment, such that $\widetilde{Q}_w^2(c) \geq \overline{Q}_w^2(c)$ for all $c \in C$. In other words, $\widetilde{\Gamma}$ gives more advantage to the first period winner at the second procurement than $\overline{\Gamma}$. Assume also that this variable can't be observed. Therefore, for any fixed investment level I we have

$$\int_C \widetilde{Q}_w^2(c) \frac{\partial G}{\partial I}(c, I) dc - \Psi'(I) \geq \int_C \overline{Q}_w^2(c) \frac{\partial G}{\partial I}(c, I) dc - \Psi'(I) \tag{26}$$

and, as a consequence, the level chosen by the first period winner when this variable is not observable is higher under $\widetilde{\Gamma}$ (recall the first order condition of the winner's problem, assumption **??** and Ψ's convexity). It is quite interesting to observe that when mechanisms are extremely inefficient, in terms of assigning the last project to the first period winner all the time (i.e. $Q_w^2(c) = 1$), there are still investment incentives and, moreover, due to the above inequality the induced level is the highest possible. This result can seem counterintuitive, since an advantage gap can make the first period winner "relax", knowing he owns a big advantage over competitors and likely to win anyway. In order to understand this result we must consider two effects. First, cost reduction investment increases the winner's second period expected costs through Ψ's effect. On the other side, giving more advantage to this agent raises his second period expected utility, which enables him to invest more in cost reduction, and a a consequence, expected second period costs reduce through the distributional upgrade effect. Therefore, it must be that latter effect dominates always the former.

7. DISCUSSION

7.1 Investment Observability "Irrelevancy"

The fact that investment observability cannot improve the buyer's ability to reduce expected costs in the case of full commitment is interesting. Moreover, this is also the case when the buyer's objective is ex-post efficiency, since the ex-post efficient stage mechanisms induce the efficient investment level. A natural question that arises is whether this is true for any mechanism used by the buyer. The answer is no, as shown in the next result. When the buyer sets, for instance, a mechanism that gives less advantage to the first period winner at the second stage than the one in the cost-minimizing mechanism, the investment level chosen by the auctioneer (in the observable case) is higher than the one induced when investment is not observable. The intuition is as follows: a less advantageous mechanism for the first period winner implies that he agent invests less (recall the discussion at the

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end of the efficiency section). On the other hand, the buyer compensates this sub-optimal mechanism by imposing a higher cost-reducing investment level than the one in the optimal mechanism.

Proposition 21 Set $n = 2$ and assume that the buyer wants to implement the following incentive compatible mechanism:

$$\tilde{q}_{w,I}^2(c_w, c_l) = \begin{cases} 1 & c_w < g(c_l) \\ 0 & \sim \end{cases} \tag{27}$$

with $g(\cdot)$ an increasing function that satisfies

$$g(c) \le g_2(c) = c + 2\frac{F(c)}{f(c)}, \ \forall c \in C$$

with strict inequality on a positive-measure subset of C. Denote by $\tilde{I}_b$ and $\tilde{I}_w$ the investment levels chosen by the buyer and first period winner respectively when facing this mechanism. Then, $\tilde{I}_b > \tilde{I}_w$

Proof: Appendix A.

$\square$

7.2 Number of Competitors

Let $n < m \in \mathbb{N}$ and $I^*(n), I^*(m)$ the *full commitment* solutions when there are n and m competitors respectively. Consider the functions

$$g_l(c) = c + \left(1 + \frac{1}{l-1}\right)\frac{F(c)}{f(c)}$$

and recall that $g_l(\cdot)$ is exactly the function $g(\cdot)$ used in the full commitment section for the case of l competitors. Because both functions are increasing in c and $g_n(\cdot) > g_m(\cdot)$, it is easy to see that

$$Q_{w,n}^{2*}(c) = \left[1 - F(g_n^{-1}(c))\right]^{n-1} > \left[1 - F(g_m^{-1}(c))\right]^{m-1} = Q_{w,m}^{2*}(c)$$

that is, the first period winner has more chances of winning the second procurement when the number of competitors decreases. This is due to two factors: as the number of competitors grows, the advantage to the winner decreases, and also it is more difficult to obtain costs low enough relative to *all* rivals. Thus, (??) implies that $I^*(n) > I^*(m)$, and since

$$\lim_{n \to \infty} Q_{w,n}^{2*}(c) = 0, \ \forall c \in C$$

we conclude that $I^*(n) \to 0$.

Two effects induce this result. From the first period winner's point of view, the incentives to invest decrease with the number of players because mechanisms become more disadvantageous, and therefore the probability of winning decreases, and with it the probability of effectively gaining from a

cost reduction.[6] On the other hand, from the buyer's perspective, expected costs are reduced due to the fact that more competition is generated when the number of players increases, so inducing cost reduction becomes relatively expensive (recall that the buyer ensures participation for the first period winner at the investment level he chooses) in comparison to the marginal reduction of expected expenditures through the distributional upgrade.

If the number of players is large enough, the buyer would not want to induce cost reduction eventually. In fact, denoting by $\hat{I}(n)$, $\hat{\hat{I}}(n)$, $I^e(n)$ the investment levels for the corresponding cases when there are n competitors, Proposition **??** still holds, that is, $0 \leq \hat{I}(n) \leq \hat{\hat{I}}(n) < I^e(n) < I^*(n)$, so they all collapse to zero when $n \to \infty$ (eventually for $n \geq \bar{n}$, some $\bar{n} \in I\!N$).

8. Conclusions

Throughout this paper we have analyzed the interaction between sequential procurements auctions and cost reduction investment, from a mechanism design approach. Our main finding is the role of mechanisms not only as optimal rules to assign tasks, but as investment-incentive tools in a dynamic context. As the previous literature, we find that the buyer's lack of commitment reduces the investment levels induced below efficiency. To the contrary, we showed that when the buyer can commit to contracts, introducing sequentiality raises investment over the efficient level, which becomes relevant if the objective is to reduce future expenditures.

From a theoretical point of view we obtain two results: complementarity between projects is determined endogenously in the model and the cost-minimizing contract introduces *memory* when the buyer can pre-commit. In the first case, mechanisms that give more advantage to the first period winner at the last procurement provide more investment incentives, and therefore, more complementarity among tasks. The extreme case is the following: the buyer can specify rules so that *ex ante* substitutes projects become complements endogenously. With respect to the second point, in a sequential-procurement setting, the *full commitment* solution introduces *memory* in the contract, expressed in an *advantage gap* granted to the first period winner at the last procurement. This advantage decreases with the number of competitors but never disappears. Moreover, it is optimal even when investment chances are not allowed since the buyer can distribute incentives inter-temporally in a better way than in the two-independent-contracts solution.

We showed that, in a context of full commitment, the cost minimizing and efficient investment levels can achieved regardless of the observability of this variable. This can be done by setting in both cases (observable and non-observable) the corresponding optimal mechanism derived when investment can be observed. This result states that there is no need in observing the investment carried out by the first period winner: providing the right incentives will lead to the correct investment levels. This issue takes especial relevance when monitoring investment is expensive.

Finally, it could be useful that institutions that make use periodically of procurement auctions

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[6]This dominates a force with the opposite sign: as a mechanism becomes less advantageous, a bigger invest is needed to have a good probability of winning

establish dynamic contracts in order to reduce costs. If they can credibly commit to these type of contracts, investment levels above efficiency can be achieved within an optimal rule. All together, sequentiality and commitment, will lead to lower expected expenditures because of inter-temporal distribution of incentives and distributional upgrades respectively. To conclude, the essential message of this paper is that, in a context of dynamic procurements, mechanisms play a investment-incentive role which can be used by the buyer in order to reduce expected costs. As a consequence, in setting the appropriate incentives, these institutions can lower their expected expenditures when compared to static contracts.

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9. APPENDIX A: PROOFS

Example ??: Assume that $F(\cdot)$ is a concave twice differentiable distribution. Define $G(c, 0) = F(c)^{\eta}$, $0 < \eta < 1$ and $G(c, I) = 1 - (1 - G(c, 0))^{\gamma I + 1}$, $\gamma > 0$. It is straightforward that the first inequality in assumption **??** is checked when $I' = 0$. Also, it is clear that

$$\frac{\partial G}{\partial c}(c, I) = (\gamma I + 1)(1 - G(c, 0))^{\gamma I} \frac{\partial G}{\partial c}(c, 0)$$

Now, setting $0 \leq I' < I$ and $c' < c$, simple algebra shows that the second inequality in assumption **??** is equivalent to

$$(1 - G(c, 0))^{\gamma(I - I')} < (1 - G(c', 0))^{\gamma(I - I')}$$

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which is obviously true since $c' < c$.

Assumption **??** holds since

$$\frac{\partial^2 G}{\partial I^2}(c, I) = -\gamma^2 (1 - G(c, 0))^{\gamma I + 1} log^2 (1 - G(c, 0)) < 0$$

Assumption **??** holds since

$$\left| \frac{\partial G}{\partial I}(c, I) \right| = \gamma (1 - G(c, 0))^{\gamma I + 1} |log(1 - G(c, 0))|$$

$$< \gamma (1 - G(c, 0)) |log(1 - G(c, 0))| \in L^1(I\!R)$$

It remains to show that $c \mapsto \frac{G(c,I)}{\frac{\partial G}{\partial c}(c,I)}$ is increasing in c. This is equivalent to

$$\left(\frac{\partial G}{\partial c}(c, I) \right)^2 - G(c, I) \frac{\partial^2 G}{\partial c^2}(c, I) > 0$$

so, it suffices to show that $\frac{\partial^2 G}{\partial c^2}(c, I) < 0$. Since

$$\frac{\partial^2 G}{\partial c^2}(c, I) = -(\gamma I + 1)\gamma I (1 - G(c, 0))^{\gamma I - 1} \left(\frac{\partial G}{\partial c}(c, 0) \right)^2 + (\gamma I + 1)(1 - G(c, 0))^{\gamma I} \frac{\partial^2 G}{\partial c^2}(c, 0)$$

, $\eta < 1$, $F'' \leq 0$ and

$$\frac{\partial^2 G}{\partial c^2}(c, 0) = \eta(\eta - 1)F^{\eta - 2}(c)(F'(c))^2 + \eta F'(c)^{\eta - 1}F''(c) < 0$$

the result follows.

$\square$

Proof of Lemma **??**:

(i) Multiplying the second inequality in assumption **??** by $\frac{\partial G}{\partial c}(c, I)\frac{\partial G}{\partial c}(c, I')$ and integrating with respect to c between c' and $\bar{c}$ yields the inequality.

(ii) Integrating the same inequality in assumption **??** with respect to c' between 0 and c yields the result.

(iii) Combining (i) and (ii) we obtain

$$\frac{1 - G(c, I')}{1 - G(c, I)} < \frac{\frac{\partial G}{\partial c}(c, I')}{\frac{\partial G}{\partial c}(c, I)} < \frac{G(c, I')}{G(c, I)}, \text{ for all } I < I'$$

so, $G(c, I) < G(c, I')$ if $I < I'$.

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Proof Lemma **??**: Only (i). Inequalities $\Pi^1_{i,\bar{I}}(c_i, c_i) \geq \Pi^1_{i,\bar{I}}(c_i, c'_i)$ and $\Pi^1_{i,\bar{I}}(c'_i, c'_i) \geq \Pi^1_{i,\bar{I}}(c'_i, c_i)$ yield

$$Q^1_i(c'_i)(c'_i - c_i) \leq \Pi^1_{i,\bar{I}}(c_i, c_i) - \Pi^1_{i,\bar{I}}(c'_i, c'_i) \leq Q^1_i(c_i)(c'_i - c_i)$$

Without loss of generality, suppose $c'_i > c_i$. Therefore $Q^1_i(\cdot)$ is non-increasing and

$$Q^1_i(c'_i) \leq \frac{\Pi^1_{i,\bar{I}}(c_i, c_i) - \Pi^1_{i,\bar{I}}(c'_i, c'_i)}{c'_i - c_i} \leq Q^1_i(c_i)$$

Taking the limit when c'_i goes to c_i, we obtain

$$\frac{d\Pi^1_{i,\bar{I}}}{dc_i}(c_i, c_i) = -Q^1_i(c_i)$$

and, as a consequence,

$$\Pi^1_{i,\bar{I}}(\bar{c}, \bar{c}) - \Pi^1_{i,\bar{I}}(c, c) = -\int_c^{\bar{c}} Q^1_i(c)dc$$

concluding the proof. For (ii), the reasoning is the same.

$\square$

Proof Theorem **??**: Assume that the buyer wants to induce an investment level I and that $\Gamma(I)$ is optimal for $\mathcal{P}_o$. Rearranging terms in (**??**), (**??**), (**??**) and using Lemma **??** we obtain:

$$T^2_{w,I}(c) = \Pi^2_{w,I}(\bar{c}, \bar{c}) + \int_c^{\bar{c}} Q^2_{w,I}(c)dc + cQ^2_{w,I}(c) + \Psi(I) \tag{28}$$

$$T^2_{l,I,i}(c) = \Pi^2_{l,I,i}(\bar{c}, \bar{c}) + \int_c^{\bar{c}} Q^2_{l,I,i}(c)dc + cQ^2_{l,I,i}(c), \ i \neq w, \ i \in N \tag{29}$$

$$
\begin{aligned}
T^1_i(c) &= \Pi^1_{i,I}(\bar{c}, \bar{c}) + \int_c^{\bar{c}} Q^1_i(c)dc + cQ^1_i(c) - \beta Q^1_i(c'_i) \int_C \Pi^2_{w,I}(c, c) \frac{\partial G}{\partial c}(c, I)dc \\
&\quad - \beta[1 - Q^1_i(c'_i)] \int_C \Pi^2_{l,I,i}(c, c)f(c)dc.
\end{aligned} \tag{30}
$$

Then, integrating by parts

$$\int_C T^2_{w,I}(c)\frac{\partial G}{\partial c}(c, I)dc = \Pi^2_{w,I}(\bar{c}, \bar{c}) + \int_C Q^2_{w,I}(c)G(c, I)dc + \int_C cQ^2_{w,I}(c)\frac{\partial G}{\partial c}(c, I)dc + \Psi(I) \tag{31}$$

$$\int_C T^2_{l,I,i}(c)f(c)dc = \Pi^2_{l,I,i}(\bar{c}, \bar{c}) + \int_C Q^2_{l,I,i}(c)F(c)dc + \int_C cQ^2_{l,I,i}(c)f(c)dc, \ i \neq w, \ i \in N \tag{32}$$

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$$\int_C T_i^1(c)f(c)dc \;=\; \Pi_{i,I}^1(\bar{c},\bar{c}) + \int_C Q_i^1(c)F(c)dc + \int_C cQ_i^1(c)f(c)dc - \beta Q_i^1 \int_C \Pi_{w,I}^2(c,c)\frac{\partial G}{\partial c}(c,I)dc$$

$$-\beta[1-Q_i^1]\int_C \Pi_{l,I,i}^2(c,c)f(c)dc. \tag{33}$$

with $Q_i^1 = \int_C Q_i^1(c)f(c)dc$ (observe that $\sum_{i=1}^n Q_i^1 = 1$).

Replacing this last expressions in (**??**) yields a procurement cost of

$$\mathcal{C} \;=\; \sum_{i=1}^n \left[\Pi_{i,I}^1(\bar{c},\bar{c}) + \int_C Q_i^1(c)F(c)dc + \int_C cQ_i^1(c)f(c)dc \right] - \beta\left[\Pi_{w,I}^2(\bar{c},\bar{c}) + \int_C Q_{w,I}^2(c,c)G(c,I)dc \right]$$

$$-\beta\sum_{i=1}^n \left[\Pi_{l,I,i}^2(\bar{c},\bar{c}) + \int_C Q_{l,I,i}^2(c)F(c)dc \right] + \sum_{i=1}^n Q_i^1 \left[\Pi_{l,I,i}^2(\bar{c},\bar{c}) + \int_C Q_{l,I,i}^2(c)F(c)dc \right]$$

$$+\beta\left[\Pi_{w,I}^2(\bar{c},\bar{c}) + \int_C Q_{w,I}^2(c)G(c,I)dc \right] + \beta \int_C cQ_{w,I}^2(c)\frac{\partial G}{\partial c}(c,I)dc + \beta\Psi(I)$$

$$+\beta\sum_{i\neq w} \left[\Pi_{l,I,i}^2(\bar{c},\bar{c}) + \int_C Q_{l,I,i}^2(c)F(c)dc \right] + \beta\sum_{i\neq w} \int_C cQ_{l,I,i}^2(c)f(c)dc \tag{34}$$

Since the second and fifth term cancel each other we get

$$\mathcal{C} \;=\; \sum_{i=1}^n \left[\Pi_{i,I}^1(\bar{c},\bar{c}) + \int_C Q_i^1(c)F(c)dc + \int_C cQ_i^1(c)f(c)dc \right]$$

$$+\beta\left[\int_C cQ_{w,I}^2(c)\frac{\partial G}{\partial c}(c,I)dc + \Psi(I) + \sum_{i\neq w}\int_C cQ_{l,I,i}^2(c)f(c)dc \right]$$

$$-\beta\sum_{i=1}^n \left[\Pi_{l,I,i}^2(\bar{c},\bar{c}) + \int_C Q_{l,I,i}^2(c)F(c)dc \right] + \beta\sum_{i\neq w} \left[\Pi_{l,I,i}^2(\bar{c},\bar{c}) + \int_C Q_{l,I,i}^2(c)F(c)dc \right]$$

$$+\sum_{i=1}^n Q_i^1 \left[\Pi_{l,I,i}^2(\bar{c},\bar{c}) + \int_C Q_{l,I,i}^2(c)F(c)dc \right] \tag{35}$$

Consider the last term in the above expression. Define

$$\mathcal{U}_{l,i}^2 = \int_C \Pi_{l,I,i}^2(c,c)f(c)dc = \Pi_{l,I,i}^2(\bar{c},\bar{c}) + \int_C Q_{l,I,i}^2(c)F(c)dc$$

that is, the expected utility for player i at $t = 2$ conditional on being a first-period loser. At the optimum, it must be that $Q_i^1, Q_j^1 > 0 \Rightarrow \mathcal{U}_{l,i}^2 = \mathcal{U}_{l,j}^2$ for all $i \neq j$, $i,j \in N$. Suppose this is not true,

then, there are $i \neq j$, $i, j \in N\backslash\{w\}$ such that, for instance, $\mathcal{U}^2_{l,i} > \mathcal{U}^2_{l,j}$ and $Q^1_i, Q^1_j > 0$. Define

$$\tilde{q}^1_k(\cdot) = \begin{cases} q^1_k(\cdot) & k \neq j, i. \\ q^1_j(\cdot) + q^1_i(\cdot) & k = j. \\ 0 & k = i. \end{cases}$$

that induces the family of expected probabilities $(\tilde{Q}^1_i(\cdot); i = 1, ..., n)$. It is easy to see that $\tilde{Q}^1_i(c)$ is not increasing in c for all $i \in N$. For all $k \neq i, j$ keep transfers unchanged. Then $\tilde{\Pi}^1_{k,I}(c,c) = \Pi^1_{k,I}(c,c)$, so incentive compatibility and participation constraints for this players are satisfied. For player i, because Γ is feasible, $\Pi^1_i(\bar{c}, \bar{c}) \geq \beta \int_C \Pi^2_{l,I,i}(c)f(c)dc$. Define $\tilde{T}^1_i(\bar{c})$ such that $\tilde{\Pi}^1_{i,I}(\bar{c}, \bar{c}) = \Pi^1_{i,I}(\bar{c}, \bar{c})$ and compute transfers according to Lemma **??**. We obtain $\tilde{\Pi}^1_{i,I}(c,c) = \tilde{\Pi}^1_{i,I}(\bar{c}, \bar{c})$, $\forall c$ and so all restrictions are fulfilled. Finally, for player j define $\tilde{T}^1_j(\bar{c})$ so $\tilde{\Pi}^1_{j,I}(c,c) = \tilde{\Pi}^1_{j,I}(\bar{c}, \bar{c})$ holds and calculate transfers using the same lemma. As a consequence, no restriction is violated for this competitor. Henceforth, this new mechanism $\tilde{\Gamma}$ is feasible (it is direct that probabilities sum one). Moreover, since $\sum_{i=1}^n \tilde{Q}^1_i(c) = \sum_{i=1}^n Q^1_i(c)$ for all c and $\sum_{i=1}^n \tilde{\Pi}^1_{i,I}(\bar{c}, \bar{c}) = \sum_{i=1}^n \Pi^1_{i,I}(\bar{c}, \bar{c})$ under $\tilde{\Gamma}$ the first term in $\mathcal{C}$ keeps unaltered and the last one falls, a contradiction with $\Gamma(I)$'s optimality (the remaining terms are not affected by this change). Thus, at the optimum, $\mathcal{U}^2_{l,i} = \mathcal{U}_l$ for all $i \neq w$.

This last result enables the buyer to pay attention only to mechanisms that satisfy

$$Q^2_{l,I,i}(\cdot) \equiv Q^2_{l,I}(\cdot)$$

$$T^2_{l,I,i}(\cdot) \equiv T^2_{l,I}(\cdot)$$

Therefore, total expected cost reduces to

$$\begin{aligned}
\mathcal{C} &= \sum_{i=1}^n \left[\Pi^1_{i,I}(\bar{c}, \bar{c}) + \int_C Q^1_i(c)F(c)dc + \int_C cQ^1_i(c)f(c)dc \right] \\
&+ \beta \left[\int_C cQ^2_{w,I}(c)\frac{\partial G}{\partial c}(c,I)dc + \Psi(I) + (n-1)\int_C cQ^2_{l,I}(c)f(c)dc \right] - \beta n\mathcal{U}_l + \beta(n-1)\mathcal{U}_l + \mathcal{U}_l \\
&= \sum_{i=1}^n \left[\Pi^1_{i,I}(\bar{c}, \bar{c}) + \int_C Q^1_i(c)F(c)dc + \int_C cQ^1_i(c)f(c)dc \right] \\
&+ \beta \left[\int_C cQ^2_{w,I}(c)\frac{\partial G}{\partial c}(c,I)dc + \Psi(I) + (n-1)\int_C cQ^2_{l,I}(c)f(c)dc \right] \qquad (36)
\end{aligned}$$

To conclude, observe that at the optimum

$$\beta \sum_{i=1}^n \int_C \Pi^2_{l,I}(c,c)f(c)dc = \beta \sum_{i \neq w} \left(1 + \frac{1}{n-1} \right) \left[\Pi^2_{l,I}(\bar{c}, \bar{c}) + \int_C Q^2_{l,I}(c)F(c)dc \right]$$

29

Now add and subtract the last and first term respectively. We obtain

$$
\begin{aligned}
\mathcal{C} &= \sum_{i=1}^{n}\left[\int_{C} Q_{i}^{1}(c)F(c)dc + \int_{C} cQ_{i}^{1}(c)f(c)dc\right] + \sum_{i=1}^{n}\left[\Pi_{i,I}^{1}(\bar{c},\bar{c}) - \beta\int_{C}\Pi_{l,I}^{2}(c,c)f(c)dc\right] \\
&\quad + \beta\int_{C} cQ_{w,I}^{2}(c)\frac{\partial G}{\partial c}(c,I)dc + \beta\Psi(I) + \beta\sum_{i=1}^{n}\Pi_{l,I}^{2}(\bar{c},\bar{c}) \\
&\quad + \sum_{i\neq w}\left[\int_{C} cQ_{l,I}^{2}(c)f(c)dc + \left(1+\frac{1}{n-1}\right)\int_{C} Q_{l,I}^{2}(c)F(c)dc\right]
\end{aligned}
\tag{37}
$$

Note that, in an optimal mechanism, $T_{l,I}^{2}(\bar{c})$ will be set up such that $\Pi_{l,I}^{2}(\bar{c},\bar{c})=0$. On the other hand, $T_{i,I}^{1}(\bar{c})$ will be designed such that $\Pi_{i,I}^{1}(\bar{c},\bar{c}) = \beta\int_{C}\Pi_{l,I}^{2}(c,c)f(c)dc$ for all $i\in N$. As a consequence,

$$
\begin{aligned}
\mathcal{C} &= \sum_{i=1}^{n}\left[\int_{C} Q_{i}^{1}(c)F(c)dc + \int_{C} cQ_{i}^{1}(c)f(c)dc\right] + \beta\Psi(I) \\
&\quad + \beta\left[\int_{C} cQ_{w,I}^{2}(c)\frac{\partial G}{\partial c}(c,I)dc + (n-1)\int_{C} cQ_{l,I}^{2}(c)f(c)dc + n\int_{C} Q_{l,I}^{2}(c)F(c)dc\right]
\end{aligned}
\tag{38}
$$

which can be written as

$$
\begin{aligned}
\mathcal{C} &= \int_{C^{n}}\sum_{i=1}^{n}\left[c_{i}+\frac{F(c_{i})}{f(c_{i})}\right]q_{i}^{1}(c)f^{n}(c)dc + \beta\Psi(I) \\
&\quad + \beta\int_{C^{n}}\left[c_{w}q_{w,I}^{2}(c) + \sum_{i\neq w}\left[c_{i}+\left(1+\frac{1}{n-1}\right)\frac{F(c_{i})}{f(c_{i})}\right]q_{l,I,i}^{2}(c)\right]f^{n-1}(c_{-w})\frac{\partial G}{\partial c_{w}}(c_{w},I)dc
\end{aligned}
\tag{39}
$$

Pointwise maximization yields the following rules

$$
q_{i}^{1*}(c_{1},...,c_{n}) = \begin{cases} 1 & c_{i}+\frac{F(c_{i})}{f(c_{i})} < c_{j}+\frac{F(c_{j})}{f(c_{j})} \quad \forall j\neq i \\ 0 & \sim \end{cases}
$$

$$
q_{w}^{2*}(c_{w},c_{-w}) = \begin{cases} 1 & c_{w} < g(c_{i}) \quad \forall i\neq w \\ 0 & \sim \end{cases}
$$

$$
q_{l,i}^{2*}(c_{i},c_{-i}) = \begin{cases} 1 & g(c_{i}) = min\{c_{w},g(c_{j}); \quad \forall j\neq w\} \\ 0 & \sim \end{cases}
$$

with $g(c) = c + \left(1+\frac{1}{n-1}\right)\frac{F(c)}{f(c)}$. Finally, because this last function and $c+\frac{F(c)}{f(c)}$ are increasing (assumption **??**), the expected probability functions in each period are non increasing. If transfers are computed according to Lemma **??**, the mechanism is incentive compatible and no participation constraint is violated. Because this mechanism is optimal regardless of the investment level the buyer may want to induce, it is also optimal for $\mathcal{P}_{o}$. This concludes the proof.

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$\square$

Proof Theorem **??**: Recall that under full commitment and investment observability the rules defined by Γ^* minimize the expected cost of both projects for any level of investment $I \geq 0$ chosen by the first period winner. Thus, in this case, to obtain the optimal level of investment for the buyer, I^*, we replace the mentioned mechanism in the procurement cost expression and minimize with respect to I. Remember that, at the optimum of this problem, the expected procurement cost can be written as (see expression (**??**) in the proof of theorem **??**)

$$
\begin{aligned}
\mathcal{C} \;=\; & \sum_{i=1}^{n}\left[\int_C cQ_i^{1*}(c)F'(c)dc + \int_C Q_i^{1*}(c)F(c)dc\right] + \beta\Psi(I) \\
& + \beta\left[\int_C cQ_w^{2*}(c)\frac{\partial G}{\partial c}(c,I)dc + n\int_C Q_l^{2*}(c)F(c)dc + (n-1)\int_C cQ_l^{2*}(c)f(c)dc\right] \\
& = \mathcal{K}(Q^{1*}) + \beta\mathcal{T}(Q^{2*},I) + \beta\Psi(I)
\end{aligned}
\tag{40}
$$

with

$$
\mathcal{K}(Q^{1*}) = \int_C cQ_i^{1*}(c)F'(c)dc + \int_C Q_i^{1*}(c)F(c)dc
\tag{41}
$$

$$
\mathcal{T}(Q^{2*},I) = \int_C cQ_w^{2*}(c)\frac{\partial G}{\partial c}(c,I)dc + n\int_C Q_l^{2*}(c)F(c)dc + (n-1)\int_C cQ_l^{2*}(c)f(c)dc
\tag{42}
$$

Since $\mathcal{K}(Q^{1*})$ does not depend on investment we can only pay attention to $\mathcal{T}(Q^{2*},I)$. Then, in order to obtain the optimal investment level, the buyer solves

$$
\min_{I \geq 0} \; \mathcal{T}(Q^{2*},I) + \Psi(I)
\tag{43}
$$

It can be easily checked that

$$
Q_w^{2*}(c) = \begin{cases} 1 & c \leq g(\underline{c}) \\ (1-F(g^{-1}(c)))^{n-1} & \sim \end{cases}
\tag{44}
$$

$$
Q_l^{2*}(c) = \begin{cases} 0 & c > g^{-1}(\bar{c}) \\ (1-G(g(c),I))(1-F(c))^{n-2} & \sim \end{cases}
\tag{45}
$$

are the expected probabilities of winning the second project for a winner and any loser respectively,

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conditional on the report c. Then, integrating by parts,

$$\int_S cQ_w^{2*}(c)\frac{\partial G}{\partial c}(c,I)dc = \int_{\underline{c}}^{g(\underline{c})} c\frac{\partial G}{\partial c}(c,I)dc + \int_{g(\underline{c})}^{\bar{c}} c[1-F(g^{-1}(c))]^{n-1}\frac{\partial G}{\partial c}(c,I)dc$$

$$= g(\underline{c})G(g(\underline{c}),I) - \int_{\underline{c}}^{g(\underline{c})} G(c,I)dc + \bar{c}[1-F(g^{-1}(\bar{c}))]^{n-1} - g(\underline{c})G(g(\underline{c}),I)$$

$$- \int_{g(\underline{c})}^{\bar{c}} [1-F(g^{-1}(c))]^{n-1}G(c,I)dc$$

$$+(n-1)\int_{g(\underline{c})}^{\bar{c}} c[1-F(g^{-1}(c))]^{n-2}f(g^{-1}(c))\frac{G(c,I)}{g'(g^{-1}(c))}dc \qquad (46)$$

Using $t = g^{-1}(c)$ in the last integral we obtain

$$\int_S cQ_w^{2*}(c)\frac{\partial G}{\partial c}(c,I)dc = \bar{c}[1-F(g^{-1}(\bar{c}))]^{n-1} - \int_{\underline{c}}^{g(\underline{c})} G(c,I)dc$$

$$- \int_{g(\underline{c})}^{\bar{c}} [1-F(g^{-1}(c))]^{n-1}G(c,I)dc$$

$$+(n-1)\int_{\underline{c}}^{g^{-1}(\bar{c})} g(t)[1-F(t)]^{n-2}f(t)G(g(t),I)dt \qquad (47)$$

and replacing $g(c) = c + \left(1 + \frac{1}{n-1}\right)\frac{F(c)}{f(c)}$ in the same term we get

$$\int_S cQ_w^{2*}(c)\frac{\partial G}{\partial c}(c,I)dc = \bar{c}[1-F(g^{-1}(\bar{c}))]^{n-1} - \int_C [1-F(g^{-1}(c))]^{n-1}G(c,I)dc$$

$$+(n-1)\int_{\underline{c}}^{g^{-1}(\bar{c})} c[1-F(c)]^{n-2}f(c)G(g(c),I)dc$$

$$+n\int_{\underline{c}}^{g^{-1}(\bar{c})} [1-F(c)]^{n-2}G(g(c),I)F(c)dc \qquad (48)$$

Also, we have that

$$n\int_C Q_l^{2*}(c)F_l(c)dc = n\int_{\underline{c}}^{g^{-1}(\bar{c})} [1-F(c)]^{n-2}[1-G(g(c),I)]F(c)dc$$

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$$(n-1)\int_C cQ_l^{2*}(c)f(c)dc = (n-1)\int_{\underline{c}}^{g^{-1}(\bar{c})} c[1-F(c)]^{n-2}[1-G(g(c),I)]f(c)dc$$

Therefore

$$
\begin{aligned}
\mathcal{T}(Q^{2*},I) &= \bar{c}[1-F(g^{-1}(\bar{c}))]^{n-1} - \int_C [1-F(g^{-1}(c))]^{n-1}G(c,I)dc \\
&\quad +n\int_{\underline{c}}^{g^{-1}(\bar{c})}[1-F(c)]^{n-2}F(c)dc + (n-1)\int_{\underline{c}}^{g^{-1}(\bar{c})} c[1-F(c)]^{n-2}f(c)dc \qquad (49)
\end{aligned}
$$

Only the second term of the last expression depends on I, so the level of investment that minimizes the expected cost under full observability and commitment is the solution to

$$\max_{I\geq 0}\int_C [1-F(g^{-1}(c))]^{n-1}G(c,I)dc - \Psi(I)$$

$\square$

*Proof Proposition **??**.*

Since (Γ^*,I^*), defined by expressions (**??**), (**??**) and (**??**), does not depend on the level of investment I, the rules in Γ^* are feasible when the investment level is not observable. When the winner of the first procurement faces the rules in Γ^*, he decides his level of investment by solving

$$\max_{I\geq 0}\int_C \Pi_w^2(c,c)\frac{\partial G}{\partial c}(c,I)dc - \Psi(I)$$

Since $\int_C \Pi_w^{2*}(c)\frac{\partial G}{\partial c}(c,I)dc = T_w^{2*}(\bar{c}) - \bar{c}Q_w^{2*}(\bar{c}) + \int_C Q_w^{2*}(c)G(c,I)dc - \Psi(I)$, this is equivalent to solve

$$\max_{I\geq 0}\int_C Q_w^{2*}(c)G(c,I)dc - \Psi(I)$$

and using the definition of $Q_w^{2*}(\cdot)$, this is equivalent to solve

$$\max_{I\geq 0}\int_S [1-F(g^{-1}(c))]^{n-1}G(c,I)dc - \Psi(I)$$

Since I^* satisfies the same optimization problem, we conclude that $\mathcal{C}(\Gamma_{no}^*,I_{no}^*)$ is incentive compatible, and therefore feasible for the non-observable case.

It is obvious that $\mathcal{C}(\Gamma^*,I^*) \leq \mathcal{C}(\Gamma_{no}^*,I_{no}^*)$, so the feasibility of (Γ^*,I^*) for the non-observable case implies that $\mathcal{C}(\Gamma_{no}^*,I_{no}^*) = \mathcal{C}(\Gamma^*,I^*)$.

$\square$

Proof Proposition **??**:

Suppose the first period winner had invested an amount I. It is easy to see that the second procurement expected cost corresponds to

$$\mathcal{C}^2(I) = \int_{C^n} \left[\left(c_w + \frac{G(c_w, I)}{\frac{\partial G}{\partial c}(c_w, I)} \right) q^2_{w,I}(c) + \sum_{j \neq w} \left(c_{j,l} + \frac{F(c_{j,l})}{f(c_{j,l})} \right) q^2_{j,l}(c) \right] f^{n-1}(c_{-w}) \frac{\partial G}{\partial c}(c_w, I) dc \quad (50)$$

Therefore, the second period cost-minimizing mechanism, call it $\hat{\Gamma}^2(I)$, is defined by

$$\hat{q}^2_{w,I}(c_w, c_{-w}) = \begin{cases} 1 & c_w + \frac{G(c_w, I)}{\frac{\partial G}{\partial c}(c_w, I)}) < \min_{i \neq w} \left\{ c_i + \frac{F(c_i)}{f(c_i)} \right\} \\ 0 & \sim \end{cases}$$

$$\hat{q}^2_{l,I,i}(c_i, c_{-i}) = \begin{cases} 1 & c_i + \frac{F(c_i)}{f(c_i)} < \min \left\{ c_w + \frac{G(c_w, I)}{\frac{\partial G}{\partial c}(c_w, I)}), c_j + \frac{F(c_j)}{f(c_j)} \middle| j \neq i, w \right\} \\ 0 & \sim \end{cases}$$

This mechanism is incentive compatible due to the regularity conditions imposed in assumption **??**. Anticipating that the selection of $\hat{\Gamma}^2(I)$, a first period winner will solve

$$\max_{I \geq 0} \int_C \widehat{Q}^2_{w,I}(c_w) G(c_w, I) dc_w - \Psi(I)$$

with $\widehat{Q}^2_{w,I}(c) = \int_{C^{n-1}} \hat{q}^2_{w,I}(c_w, I) f^{n-1}(c_{-w}) dc_{-w}$. If we define $h(c, I) = c + \frac{G(c,I)}{\frac{\partial G}{\partial c}(c,I)}$ and $g_1(c) = c + \frac{F(c)}{f(c)}$ we have that $\hat{q}^2_{w,I}(c_w, c_{-w}) = 1 \Leftrightarrow g_1^{-1}(h(c, I)) < c_i, \forall i \neq w$.

Because of assumption **??**, $g_1^{-1}(\cdot)$ exists, and because of Lemma **??**, $h(\cdot, I) > g(\cdot)$, thus $g_1^{-1}(h(c, I)) > \underline{c}$ for all $c > 0$. Hence, we obtain

$$\widehat{Q}^2_{w,I}(c) = \begin{cases} (1 - F(g_1^{-1}(h(c, I))))^{n-1} & g_1^{-1}(h(c, I)) < \bar{c} \\ 0 & \sim \end{cases}$$

and as a direct consequence

$$\int_C \widehat{Q}^2_{w,I}(c) G(c, I) dc - \Psi(I) = \int_C [1 - F(g_1^{-1}(h(c, I)))]^{n-1} G(c, I) dc - \Psi(I)$$

Therefore, in absence of investment observability and buyer's commitment, the level of investment carried out by the first period winner, $\widehat{I}$, is the solution to

$$\max_{I \geq 0} \int_S [1 - F(g_1^{-1}(h(c, I)))]^{n-1} G(c, I) dc - \Psi(I)$$

$\square$

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Proof Proposition **??***:* Remember the function

$$V(I) = \int_C [1 - F(g_1^{-1}(h(c, I)))]^{n-1} G(c, I) dc - \Psi(I)$$

Define

$$L(I) = \int_C [1 - F(g_1^{-1}(h(c, I)))]^{n-1} \frac{\partial G}{\partial I}(c, I) dc - \Psi'(I)$$

It is continuous because of: (i) $[1 - F(g_1^{-1}(h(c, I)))]^{n-1} \frac{\partial G}{\partial I}(c, I)$ is continuous in I for each c and, (ii)
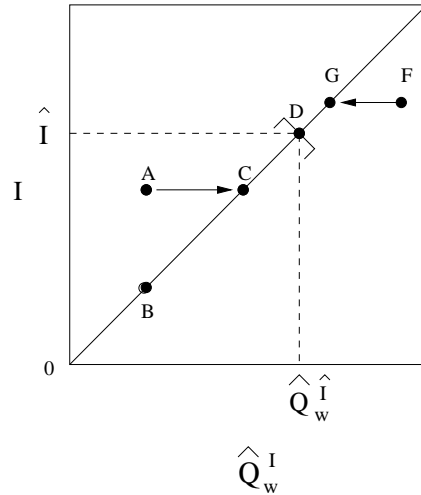
$$\left| [1 - F(g_1^{-1}(h(c, I)))]^{n-1} \frac{\partial G}{\partial I}(c, I) \right| < \frac{\partial G}{\partial I}(c, I) < f(c) \in L^1(\mathbb{R})$$

thanks to assumption **??** (Dominated Convergence Theorem in $L^1(\mathbb{R})$). Also, $L(\cdot)$ is decreasing because of assumptions **??**, **??** and since $[1 - F(g_1^{-1}(h(c, I)))]^{n-1}$ decreases with I. On the other hand, condition (**??**), that is

$$\lim_{I \to \infty} L(I) = \int_C [1 - F(g_1^{-1}(h(c, I)))]^{n-1} \frac{\partial G}{\partial I}(c, I) dc - \Psi'(I)$$

ensures the existence of $\tilde{I}$ such that $\forall I > \tilde{I}$, $L(I) < 0$.

Suppose that $V(\cdot)$ is one of *type 1*. Consider the following figure:

It s clear that there is no equilibrium off the diagonal $\{(I, \widehat{Q}_{w,I}) | I \in [0, \tilde{I}]\}$: Given I fixed, the buyer has incentive to impose $\hat{\Gamma}^2(I)$ that induces $\hat{Q}^2_{w,I}(\cdot)$. In the figure, the buyer prefers points C and G over A and F, respectively.

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Points like $(I, \widehat{Q}_{w,I}(\cdot))$ with $0 \leq I \leq \hat{I}$ can't be equilibria as well: In the figure, the first period winner prefers C over B because the former gives more utility (recall that in this zone $V(\cdot)$ increases with I). On the other side, given C's investment level, the first period winner prefers A: if $\bar{I} > \bar{\bar{I}}$ then $U_w(I \mid \bar{\bar{I}}) > U_w(I \mid \bar{I})$. Transitivity ensures that A is preferred over B, hence, the winner has an incentive to deviate.

The point $(\widehat{I}, \widehat{Q}_{w,\hat{I}})$ is neither an equilibrium: remember that $\hat{I}$ satisfies $V(\hat{I}) = 0$ so

$$\frac{\partial U_w}{\partial I}(I \mid \hat{I})\Big|_{I=\hat{I}} > 0$$

This occurs since in U_w the negative effect due to $[1 - F(g_1^{-1}(h(I,c)))]^{n-1}$ is fixed, which does not happen with $V(\cdot)$. Therefore, the first period winner would like to invest more.

As $L(\hat{I}) = \frac{\partial U_w}{\partial I}(I \mid \hat{I})\Big|_{I=\hat{I}} > 0$ and $L(I) < 0$ if $I > \tilde{I}$, because of $L(\cdot)$'s continuity and monotonicity, there exists a unique $\hat{\hat{I}} \in (\hat{I}, \tilde{I})$ such that

$$L(\hat{\hat{I}}) = \frac{\partial U_w}{\partial I}(I \mid \hat{\hat{I}})\Big|_{I=\hat{\hat{I}}} = 0$$

To see that $\hat{\hat{I}}$ is a maximum, note that for each $\bar{I}$ fixed, the function $U_w(I \mid \bar{I})$ is strictly concave in I (assumption **??**). Taking $\bar{I} = \hat{\hat{I}}$, we conclude that $\hat{\hat{I}}$ is a maximum since it verifies $\frac{\partial U_w}{\partial I}(I \mid \hat{\hat{I}})\Big|_{I=\hat{\hat{I}}} = 0$. Then, given the mechanism $\widehat{Q}_{w,\hat{\hat{I}}}(\cdot)$, the winner has the incentive to invest $\hat{\hat{I}}$, that is, we've proved that $(\hat{\hat{I}}, \widehat{Q}_{w,\hat{\hat{I}}}(\cdot))$ is the unique pure strategy equilibrium (clearly, the buyer has as dominant strategy to choose mechanisms over the diagonal corresponding to the investment level carried out).

If $V(\cdot)$ is belongs to the second class, along the diagonal $(I, \widehat{Q}_{w,I}(\cdot))$ this function decreases and, as a direct consequence, the investment level induced under *non-commitment* and observability, $\hat{I}$, satisfies $\hat{I} = 0$. Thus, we can no longer ensure the existence of a point $\hat{I}$ such that $L(\hat{I}) > 0$, which was the main argument to show the existence of $\hat{\hat{I}}$. Therefore, we have two cases:

(i) Exists $\hat{I} \geq 0$ such that $L(\hat{I}) > 0$. In this case the existence and uniqueness of $\hat{\hat{I}}$ is ensured using the same arguments as before. Also we have $\hat{I} < \hat{\hat{I}}$ because $L(\cdot)$ is decreasing.

(ii) There is no $I > 0$ such that $L(I) = 0$. Because of (**??**), continuity and monotonicity of $L(\cdot)$ (decreasing), it must be that $L(I) < 0$ if $I > 0$, hence, $L(0) \leq 0$. In this case $(0, \widehat{Q}_{w,0}(\cdot))$ is an equilibrium: when the winner makes no investment, the buyer has no incentive to deviate from $\widehat{Q}_{w,0}(\cdot)$. On the other side, when facing this last mechanism, the first period winner has no incentive to invest because his marginal utility under this mechanism, $L(0)$, is always negative. It is unique because (i) $L(I) < 0$ if $I > 0$, in other words, for every $I > 0$ the first period winner has the incentive to invest a lower level that I and, (ii) there are no equilibria off the diagonal. This concludes the proof.

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□

Proof Proposition **??**: Recall the function

$$V(I) = \int_C [1 - F(g_1^{-1}(h(c,I)))]^{n-1} G(c,I) dc - \Psi(I)$$

Necessary conditions under which the first inequality becomes equality were exposed in Proposition **??**'s proof. Therefore, let's concentrate on the interesting case, that is, when $V(\cdot)$ is of *type 1*. In this setting, strict inequality holds thanks to the previous proposition. It remains to show that $\hat{I} < I^*$. Remember that I^* is the solution to

$$\max_{I \geq 0} \int_C [1 - F(g^{-1}(c))]^{n-1} G(c,I) dc - \Psi(I)$$

whose first order condition is

$$(FOC): \quad \int_C [1 - F(g^{-1}(c))] \frac{\partial G}{\partial I}(c, I^*) dc - \Psi'(I^*) = 0$$

(we used assumption **??** and Dominated Convergence Theorem). From Proposition **??**, $\hat{I}$ satisfies

$$L(\hat{I}) = \int_C [1 - F(g_1^{-1}(h(c,\hat{I})))]^{n-1} \frac{\partial G}{\partial I}(c, \hat{I}) dc - \Psi'(\hat{I}) = 0$$

As $h(c,I) > c \ \forall c \in S$, we have $g_1^{-1}(h(c,I)) > g_1^{-1}(c)$, therefore

$$0 = L(\hat{I}) = \int_C [1 - F(g_1^{-1}(h(c,\hat{I})))]^{n-1} \frac{\partial G}{\partial I}(c, \hat{I}) dc - \Psi'(\hat{I}) < \int_C [1 - F(g_1^{-1}(c))]^{n-1} \frac{\partial G}{\partial I}(c, \hat{I}) dc - \Psi'(\hat{I})$$

Observing that $g^{-1}(c) < g_1^{-1}(c) \ \forall c \in S$ (because $g_2(\cdot) = c + \left(1 + \frac{1}{n-1}\right) \frac{F(c)}{f(c)} > c + \frac{F(c)}{f(c)} = g_1(\cdot)$ and these are increasing functions) we obtain

$$0 < \int_C [1 - F(g^{-1}(c))]^{n-1} \frac{\partial G}{\partial I}(c, \hat{I}) dc - \Psi'(\hat{I})$$

Since

$$I \mapsto \int_C [1 - F(g^{-1}(c))]^{n-1} \frac{\partial G}{\partial I}(c, I) dc - \Psi'(I)$$

is continuous and decreasing (consequence of assumption **??**, assumption **??** and Dominated Convergence Theorem), and I^* satisfies (FOC), it must be that $\hat{I} < I^*$, concluding the proof.

□

Proof Proposition **??**: I^e solves

$$\min_{I \geq 0} \mathcal{C}(\Gamma^e, I)$$

with

$$
\begin{aligned}
\mathcal{C}(\Gamma^e, I) &= n \int_C c[1 - F(c)]^{n-1} f(c) dc + \beta \int_C c[1 - F(c)]^{n-1} \frac{\partial G}{\partial c}(c, I) dc \\
&\quad + \beta(n-1) \int_C c[1 - F(c)]^{n-2} [1 - G(c, I)] f(c) dc + \beta \Psi(I)
\end{aligned}
\tag{51}
$$

Then I^e is the solution to

$$\min_{I \geq 0} \int_C c[1 - F(c)]^{n-1} \frac{\partial G}{\partial c}(c, I) dc + (n-1) \int_C c[1 - F(c)]^{n-2} [1 - G(c, I)] f(c) dc + \Psi(I) \tag{52}$$

Integrating by parts,

$$
\begin{aligned}
\int_C c[1 - F(c)]^{n-1} \frac{\partial G}{\partial c}(c, I) dc &= c[1 - F(c)]^{n-1} \frac{\partial G}{\partial c}(c, I) \Big|_{\underline{c}}^{\bar{c}} + (n-1) \int_C c[1 - F(c)]^{n-2} f(c) G(c, I) dc \\
&\quad - \int_C [1 - F(c)]^{n-1} G(c, I) dc
\end{aligned}
\tag{53}
$$

The first term vanishes. Replacing this expression in the problem just mentioned we obtain

$$\min_{I \geq 0} \int_C c[1 - F(c)]^{n-2} f(c) dc - \int_C [1 - F(c)]^{n-1} G(c, I) dc + \Psi(I)$$

and because the first term does not depends on I we conclude that I^e is the solution to

$$\max_{I \geq 0} \int_C [1 - F(c)]^{n-1} G(c, I) dc - \Psi(I)$$

concluding the first part. For the last one, recall that under a second price sealed bid auction there are no incentive compatibility problems since truth-telling is a dominant strategy. Because each project is assigned to the least-cost competitor, in the second period the expected probability function for the first period winner is $Q_w^{2;e}(c) = [1 - F(c)]^{n-1}$. Therefore, if the buyer is full committed and imposes two second price sealed bid auctions we will achieve: (i) Efficiency, in terms of giving the project to the competitor with the lowest cost, (ii) I^e, if investment is observable (the mechanism designer will solves the above problem). If it is not observable, the first period winner will solve

$$\max_{I \geq 0} \int_C Q_w^{2;e}(c) G(c, I) dc - \Psi(I) = \int_C [1 - F(c)]^{n-1} G(c, I) dc - \Psi(I)$$

and, as a consequence, I^e will be selected as well. Therefore, I^e is achieved in a context of full commitment regardless of investment observability.

□

Proof Proposition **??**: We only need to show that $\hat{\hat{I}} < I^e < I^*$. Recall that I^* solves

$$\max_{I \geq 0} \int_C [1 - F(g^{-1}(c))]^{n-1} G(c, I) dc - \Psi(I)$$

On the other hand, the efficient mechanism verifies $Q_w^{2,e}(c) = [1 - F(c)]^{n-1}$. It is clear that $Q_w^{2*}(c) = [1 - F(g^{-1}(c))]^{n-1} > Q_w^{2,e}(c)$ if $c \neq \underline{c}$, that is, the cost minimizing mechanism gives more advantage to the first period winner at $t = 2$ than the efficient one. When facing a mechanism $Q_w^2(\cdot)$, the first order condition derived by the first period winner is

$$\int_C Q_w^2(c) \frac{\partial G}{\partial I}(c, I) dc - \Psi'(I) = 0$$

Because the function

$$I \mapsto \int_C Q_w^2(c) \frac{\partial G}{\partial I}(c, I) dc - \Psi'(I)$$

is decreasing (assumption **??** and assumption **??**), the fact that $Q_w^{2*}(c) > Q_w^{2,e}(c)$ implies $I^e < I^*$.

To conclude we must prove that $\hat{\hat{I}} < I^e$. Recall that $\hat{\hat{I}}$ satisfies

$$L(\hat{\hat{I}}) = \int_C [1 - F(g_1^{-1}(h(c, \hat{\hat{I}})))]^{n-1} \frac{\partial G}{\partial I}(c, \hat{\hat{I}}) dc - \Psi'(I) = 0$$

Because $h(c, I) > g_1(c) \; \forall c \in S$ and $g_1(\cdot)$ is increasing, we have that

$$L(I) = \int_C [1 - F(g_1^{-1}(h(c, I)))]^{n-1} \frac{\partial G}{\partial I}(c, I) dc - \Psi'(I) < \int_C [1 - F(c)] \frac{\partial G}{\partial I}(c, I) dc - \Psi'(I)$$

The function

$$T(I) = \int_C [1 - F(c)]^{n-1} \frac{\partial G}{\partial I}(c, I) dc - \Psi'(I)$$

satisfies $T(I^e) = 0$ and it is decreasing (assumption **??**). Because $\hat{\hat{I}}$ satisfies $L(\hat{\hat{I}}) = 0$, it must be that $\hat{\hat{I}} < I^e$, concluding the proof.

□

Proof Proposition **??**: For any investment level I, total expected costs corresponds to

$$\mathcal{C}(I) = \int_C \int_C \left[\sum_{i=1,2} (c_i + \frac{F(c_i)}{F'(c_i)}) q_i^1(c_1, c_2) \right] f(c_1) f(c_2) dc_1 dc_2$$

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$$+\beta \int_C \int_C \left[c_w q_{w,I}^2(c_w, c_l) + \left(c_l + 2\frac{F(c_l)}{f(c_l)} \right) q_l^2(c_w, c_l) \right] \frac{\partial G}{\partial c_w}(c_w, I) f(c_l) dc_w dc_l + \beta \Psi(I)$$

and the optimal second period rule is given by

$$q_{w,I}^2(c_w, c_l) = \begin{cases} 1 & c_w < c_l + 2\frac{F(c_l)}{f(c_l)} \\ 0 & \sim \end{cases}$$

Define $g_2(c) = c + 2\frac{F(c)}{f(c)}$ and consider the following incentive compatible mechanism:

$$\tilde{q}_{w,I}^2(c_w, c_l) = \begin{cases} 1 & c_w < g(c_l) \\ 0 & \sim \end{cases} \tag{54}$$

with $g(\cdot)$ an increasing function. Using this mechanism, the second period expected cost can be written as

$$\mathcal{C}^2(I) = \int_C f(c_l) \left(\int_C h(c_l, c_w) \frac{\partial G}{\partial c_w}(c_w, I) dc_w \right) dc_l$$

with

$$h(c_l, c_w) = \begin{cases} g_2(c_l) & g(c_l) < c_{w,I} \\ c_w & \sim \end{cases}$$

Therefore, under this mechanism, the buyer chooses the investment level by solving

$$\min_{I \geq 0} \ \mathcal{C}^2(I) + \Psi(I)$$

Now, given $c_l \in C$

$$\int_C h(c_l, c_{w,I}) \frac{\partial G}{\partial c_w}(c_w, I) dc_w = \int_{g(c_l)}^{\bar{c}} g_2(c_l) \frac{\partial G}{\partial c_w}(c_w, I) dc_w + \int_{\underline{c}}^{g(c_l)} c_w \frac{\partial G}{\partial c_w}(c_w, I) dc_w$$

But,

$$\int_{g(c_l)}^{\bar{c}} g_2(c_l) \frac{\partial G}{\partial c}(c_w, I) dc_w = \begin{cases} g_2(c_l)(1 - G(g(c_l), I)) & g(c_l) < \bar{c} \\ 0 & \sim \end{cases}$$

$$\int_{\underline{c}}^{g(c_l)} c_w \frac{\partial G}{\partial c_w}(c_w, I) dc_w = \begin{cases} g(c_l) G(g(c_l), I) - \int_{\underline{c}}^{g(c_l)} G(c_w, I) dc_w & g(c_l) < \bar{c} \\ \bar{c} - \int_C G(c_w, I) dc_w & \sim \end{cases}$$

and, as a consequence,

$$\int_C h(c_l, c_{w,I}) \frac{\partial G}{\partial c_w}(c_w, I) dc_w = \begin{cases} g_2(c_l) + [g(c_l) - g_2(c_l)] G(g(c_l), I) - \int_{\underline{c}}^{g(c_l)} G(c_w, I) dc_w & g(c_l) < \bar{c} \\ \bar{c} - \int_C G(c_w, I) dc_w & \sim \end{cases}$$

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Therefore, the buyer minimizes

$$\int_{\underline{c}}^{g^{-1}(\bar{c})} \left[g_2(c_l) + (g(c_l) - g_2(c_l))G(g(c_l), I) \right] f(c_l)dc_l - \int_{\underline{c}}^{g^{-1}(\bar{c})} \int_{\underline{c}}^{g(c_l)} G(c_w, I)f(c_l)dc_w dc_l + \Psi(I)$$

$$+ \int_{g^{-1}(\bar{c})}^{\bar{c}} \left[\bar{c} - \int_C G(c_w, I)dc_w \right] f(c)dc_l$$

which is equivalent to minimize

$$\int_{\underline{c}}^{g^{-1}(\bar{c})} (g(c_l) - g_2(c_l))G(g(c_l), I)f(c_l)dc_l - \int_{\underline{c}}^{g^{-1}(\bar{c})} \int_{\underline{c}}^{g(c_l)} G(c_w, I)f(c_l)dc_w dc_l$$

$$- \int_{g^{-1}(\bar{c})}^{\bar{c}} \int_C G(c_w, I)f(c_l)dc_w dc_l + \Psi(I)$$

This last expression can be written as

$$\int_{\underline{c}}^{g^{-1}(\bar{c})} (g(c_l) - g_2(c_l))G(g(c_l), I)f(c_l)dc_l - \int_C \left(\int_{\underline{c}}^{min\{g(c_l), \bar{c}\}} G(c_w, I)dc_w \right) f(c_l)dc_l + \Psi(I) \qquad (55)$$

Define

$$\mathcal{H}(I) = \int_C \left(\int_{\underline{c}}^{min\{g(c_l), \bar{c}\}} G(c_w, I)dc_w \right) f(c_l)dc_l$$

Then, the buyer solves

$$\max_{I \geq 0} \; \mathcal{H}(I) - \Psi(I) - \int_{\underline{c}}^{g^{-1}(\bar{c})} (g(c_l) - g_2(c_l))G(g(c_l), I)f(c_l)dc_l \qquad (56)$$

We denote the solution to this problem $\tilde{I}_b$. Also note that

$$\int_{\underline{c}}^{min\{\bar{c}, g(c_l)\}} G(c_w, I)dc_w = \begin{cases} \int_{\underline{c}}^{g(c_l)} G(c_w, I)dc_w & c_l < g^{-1}(\bar{c}) \\ \int_C G(c_w, I)dc_w & \sim \end{cases}$$

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and therefore

$$\mathcal{H}(I) \;=\; \int\limits_{\underline{c}}^{C} \left(\int\limits_{\underline{c}}^{min\{\bar{c},g(c_l)\}} G(c_w,I)dc_w \right) f(c_l)dc_l$$

$$=\; \int\limits_{\underline{c}}^{g^{-1}(\bar{c})} \left(\int\limits_{\underline{c}}^{g(c_l)} G(c_w,I)dc_w \right) f(c_l)dc_l + [1 - F(g^{-1}(\bar{c}))] \int\limits_{C} G(c_w,I)dc_w \qquad (57)$$

Integrating by parts,

$$\int\limits_{\underline{c}}^{g^{-1}(\bar{c})} \left(\int\limits_{\underline{c}}^{g(c_l)} G(c_w,I)dc_w \right) f(c_l)dc_l = F(g^{-1}(\bar{c})) \int\limits_{C} G(c_w,I)dc_w - \int\limits_{\underline{c}}^{g^{-1}(\bar{c})} F(c_l)G(g(c_l),I)g'(c_l)dc_l$$

which leads to

$$\mathcal{H}(I) = \int\limits_{C} G(c,I)dc - \int\limits_{\underline{c}}^{g_2^{-1}(\bar{c})} F(c)G(g_2(c),I)g_2'(c)dc \qquad (58)$$

When facing the mechanism defined by (**??**) the first period winner solves

$$\max_{I \geq 0} \int\limits_{S} \tilde{Q}_w^2(c)G(c,I)dc - \Psi(I)$$

with

$$\tilde{Q}_w^2(c) = \int\limits_{C} \tilde{q}_w^2(c,c_l)f(c_l)dc_l = \begin{cases} \int\limits_{g^{-1}(c)}^{\bar{c}} f(c_l)dc_l = 1 - F(g^{-1}(c)), & g(\underline{c}) < c \\[2mm] \int\limits_{\underline{c}}^{\bar{c}} f(c_l)dc_l = 1 & \sim \end{cases}$$

As,

$$\int\limits_{S} \tilde{Q}_w^2(c)G(c,I)dc \;=\; \int\limits_{\underline{c}}^{g(\underline{c})} \tilde{Q}_w^2(c)G(c,I)dc + \int\limits_{g(\underline{c})}^{\bar{c}} \tilde{Q}_w^2(c)G(c,I)dc$$

$$=\; \int\limits_{\underline{c}}^{g(\underline{c})} G(c,I)dc + \int\limits_{g(\underline{c})}^{\bar{c}} [1 - F(g^{-1}(c))]G(c,I)dc$$

$$=\; \int\limits_{C} G(c,I)dc - \int\limits_{g(\underline{c})}^{\bar{c}} F(g^{-1}(c))G(c,I)dc \qquad (59)$$

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and using $t = g_2(c)$, the first period winner solves

$$\max_{I \geq 0} \mathcal{H}(I) - \Psi(I) = \int_C G(c, I)dc - \int_{\underline{c}}^{g^{-1}(\bar{c})} F(t))G(g(t), I)g'(t)dt - \Psi(I) \tag{60}$$

Call the solution to this problem $\tilde{I}_w$. Finally, assume that the first order conditions for (**??**) and (**??**) are satisfied. As a consequence, the following holds:

$$\mathcal{H}'(\tilde{I}_b) - \Psi'(\tilde{I}_b) + \int_{\underline{c}}^{g^{-1}(\bar{c})} (g_2(c_l) - g(c_l))\frac{\partial G}{\partial I}(g(c_l), \tilde{I}_b)f(c_l)dc_l = 0 \tag{61}$$

$$\mathcal{H}'(\tilde{I}_w) - \Psi'(\tilde{I}_w) = 0 \tag{62}$$

Assumptions **??** and **??** ensure that $\mathcal{H}'(\cdot)$ and $-\Psi'(\cdot)$ are decreasing. Since $g_2 \geq g$ with strict inequality on a set of positive measure, we conclude that

$$\tilde{I}_w < \tilde{I}_b$$

using the fact that $\frac{\partial^2 G}{\partial I^2} < 0$.

$\square$

10.   APPENDIX B: HISTORY-DEPENDENT MECHANISMS

In this section we prove that the buyer cannot do better by specifying contracts in which the second period rules and investment levels differ according to the first period winner's identity.

Assume that investment is observable. At $t = 1$ denote firms by subscripts $i \in N$ and define analogously $q_i^1(c')$, $Q_i^1(c_i')$, $t_i^1(c')$ and $T_i^1(c_i')$, $i \in N$.

At $t = 2$ we allow the second period mechanisms to depend on the winner's identity. Consider the following history: (*history i,I_i*) "firm i was the first period winner and invested an amount I_i", $i \in N$. In this setting we denote firm i by the subscript i, w, I_i at $t = 2$, and any other competitor $j \neq i$ will be denoted by subscripts j, l, i[7] which means that j is a loser under *history i,I_i*, $i \neq j$, $i, j \in N$ (now the identity of a loser is the first letter in the subscript, not as in the model presented in the paper). Conditional on this history, we define $q_{i,w,I_i}^2(c')$ as the probability that firm i wins the second project conditional on the report vector c' ($q_{j,l,i}^2(c')$ is defined analogously, $j \neq i$, $i, j \in N$). Transfers are defined in the same way, i.e., they depend on the realized history and on player's reports. Now we are able to define a *mechanism* in the case of investment observability:

---

[7]Actually the correct subscript would be j, l, I_i, i because it depends explicitly on the realized history. Nevertheless for notational convenience will be reduced to j, l, i

In this setting, a *mechanism* Γ is a family of functions defined in C^n

$$\Gamma = \{(q_i^1(\cdot), t_i^1(\cdot))|\ i \in N\} \cup \left(\bigcup_{history\ j, I_j} \{(q_i^2(\cdot), t_i^2(\cdot))|\ i = (j, w, I), (k, l, j),\ j \neq k,\ k \in N\} \right)$$

such that

(i) For all $i \in N$, $0 \le q_i^1(\cdot) \le 1$ and $\sum_{i=1}^{n} q_i^1(c) = 1$, $\forall\ c \in C^n$.

(ii) For every *history* i, I_i, $i \in N$, $I_i \ge 0$, $0 \le q_{i,w,I_i}^2(\cdot) \le 1$, $0 \le q_{j,l,i}^2(\cdot) \le 1\ \forall\ j \neq i,\ j \in N$ and
$q_{i,w,I_i}^2(c) + \sum_{j \neq i}^{n} q_{j,l,i}^2(c) = 1$, $\forall\ c \in C^n$.

That is, it is a contingent plan of transfers and probabilities, which depend on every possible history. Probabilities must sum 1 because the buyer is compelled to procure the two projects.

With the corresponding notational changes consider the one-variable functions $Q_{i,w,I_i}^2(c_i')$, $Q_{j,l,i}^2(c_j')$ $T_{i,w,I_i}^2(c_i')$, $T_{j,l,i}^2(c_i')$, $\Pi_{i,w,I_i}^2(c_i, c_i')$ and $\Pi_{j,l,i}^2(c_j, c_j')$ for firm $j \neq i$, $j \in N$. This last two satisfy

$$\Pi_{i,w,I_i}^2(c_i, c_i') = T_{i,w,I_i}^2(c_i') - c_i Q_{i,w,I_i}^2(c_i') - \Psi(I_i),\ i \in N. \tag{63}$$

$$\Pi_{j,l,i}^2(c_j, c_j') = T_{j,l,i}^2(c_j') - c_j Q_{j,l,i}^2(c_j'),\ i \neq j,\ i, j \in N. \tag{64}$$

Set $Q_{i,j}^1(c_i')$ as the expected probability that player j wins conditional on player i's report c_i'. Corresponds to

$$Q_{i,j}^1(c_i') = \int_{C^{n-1}} q_j^1(c_i', c_{-i}) f^{n-1}(c_{-i}) dc_{-i} \tag{65}$$

and it is clear that

$$\int_C Q_{i,j}^1(c_i') dc_i' = \int_{C^n} q_j^1(c) dc := Q_j^1 \tag{66}$$

in other words, for firm i, the expected probability of being defeated by firm j is exactly the expected probability that this firm wins the first procurement, $i \neq j$, $i, j \in N$.

Assume that the buyer wants to induce firm i to invest $I_i \ge 0$. Define $\vec{I} = (I_1, ..., I_n)$. We denote by $\Pi_{i,\vec{I}}^1(c_i, c_i')$ the discounted expected utility at $t = 1$ for firm i with cost c_i and reported cost c_i', conditional on revealing real costs at $t = 2$ and on the investment profile $\vec{I}$. It satisfies

$$\Pi_{i,\vec{I}}^1(c_i, c_i') = T_i^1(c_i') - c_i Q_i^1(c_i') + \beta Q_i^1(c_i') \int_C \Pi_{i,w,I_i}^2(c, c) \frac{\partial G}{\partial c}(c, I_i) dc$$

$$+ \beta \sum_{j \neq i} Q_{i,j}(c_i') \int_C \Pi_{i,l,j}^2(c, c) f(c) dc. \tag{67}$$

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Incentive compatibility is characterized by Lemma **??** with the corresponding notational changes.

Denote by $\mathcal{C} = \mathcal{C}(\Gamma, \vec{I})$ the expected procurement cost when the buyer uses mechanism Γ and wants to induce the investment profile $\vec{I}$. It is clear that

$$\mathcal{C} = \sum_{i=1}^{n} \int_C T_i^1(c)f(c)dc + \beta \sum_{i=1}^{n} \int_C Q_i^1(c)f(c)dc \left[\int_C T_{i,w,I_i}^2(c)\frac{\partial G}{\partial c}(c,I)dc + \sum_{j\neq i} \int_C T_{j,l,i}^2(c)f(c)dc \right] \quad (68)$$

The following lemma is an expression for $\mathcal{C}$ under incentive compatible mechanisms:

Lemma: Suppose the buyer wants to induce an investment profile $\vec{I} = (I_1, ..., I_n)$ using a incentive compatible mechanism Γ. The expected procurement cost corresponds to

$$\begin{aligned}
\mathcal{C} &= \sum_{i=1}^{n} \left[\int_C cQ_i^1(c)f(c)dc + \int_C Q_i^1(c)F(c)dc + \Pi_{i,\vec{I}}^1(\bar{c},\bar{c}) \right] \\
&+ \beta \sum_{i=1}^{n} Q_i^1 \left[\int_C cQ_{i,w,I_i}^2(c)\frac{\partial G}{\partial c}(c,I_i)dc + \sum_{j\neq i} \int_C cQ_{j,l,i}^2(c)f(c)dc + \Psi(I_i) \right] \quad (69)
\end{aligned}$$

Proof: Using Lemma **??**, rearranging terms in **(??)**, **(??)** **(??)**, and integrating by parts in the last expression, we obtain

$$T_{i,w,I_i}^2(c) = cQ_{i,w,I_i}^2(c) + \int_c^{\bar{c}} Q_{i,w,I_i}^2(x)dx + \Pi_{i,w,I_i}^2(\bar{c},\bar{c}) - \Psi(I_i) \quad (70)$$

$$T_{j,l,i}^2(c) = cQ_{j,l,i}^2(c) + \int_c^{\bar{c}} Q_{j,l,i}^2(x)dx + \Pi_{j,l,i}^2(\bar{c},\bar{c}) \quad (71)$$

$$\begin{aligned}
T_i^1(c) &= cQ_i^1(c) + \int_c^{\bar{c}} Q_i^1(x)dx + \Pi_{i,\vec{I}}^1(\bar{c},\bar{c}) - \beta Q_i^1(c) \left[\int_C Q_{i,w,I_i}^2(c)G(c,I_i)dc + \Pi_{i,w,I_i}^2(\bar{c},\bar{c}) \right] \\
&- \beta \sum_{i\neq j} Q_{i,j}^1(c) \left[\int_C Q_{i,l,j}^2(c)F(c)dc + \Pi_{i,l,j}^2(\bar{c},\bar{c}) \right] \quad (72)
\end{aligned}$$

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Replacing this terms in $\mathcal{C}$ we get

$$
\begin{aligned}
\mathcal{C} \;=\; & \sum_{i=1}^{n}\left[\int_{C} cQ_i^1(c)f(c)dc + \int_{C} Q_i^1(c)F(c)dc + \Pi^1_{i,\vec{I}}(\bar c,\bar c)\right] \\[2mm]
& -\beta\sum_{i=1}^{n}\int_{c} Q_i^1(c)f(c)dc\left[\int_{C} Q^2_{i,w,I_i}(c)G(c,I_i)dc + \Pi^2_{i,w,I_i}(\bar c,\bar c)\right] \\[2mm]
& -\beta\sum_{i=1}^{n}\sum_{j\neq i}\int_{C} Q_{i,j}^1(c)f(c)dc\left[\int_{C} Q^2_{i,l,j}(c)F(c)dc + \Pi^2_{i,l,j}(\bar c,\bar c)\right] \\[2mm]
& +\beta\sum_{i=1}^{n}\int_{C} Q_i^1(c)f(c)dc\left[\int_{C} cQ^2_{i,w,I_i}(c)\frac{\partial G}{\partial c}(c,I_i)dc\right] \\[2mm]
& +\beta\sum_{i=1}^{n}\int_{C} Q_i^1(c)f(c)dc\left[\int_{C} Q^2_{i,w,I_i}(c)G(c,I_i)dc + \Pi^2_{i,w,I_i}(\bar c,\bar c)\right] \\[2mm]
& +\beta\sum_{i=1}^{n}\left(\int_{C} Q_i^1(c)f(c)dc\right)\Psi(I_i) \\[2mm]
& +\beta\sum_{i=1}^{n}\int_{C} Q_i^1(c)f(c)dc\sum_{j\neq i}\left[\int_{C} cQ^2_{j,l,i}(c)f(c)dc\right] \\[2mm]
& +\beta\sum_{i=1}^{n}\int_{C} Q_i^1(c)f(c)dc\sum_{j\neq i}\left[\int_{C} Q^2_{j,l,i}(c)F(c)dc + \Pi^2_{j,l,i}(\bar c,\bar c)\right] \qquad (73)
\end{aligned}
$$

The first term correspond to the expected transfers standing at $t=1$ and the rest consist of second period payments. Recall that $\int_{S} Q_{i,j}(c)f(c)dc = \int_{S} Q_j(c)f(c)dc = Q_j^1$, and with this the third and eighth terms can be written as $\sum_{i=1}^{n}\sum_{j\neq i} Q_j^1 a_{i,j}$ and $\sum_{i=1}^{n} Q_i^1 \sum_{j\neq i} a_{j,i}$ respectively. Clearly they are equal, so they cancel. The second and fifth expression cancel each other as well. Thus, the expected cost reduces to

$$
\begin{aligned}
\mathcal{C} \;=\; & \sum_{i=1}^{n}\left[\int_{S} cQ_i^1(c)f(c)dc + \int_{S} Q_i^1(c)F(c)dc + \Pi^1_{i,\vec{I}}(\bar c,\bar c)\right] \\[2mm]
& +\beta\sum_{i=1}^{n} Q_i^1\left[\int_{C} cQ^2_{i,w,I_i}(c)\frac{\partial G}{\partial c}(c,I_i)dc + \sum_{j\neq i}\int_{C} cQ^2_{j,l,i}(c)f(c)dc + \Psi(I_i)\right] \qquad (74)
\end{aligned}
$$

concluding the proof.

$\square$

The next result states that the buyer may restrict to mechanisms that do not depend on the first period winner's identity.

Theorem (*History i*-independence): When the buyer is fully committed and investment is observable, this agent may restrict mechanisms independent of the first period winner's identity. In other words, second period rules and the investment level he wishes to implement may not depend on who was the first period winner. In this setting, when the buyer wants to induce an investment level I, total expected cost $\mathcal{C}$ can be written as

$$\mathcal{C}(I) \;=\; \sum_{i=1}^{n} \left[\int_S cQ_i^1(c)f(c)dc + \int_C Q_i^1(c)F(c)dc + \Pi_{i,I}^1(\bar{c},\bar{c}) \right.$$

$$\left. + \beta \left[\int_C cQ_{w,I}^2(c)\frac{\partial G}{\partial I}(c,I)dc + \sum_{j\neq w} \int_C cQ_{j,l}^2(c)f(c)dc + \Psi(I) \right] \right] \tag{75}$$

Moreover, the expected probability of winning the second project, as a function, is equal for all losers.

Proof: Assume that $\Gamma^*(\vec{I})$ is a cost-minimizer incentive compatible mechanism when the buyer wants to implement an investment profile $\vec{I} = (I_1, ..., I_n)$. We do not specify under which *usual*[8] participation constraints this mechanism is optimal since the result is quite robust to this feature. Therefore, suppose that that the buyer impose some of this constraints and that $\Gamma^*(\vec{I})$ satisfy them. To begin with, two assertions:

(i) Exists $i \in N$ and $\epsilon > 0$ such that $Q_i^1(c) > 0$ in $(\underline{c}, \underline{c}+\epsilon)$.

Ans: Suppose not. As $Q_i^1(\cdot)$ is non increasing and non negative, it will be identically null in C, hence, $q_i^1(\cdot) = 0$ a.e. in C for all $i \in N$, which contradicts feasibility. Then exists i such that $Q_i^1 = \int_S Q_i^1(c)f(c)dc > 0$.

(ii) Define

$$\mathcal{C}_i^2 = \int_C cQ_{i,w,I_i}^2(c)\frac{\partial G}{\partial c}(c,I_i)dc + \sum_{j\neq i} \int_C cQ_{j,l,i}^2(c)f(c)dc.$$

If $(Q_i^{1*}(\cdot); i = 1, ..., n)$ is the first period optimal rule, then $Q_i^{1*}, Q_j^{1*} > 0 \Rightarrow \mathcal{C}_i^2 = \mathcal{C}_j^2$.

Ans: The expression $\mathcal{C}_i^2 = \int_S cQ_{i,w,I_i}^2(c)\frac{\partial G}{\partial c}(c,I_i)dc + \sum_{j\neq i} \int_S cQ_{j,l,i}^2(c)f(c)dc$ correspond to the second period expected cost conditional on *history i, I_i*. First note that, $Q_i^1 = \int_S Q_i^1(c)f(c)dc$ verifies $Q_i^1 \geq 0$ and $\sum_i Q_i^1 = 1$ (consequence of a feasible mechanism).

If exists $i \neq j$, $i, j \in N$, such that $\mathcal{C}_i^2 > \mathcal{C}_j^2$, we can define the distribution

$$\tilde{q}_k^1(\cdot) = \begin{cases} q_k^1(\cdot) & k \neq j, i. \\ q_j^1(\cdot) + q_i^1(\cdot) & k = j. \\ 0 & k = i. \end{cases}$$

---

[8]By *usual* we mean participation constraints that involve player's participation under any possible type.

that induces the family of expected probabilities $(\tilde{Q}_i^1(\cdot); i = 1, ..., n)$. It is easy to see that $\tilde{Q}_i^1(c)$ is not increasing in c for all $i \in N$. For all $k \neq i, j$ keep transfers unchanged. Then $\tilde{\Pi}_{k,\vec{I}}^1(c,c) = \Pi_{k,\vec{I}}^1(c,c)$, so incentive compatibility and participation constraints for this players are satisfied. For player i consider $\tilde{T}_i^1(\bar{c})$ such that $\tilde{\Pi}_{i,\vec{I}}^1(\bar{c},\bar{c}) = \Pi_{i,\vec{I}}^1(\bar{c},\bar{c})$ and compute transfers according to Lemma **??**. We obtain $\tilde{\Pi}_{i,\vec{I}}^1(c,c) = \tilde{\Pi}_{i,\vec{I}}^1(\bar{c},\bar{c})$, $\forall c$ and so all restrictions are fulfilled. Finally, for player j define $\tilde{T}_j^1(\bar{c})$ so that $\tilde{\Pi}_{i,\vec{I}}^1(\bar{c},\bar{c}) = \tilde{\Pi}_{i,\vec{I}}^1(\bar{c},\bar{c})$ holds and calculate transfers using the same lemma. As a consequence, no costraint is violated for competitor j. Henceforth, this new mechanism $\tilde{\Gamma}$ is feasible (is direct that probabilities sum one). Moreover, because $\sum_{i=1}^n \tilde{Q}_i^1(c) = \sum_{i=1}^n Q_i^1(c)$ for all c and $\sum_{i=1}^n \tilde{\Pi}_{i,\vec{I}}^1(\bar{c},\bar{c}) = \sum_{i=1}^n \Pi_{i,\vec{I}}^1(\bar{c},\bar{c})$, under $\tilde{\Gamma}$ the first term in $\mathcal{C}$ remains unaltered and the second one falls, a contradiction with $\Gamma^*(\vec{I})$'s optimality. Hence, at the optimum, $\mathcal{C}_i^2 = \mathcal{C}^2$, $i \in N$ if $Q_i^{1*} > 0$.

A consequence of (i) and (ii) is that the mechanism designer may restrict simultaneously to uniform investment profiles, that is, $I_i = ... = I_n = I$ and, mechanisms such that

$$Q_{i,w,I}^2(c) = Q_{w,I}^2(c), \ T_{i,w,I}^2(c) = T_{w,I}^2(c)$$

$$Q_{j,l,i}^2(c) = Q_{j,l}^2(c), \ T_{j,l,i}^2(c) = T_{j,l}^2(c)$$

and, therefore, expected total cost reduces to

$$\mathcal{C}(I) = \sum_{i=1}^n \left[\int_S cQ_i^1(c)f(c)dc + \int_C Q_i^1(c)F(c)dc + \Pi_{i,I}^1(\bar{c},\bar{c}) \right]$$

$$+\beta \left[\int_C cQ_{w,I}^2(c)\frac{\partial G}{\partial I}(c,I)dc + \sum_{j \neq w} \int_C cQ_{j,l}^2(c)f(c)dc + \Psi(I) \right]$$

concluding the proof.

$\square$

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