Nº 240

A NOTE ON THE COMPARATIVE STATICS OF OPTIMAL PROCUREMENT AUCTIONS

GONZALO CISTERNAS Y NICOLÁS FIGUEROA

DOCUMENTOS DE TRABAJO

Serie Economía

A Note on the Comparative Statics of Optimal Procurement Auctions

Gonzalo Cisternas* **Nicolás Figueroa**†

October 2007

Abstract

We find a necessary and sufficient condition such that a distributional upgrade on a seller's cost implies a lower expected procurement cost for a buyer. We also show that even under the strongest assumption about this upgrade made in the literature so far, the seller can be worse off, even if this upgrade is costless.

1. Introduction

We consider a buyer who has to procure a service from one of n potential sellers, whose production costs are private information. We study under which circumstances it is desirable for him to face one "better" seller, in the sense that she has a better cost distribution, and when it is desirable for the seller to have such a better cost distribution. In other words, we study the comparative statics of the buyer's expected cost and seller's expected profit with respect to distributional upgrades on a seller.

For the buyer, facing a better seller is good since there is a higher probability of her having low costs but, on the other hand, it may be bad since having a better distribution can imply that the informational rent she can extract is also higher.

For the seller, having a better distribution is good since, *ceteris paribus*, it increases her probabilities of winning the auction and the informational rent she can extract. However, since this better distribution is observed by the buyer and the mechanism is changed against the better seller, there is a negative effect associated to it.

We provide a natural and weak necessary and sufficient conditions on the distributional upgrade under which the buyer is better off. On the other hand, we show that for even for the strongest concept of distributional improvement used in the literature, the seller can be worse off when her cost distribution improves.

* *Centro de Economía Aplicada, DII, Universidad de Chile, República 701, Santiago, Chile; gcistern@dim.uchile.cl*
† *Centro de Economía Aplicada, DII, Universidad de Chile, República 701, Santiago, Chile; nicolasf@dii.uchile.cl*

2. Model

Consider a buyer who wants to procure a good or service and faces n potential suppliers indexed by $i = 1, ..., n$. If the buyer decides to carry out the task by himself, it would cost him an amount of money $c_0 \geq \underline{c}$. Suppliers' costs to perform the task (which are private information) are distributed independently across firms. Firm i obtains her cost from a differentiable distribution $F_i(\cdot)$, $i \geq 2$, with support $C \equiv [\underline{c}, \bar{c}]$. However, competitor 1 (from now on the *upgrader*) draws his cost from a differentiable distribution $F(\cdot, I)$ with the same support as before. I is a parameter that indexes the supplier's efficiency, and we assume that, as $I \geq 0$ increases, the distribution *improves*. For notational convenience we use $f_i(\cdot) \equiv F_i'(\cdot)$ if $i \geq 2$, and we keep using $\frac{\partial F}{\partial c}(c, I)$ in the upgrader's case.

We make the standard *regularity* assumption (first stated in [3]) and that guarantees the optimal mechanism can be found using pointwise maximization)

Assumption 1 *For every $i \geq 2$ and $I \geq 0$, the functions $J_i(c) = c + \frac{F_i(c)}{f_i(c)}$ and $J_I(c) = c + \frac{F(c,I)}{\frac{\partial F}{\partial c}(c,I)}$ are increasing.*

or technical reasons, we also need:

Assumption 2 *For every $c \in C$, $I \mapsto J_I^{-1}(c)$ is differentiable.*

There are several "distributional improvements" that may apply to the context presented here. We now introduce two widely-used notions, the first one being the most commonly used in statistics and economics:

Definition 3 ***(First Order Stochastic Dominance):*** *We will say that $\{F(\cdot, I)\}_{I \in \mathbb{R}_+}$ is a family of distributional improvements in the sense of first order stochastic dominance (FOSD) if, for every fixed $c \in C$, $F(c, \cdot)$ is increasing. In other words, the probability of obtaining a cost below $c \in C$ is increasing in I.*

The next one has been used before in the auction literature (see for example [[4]]) and was introduced first in contract theory:

Definition 4 ***(Monotone Likelihood Ratio Property):*** *We will say that $\{F(\cdot, I)\}_{I \in \mathbb{R}_+}$ is a family of distributional improvements in the sense of the monotone likelihood ratio property (MLRP) if, for every $I' < I \in \mathbb{R}_+$ and $c' < c \in C$,*

$$\frac{\frac{\partial F}{\partial c}(c', I')}{\frac{\partial F}{\partial c}(c, I')} < \frac{\frac{\partial F}{\partial c}(c', I)}{\frac{\partial F}{\partial c}(c, I)} \tag{1}$$

That is, as I increases, it is more likely to obtain lower costs relative *to higher ones. This condition is exactly to ask for $(c, I) \mapsto \frac{\partial F}{\partial c}(c, I)$ to be log-submodular.*[1]

The following well-known result relates both definitions and shows that MLRP is stronger than FOSD:

[1] A well-known result shows that MLRP implies that $\frac{F(c,I)}{\frac{\partial F}{\partial c}(c,I)}$ is increasing in I for all $c \in C$. This term corresponds to the informational rent a seller obtains when her type is c, and it increases with I, making non-trivial the comparison for the buyer: a better seller has lower costs but also extracts a higher informational rent.

Lemma 5 *If $\{F(\cdot,I)\}_{I\in I\!R_+}$ is a family of distributional improvements in the sense of MLRP, then, it is a family of distributional improvements in the sense of FOSD.*

Proof. Standard. ■

Finally, define $C^n = \{c^n = (c_1,...,c_n)|\ c_i \in C\ \forall i = 1,...,n\}$ and assume that for $i \geq 2$, $f_i(\cdot) > 0$ and $\forall I \geq 0$, $\frac{\partial F}{\partial c}(\cdot, I) > 0$, a.e. in C.

3. Basic Results

We now consider an upgrader with cost distribution $F(\cdot,I)$, and perform comparative statics over the procurement cost and the upgrader's utility with respect to the parameter I. The buyer's problem is to choose transfer functions $t_i : C^n \to I\!R$ (payments to the sellers) and winning probability functions $q_i : C^n \to [0,1]$ (probabilities of buying), $i = 1,...,n$. Under the regularity assumptions, it is direct that the expected optimal mechanism corresponds to (see [3])

$$q_1^*(c_1,...,c_n) = \begin{cases} 1 & J_I(c_1) \leq \min\{c_0, J_i(c_i)|\ i \geq 2\} \\ 0 & \sim \end{cases} \tag{2}$$

$$q_i^*(c_1,...,c_n) = \begin{cases} 1 & J_i(c_i) < \min\{c_0, J_I(c_1), J_l(c_l)|\ l \neq i,\ l \geq 2\} \\ 0 & \sim \end{cases} \tag{3}$$

$i = 2,...,n.$

which yields a procurement cost of:

$$\mathcal{C}(I) = \int\limits_{C^n} \left[J_I(c_1)q_1^*(c^n) + \sum_{l\geq 2} J_l(c_l)q_l^*(c^n) + c_0 \left(1 - \sum_{i\geq 1} q_i(c^n)\right)\right] \frac{\partial F}{\partial c_1}(c_1, I) \left(\prod_{j\geq 2} f_j(c_j)\right) dc^n \tag{4}$$

Our main purpose is to establish if conditions FOSD or MLRP on the family $\{F(\cdot,I)\}_{I\geq 0}$ imply that the expected procurement cost reduces. The main proposition, stated below, shows that even FOSD implies the result.

Proposition 6 *Suppose that for every $c \in C$ the function $F(c,\cdot)$ is differentiable. A sufficient pointwise conditions on the the family $\{F(\cdot,I)\}_{I\geq 0}$ under which the expected procurement cost reduce is:*

$$\forall I \geq 0,\ \forall c \in [\underline{c}, J_I^{-1}(c_0)],\ \frac{\partial F}{\partial I}(c, I) \geq 0 \tag{5}$$

As a consequence, if the mentioned family satisfies FOSD, the expected procurement cost decreases when facing a better competitor.

Proof. See Appendix. ■

As we can see, the tradeoff mentioned in the introduction (a buyer likes better sellers since they have in average lower costs, but on the other hand they can extract higher informational rents) always works in the buyer's favor. This is true since he modifies the mechanism in such a way that takes advantage optimally of this distributional improvement.

However, it is not true the upgrader is better off when improving his distribution. Since under some specific upgrades the buyer may also extract more rent from the seller, which may out-weight the benefits related to a lower expected cost, it is possible that the seller is worse off, even if this distributional upgrade is for free.

Example 7 *Suppose $n = 2$, $C = [0, 1]$ and $c_0 = +\infty$. Consider $F_2(c) = c$ and $F(c, I) = c^{\frac{1}{1+I}}$, $I \geq 0$. This last family of distributions satisfies MLRP and, as a consequence, FOSD. The upgrader's expected utility when his distribution is $F(\cdot, I)$ corresponds to*

$$\Pi(I) = \int_C \Pi(c, c)\frac{\partial F}{\partial c}(c, I)dc = \int_C Q^*(c)F(c, I)dc$$

with $Q^*(c) = \int_C q^*(c, s)f(s)ds$. Using that $q^*(c, s) = 1 \Leftrightarrow J_I(c) \leq J_2(s)$ (from the previous characterization of the optimal mechanism), $J_I(c) = c(2 + I)$ and $J_2(c) = 2c$, (thus $J_2^{-1}(J_I(c)) = \frac{c(2+I)}{2}$) we have

$$\begin{aligned}
\Pi(I) &= \int_{\underline{c}}^{\frac{2}{2+I}} \left[1 - \frac{c(2+I)}{2}\right] c^{\frac{1}{2+I}} dc \\
&= \frac{(1+I)^2}{(2+I)(3+2I)}\left(\frac{2}{2+I}\right)^{\frac{2+I}{1+I}}
\end{aligned}$$

To analyze $\Pi(\cdot)$'s behavior we study $log(\Pi(I))$:

$$\frac{d}{dI}(log(\Pi(I))) = \frac{1}{1+I} - \frac{1}{2+I} - \frac{2}{3+2I} + \frac{1}{(1+I)^2}\left[log\left(\frac{2+I}{2}\right)\right]$$

Finally, evaluating at $I = 0$:

$$\frac{d}{dI}(log(\Pi(I)))\Big|_{I=0} = 1 - \frac{1}{2} - \frac{2}{3} < 0$$

Therefore, for small distributional upgrades (starting from $I = 0$) the seller is worse-off, even if this upgrade is for free.

□

4. Appendix: Proofs

We first rewrite the procurement cost in the next lemma:

Lemma 8 *The expected procurement cost can be written as*

$$\begin{aligned}
\mathcal{C}(I) &= \sum_{i\geq 2} \int\limits_{\underline{c}}^{J_i^{-1}(c_0)} f_i(c)\left(\prod_{l\neq i}[1-F_l(J_l^{-1}(J_i(c)))]\right) J_I^{-1}(J_i(c))F(J_I^{-1}(J_i(c)),I)dc \\
&\quad + \left(\prod_{l\geq 2}[1-F_l(J_l^{-1}(c_0))]\right) J_I^{-1}(c_0)F(J_I^{-1}(c_0),I) \\
&\quad + \sum_{i\geq 2} \int\limits_{\underline{c}}^{J_i^{-1}(c_0)} f_i(c)\left(\prod_{l\neq i}[1-F_l(J_l^{-1}(J_i(c)))]\right) J_i(c)[1-F(J_I^{-1}(J_i(c)),I)]dc \\
&\quad + c_0\left(\prod_{l\geq 2}[1-F_l(J_l^{-1}(c_0))]\right)[1-F(J_I^{-1}(c_0),I)] \qquad (6)
\end{aligned}$$

Proof Lemma 8: Define $H(c^n, I)$ as

$$H(c^n,I) \equiv \left[J_I(c_1)q_1^*(c^n) + \sum_{l\geq 2} J_l(c_l)q_l^*(c^n) + c_0\left(1-\sum_{i\geq 1} q_i^*(c^n)\right)\right]\frac{\partial F}{\partial c_1}(c_1,I)\prod_{i\geq 2} f_i(c_i)$$

and consider the set

$$A = \{c^n \in C^n |\ J_I(c_1) \leq c_0,\ J_I(c_1) \leq J_i(c_i),\ \forall i \geq 2\}$$

That is, it is the set of cost-vectors in which the *upgrader* wins the procurement auction. Therefore,

$$\mathcal{C}(I) = \int\limits_A H(c^n,I)dc^n + \int\limits_{C^n\setminus A} H(c^n,I)dc^n$$

Set A can be written as $A = A_0 \cup \left(\bigcup\limits_{i\geq 2} A_i\right)$ with

$$\begin{aligned}
A_0 &= \{c^n \in C^n |\ J_I(c_1) \leq c_0\ \wedge\ c_0 < J_i(c_i),\ \forall i \geq 2\} \\
&= \{c^n \in C^n |\ c_1 < J_I^{-1}(c_0)\ \wedge\ J_i^{-1}(c_0) \leq c_i,\ \forall i \geq 2\}
\end{aligned}$$

$$\begin{aligned}
A_i &= \{c^n \in C^n |\ J_I(c_1) \leq J_i(c_i)\ \wedge\ J_i(c_i) \leq c_0\ \wedge\ (J_i(c_i) \leq J_l(c_l),\ l \geq i)\ \wedge\ (J_i(c_i) < J_l(c_l),\ i > l)\} \\
&= \{c^n \in C^n |\ c_1 \leq J_I^{-1}(J_i(c_i))\ \wedge c_i \leq J_i^{-1}(c_0)\ \wedge\ (J_l^{-1}(J_i(c_i)) \leq c_l,\ l \geq i)\ \wedge\ (J_l^{-1}(J_i(c_i)) < c_l,\ i > l)\}
\end{aligned}$$

and it is quite easy to see that $A_j \cap A_i = \emptyset$ if $i \neq j$ $i,j \in \{0,2,3,...,n\}$. Note that in A_i the *upgrader* wins the procurement auction and seller i reports de lowest virtual cost among all the upgrader's rivals.

On the other hand, in A_0 the same agent wins the competition but no other firm submits a bid below the reserve cost c_0. Implicitly in our above definitions, among the lowest virtual costs, the upgrader wins the procurement auction, which certainly doesn't increase expected expenditures for the buyer. As a direct consequence,

$$\int_A H(c^n, I)dc^n = \sum_{i=0, i\geq 2} \int_{A_i} H(c^n, I)dc^n$$

Now, define $t_l(\cdot) \equiv J_l^{-1}(J_i(\cdot))$ for $l \geq 2,\ l \neq i$ and $t_I(\cdot) \equiv J_I^{-1}(J_i(\cdot))$ Integrating over A_i yields

$$\int_{A_i} H(c^n, I)dc^n = \int_{\underline{c}}^{J_i^{-1}(c_0)} \int_{t_2(c_i)}^{\bar{c}} ... \int_{t_{i-1}(c_i)}^{\bar{c}} \int_{t_{i+1}(c_i)}^{\bar{c}} ... \int_{t_n(c_i)}^{\bar{c}} \int_{\underline{c}}^{t_I(c_i)} J_I(c_1)\frac{\partial F}{\partial c_1}(c_1, I) \left(\prod_{l\geq 2} f_l(c_l)\right) dc^n$$

and observing that $J_I(c_1)\frac{\partial F}{\partial c_1}(c_1, I) = \left[c_1 + \frac{F(c_1,I)}{\frac{\partial F}{\partial c_1}(c_1,I)}\right]\frac{\partial F}{\partial c_1}(c_1, I) = \frac{d}{dc_1}(c_1 F(c_1, I))$ we obtain

$$\int_{A_i} H(c^n, I)dc^n = \int_{\underline{c}}^{J_i^{-1}(c_0)} f_i(c) \left(\prod_{l\neq i}[1 - F_l(J_l^{-1}(J_i(c)))]\right) J_I^{-1}(J_i(c))F(J_I^{-1}(J_i(c)), I)dc$$

Analogously,

$$\begin{aligned}\int_{A_0} H(c^n, I)dc^n &= \int_{J_2^{-1}(c_0)}^{\bar{c}} ... \int_{J_n^{-1}(c_0)}^{\bar{c}} \int_{\underline{c}}^{J_I^{-1}(c_0)} J_I(c_1)\frac{\partial F}{\partial c_1}(c_1, I) \left(\prod_{l\geq 2} f_l(c_l)\right) dc^n \\ &= \left(\prod_{l\geq 2}[1 - F_l(J_l^{-1}(c_0))]\right) J_I^{-1}(c_0)F(J_I^{-1}(c_0), I) \qquad (7)\end{aligned}$$

Thus,

$$\begin{aligned}\int_A H(c^n, I)dc^n &= \sum_{i\geq 2} \int_{\underline{c}}^{J_i^{-1}(c_0)} f_i(c) \left(\prod_{l\neq i}[1 - F_l(J_l^{-1}(J_i(c)))]\right) J_I^{-1}(J_i(c))F(J_I^{-1}(J_i(c)), I)dc \\ &+ \left(\prod_{l\geq 2}[1 - F_l(J_l^{-1}(c_0))]\right) J_I^{-1}(c_0)F(J_I^{-1}(c_0), I) \qquad (8)\end{aligned}$$

On the other hand,

$$C^n \setminus A^n = \{c^n \in C^n| \ (\exists j \geq 2,\ J_l(c_l) < J_I(c_1)\ \wedge\ J_l(c_l) \leq c_0)\ \vee (c_0 < J_I(c_1),\ c_0 < J_i(c_i),\ \forall i \geq 2)\}$$

is the set over which the upgrader loses the procurement auction. As before, this set can be partitioned as $C^n \setminus A = B_0 \cup \left(\bigcup_{j\geq 2} B_j\right)$ with

$$B_0 = \{c^n \in C^n| \ J_I^{-1}(c_0) < c_1\ \wedge\ J_i^{-1}(c_0) < c_i,\ \forall i \geq 2\} \qquad (9)$$

$$B_i = \{c^n \in C^n |\; c_i \leq J_i^{-1}(c_0) \;\wedge\; (J_j^{-1}(J_i(c_i)) \leq c_l,\; i \leq l) \;\wedge (J_l^{-1}(J_i(c_i)) < c_l,\; i < l) \;\wedge\; J_I^{-1}(J_i(c_i)) < c_1\}$$

Set B_0 represents the zone in which the project is not assigned and B_i corresponds to the region where firm $i \geq 2$ wins the competition. Implicitly in the definition of these sets we assume that, in case of equal lowest-virtual-costs, the task is assigned to the lowest-index competitor, which certainly doesn't increase expected procurement expenditures. Then we can write

$$\int\limits_{C^n \setminus A} H(c^n, I) dc^n = \sum_{i=0, i\geq 2} \int\limits_{B_i} H(c^n, I) dc^n$$

It is direct that

$$\begin{aligned}
\int\limits_{B_i} H(c^n, I) dc^n &= \int\limits_{\underline{c}}^{J_i^{-1}(c_0)} \int\limits_{t_2(c_i)}^{\bar{c}} \dots \int\limits_{t_{i-1}(c_i)}^{\bar{c}} \int\limits_{t_{i+1}(c_i)}^{\bar{c}} \dots \int\limits_{t_n(c_i)}^{\bar{c}} \int\limits_{t_I(c_i)}^{\bar{c}} J_i(c_i) \frac{\partial F}{\partial c_1}(c_1, I) \left(\prod_{l \geq 2} f(c_l) \right) dc^n \\
&= \int\limits_{\underline{c}}^{J_i^{-1}(c_0)} \left(\prod_{l \neq i} [1 - F_l(J_l^{-1}(J_i(c_i)))] \right) [c_i f_i(c_i) + F_i(c_i)][1 - F(J_I^{-1}(J_i(c_i)), I)] dc_i \\
&= \int\limits_{\underline{c}}^{J_i^{-1}(c_0)} f_i(c) \left(\prod_{l \neq i} [1 - F_l(J_l^{-1}(J_i(c)))] \right) J_i(c)[1 - F(J_I^{-1}(J_i(c)), I)] dc \qquad (10)
\end{aligned}$$

Also,

$$\begin{aligned}
\int\limits_{B_i} H(c^n, I) dc^n &= \int\limits_{J_2^{-1}(c_0)}^{\bar{c}} \dots \int\limits_{J_n^{-1}(c_0)}^{\bar{c}} \int\limits_{J_I^{-1}(c_0)}^{\bar{c}} c_0 \frac{\partial F}{\partial c_1}(c_1, I) \left(\prod_{l \geq 2} f_l(c_l) \right) dc^n \\
&= c_0 \left(\prod_{l \geq 2} [1 - F_l(J_l^{-1}(c_0))] \right) [1 - F(J_I^{-1}(c_0), I)] \qquad (11)
\end{aligned}$$

As a consequence,

$$\begin{aligned}
\int\limits_{C^n \setminus A} H(c^n, I) dc^n &= \sum_{i \geq 2} \int\limits_{\underline{c}}^{J_i^{-1}(c_0)} f_i(c) \left(\prod_{l \neq i} [1 - F_l(J_l^{-1}(J_i(c)))] \right) J_i(c)[1 - F(J_I^{-1}(J_i(c)), I)] dc \\
&\quad + c_0 \left(\prod_{l \geq 2} [1 - F_l(J_l^{-1}(c_0))] \right) [1 - F(J_I^{-1}(c_0), I)] \qquad (12)
\end{aligned}$$

which concludes the proof.

□

Proof of Proposition 6: Define

$$\alpha_i(c) \equiv f_i(c) \left(\prod_{l \neq i} [1 - F_l(J_j^{-1}(J_i(c)))] \right)$$

thus,

$$\mathcal{C}(I) = \sum_{i\geq 2} \int_{\underline{c}}^{J_i^{-1}(c_0)} \alpha_i(c)\{J_I^{-1}(J_i(c))F(J_I^{-1}(J_i(c)),I) + [1-F(J_I^{-1}(J_i(c)),I)]J_i(c)\}dc \quad (13)$$

$$+\left(\prod_{l\geq 2}[1-F_l(J_l^{-1}(c_0))]\right)\{J_I^{-1}(c_0)F(J_I^{-1}(c_0),I) + [1-F(J_I^{-1}(c_0),I)]c_0\} \quad (14)$$

Therefore, under suitable integrability conditions

$$\mathcal{C}'(I) = \sum_{i\geq 2} \int_{\underline{c}}^{J_i^{-1}(c_0)} \alpha_i(c)\frac{\partial}{\partial I}\{F(J_I^{-1}(J_i(c)),I)[J_I^{-1}(J_i(c)) - J_i(c)]\}dc \quad (15)$$

$$+\left(\prod_{l\geq 2}[1-F_l(J_l^{-1}(c_0))]\right)\frac{\partial}{\partial I}\{F(J_I^{-1}(c_0),I)[J_I^{-1}(c_0)-c_0]\} \quad (16)$$

Define $L(c,I) \equiv F(J_I^{-1}(c),I)[J_I^{-1}(c)-c]$. Thus,

$$\begin{aligned}\frac{\partial L}{\partial I}(c,I) &= \left[\frac{\partial F}{\partial t}(J_I^{-1}(c),I)\frac{\partial}{\partial I}(J_I^{-1}(c)) + \frac{\partial F}{\partial I}(J_I^{-1}(c),I)\right][J_I^{-1}(c)-c] \\ &\quad + F(J_I^{-1}(c),I)\frac{\partial}{\partial I}(J_I^{-1}(c)) \\ &= \frac{\partial}{\partial I}(J_I^{-1}(c))\left[\frac{\partial F}{\partial t}(J_I^{-1}(c),I)[J_I^{-1}(c)-c] + F(J_I^{-1}(c),I)\right] \\ &\quad + \frac{\partial F}{\partial I}(J_I^{-1}(c),I)[J_I^{-1}(c)-c] \quad (17)\end{aligned}$$

Recall that $v_I(t) = t + \frac{F(t,I)}{\frac{\partial F}{\partial t}(t,I)}$, so, evaluating at $t = J_I^{-1}(c)$ we obtain

$$J_I^{-1}(c) - c = -\frac{F(J_I^{-1}(c),I)}{\frac{\partial F}{\partial t}(J_I^{-1}(c),I)}$$

Thus,

$$\frac{\partial L}{\partial I}(c,I) = \frac{\partial F}{\partial I}(J_I^{-1}(c),I)[J_I^{-1}(c)-c] \quad (18)$$

Therefore,

$$\mathcal{C}'(I) = \sum_{i\geq 2} \int_{\underline{c}}^{J_i^{-1}(c_0)} \alpha_i(c)\frac{\partial F}{\partial I}(J_I^{-1}(J_i(c)),I)[J_I^{-1}(J_i(c)) - J_i(c)]dc \quad (19)$$

$$+\left(\prod_{l\geq 2}[1-F_l(J_l^{-1}(c_0))]\right)\frac{\partial F}{\partial I}(J_I^{-1}(c_0),I)[J_I^{-1}(c_0)-c_0] \quad (20)$$

Since $\alpha_i(c) \geq 0$ and $J_I^{-1}(c) - c \leq 0$, $\forall c \in C$, a sufficient condition to obtain $\mathcal{C}'(I) \leq 0$ is

$$\forall i \geq 2,\ \forall c \in [\underline{c}, J_i^{-1}(c_0)],\ \frac{\partial F}{\partial I}(J_I^{-1}(J_i(c)), I) \geq 0$$

and

$$\frac{\partial F}{\partial I}(J_I^{-1}(c_0), I) \geq 0$$

which are equivalent to

$$\forall c \in [\underline{c}, J_I^{-1}(c_0)],\ \frac{\partial F}{\partial I}(c, I) \geq 0$$

since $J_I(\underline{c}) = J_i(\underline{c}) = \underline{c}$ and $J_I(\cdot)$ and $J_i(\cdot)$, $i \geq 2$, are increasing functions.

$\square$

References

[1] Cisternas, G. and Figueroa, N. (2007), "Sequential Procurement Auctions and Their Effect on Investment Decisions." *Documento de Trabajo 230, CEA.*

[2] Mas-Colell, A., Whinston, M. and Greene, J. (1995) "Microeconomic Theory." *Oxford University Press, Oxford, U.K.*

[3] Myerson, R. B. (1981). "Optimal Auction Design." *Mathematics of Operations Research.* ***6*** *58-73.*

[4] Pesendorfer, M. and Jofre-Bonet, M. (2005). "Optimal Sequential Auctions." *Mimeo, LSE.*

Centro de Economía Aplicada
Departamento de Ingeniería Industrial
Universidad de Chile

2007

240. A Note on the Comparative Statics of Optimal Procurement Auctions
Gonzalo Cisternas y Nicolás Figueroa

239. Parental choice and school markets: The impact of information approximating school effectiveness
Alejandra Mizala y Miguel Urquiola

238. Marginal Cost Pricing in Hydro-Thermal Power Industries: Is a Capacity Charge Always Needed?
M. Soledad Arellano and Pablo Serra

237. What to put in the table
Nicolas Figueroa y Vasiliki Skreta

236. Estimating Discount Functions with Consumption Choices over the Lifecycle
David Laibson, Andrea Repetto y Jeremy Tobacman

235. La economía política de la reforma educacional en Chile
Alejandra Mizala

234. The Basic Public Finance of Public-Private Partnerships
Eduardo Engel, Ronald Fischer y Alexander Galetovic

233. Sustitución entre Telefonía Fija y Móvil en Chile
M. Soledad Arellano y José Miguel Benavente

232. Note on Optimal Auctions
Nicolás Figueroa y Vasiliki Skreta.

231. The Role of Outside Options in Auction Design
Nicolás Figueroa y Vasiliki Skreta.

230. Sequential Procurement Auctions and Their Effect on Investment Decisions
Gonzalo Cisternas y Nicolás Figueroa

2006

229. Forecasting crude oil and natural gas spot prices by classification methods
Viviana Fernández

228. Copula-based measures of dependence structure in assets returns
Viviana Fernández

227. Un Análisis Econométrico del Consumo Mundial de Celulosa
José Ignacio Sémbler, Patricio Meller y Joaquín Vial

226. The Old and the New Reform of Chile's Power Industry. (Por aparecer en el International Journal of Global Energy Issues (forthcoming 2007)).
M. Soledad Arellano

225. Socioeconomic status or noise? Tradeoffs in the generation of school quality information. (Por aparecer en el Journal of Development Economics).
Alejandra Mizala, Pilar Romaguera y Miguel Urquiola.

224. Mergers and CEO power
Felipe Balmaceda

123. Task-Specific Training and Job Design.
Felipe Balmaceda

122. Performance of an economy with credit constraints, bankruptcy and labor inflexibility
Felipe Balmaceda y Ronald Fischer

121. Renegotiation without Holdup: Anticipating spending and infrastructure concessions
Eduardo Engel, Ronald Fischer y Alexander Galetovic

220. Using School Scholarships to Estimate the Effect of Government Subsidized Private Education on Academic Achievement in Chile
Priyanka Anand, Alejandra Mizala y Andrea Repetto

219. Portfolio management implications of volatility shifts: Evidence from simulated data
Viviana Fernandez y Brian M Lucey

218. Micro Efficiency and Aggregate Growth in Chile
Raphael Bergoeing y Andrea Repetto

2005

217. Asimetrías en la Respuesta de los Precios de la Gasolina en Chile
Felipe Balmaceda y Paula Soruco

216. Sunk Prices and Salesforce Competition
Alejandro Corvalán y Pablo Serra

215. Stock Markets Turmoil: Worldwide Effects of Middle East Conflicts
Viviana Fernández

214. The Competitive Role of the Transmission System in Price-regulated Power Industries
M. Soledad Arellano y Pablo Serra

213. La Productividad Científica de Economía y Administración en Chile. Un Análisis Comparativo (Documento de Trabajo Nº 301. Instituto de Economía, Pontificia Universidad Católica de Chile)
Claudia Contreras, Gonzalo Edwards y Alejandra Mizala

212. Urban Air Quality and Human Health in Latin America and the Caribbean
Luis A. Cifuentes, Alan J. Krupnick, Raúl O'Ryan y Michael A. Toman

211. A Cge Model for Environmental and Trade Policy Analysis in Chile: Case Study for Fuel Tax Increases
Raúl O'Ryan, Carlos J. de Miguel y Sebastian Millar

210. El Mercado Laboral en Chile Nuevos Temas y Desafíos
Jaime Gatica y Pilar Romaguera

209. Privatizing Highways in The United States
Eduardo Engel, Ronald Fischer y Alexander Galetovic

208. Market Power in Price-Regulated Power Industries
M. Soledad Arellano y Pablo Serra

207. Market Reforms and Efficiency Gains in Chile
Raphael Bergoeing, Andrés Hernando y Andrea Repetto

206. The Effects on Firm Borrowing Costs of Bank M&As
Fabián Duarte, Andrea Repetto y Rodrigo O. Valdés

205. Cooperation and Network Formation
Felipe Balmaceda

204. Patrones de Desarrollo Urbano: ¿Es Santiago Anómalo?
Raphael Bergoeing y Facundo Piguillem

203. The International CAPM and a Wavelet-based Decomposition of Value at Risk
Viviana Fernández

202. Do Regional Integration Agreements Increase Business-Cycle Convergence? Evidence from Apec and Nafta
Viviana Fernández y Ali M. Kutan

201. La dinámica industrial y el financiamiento de las pyme. (Por aparecer en El Trimestre Económico)
José Miguel Benavente, Alexander Galetovic y Ricardo Sanhueza

200. What Drives Capital Structure? Evidence from Chilean Panel Data
Viviana Fernández

2004

199. Spatial Peak-load Pricing
M. Soledad Arellano y Pablo Serra

198. Gas y Electricidad: ¿qué hacer ahora?. (Estudios Públicos 96, primavera 2004, 49-106)
Alexander Galetovic, Juan Ricardo Inostroza y Cristian Marcelo Muñoz

197. Reformando el sector eléctrico chileno: Diga NO a la liberalización del mercado spot
M. Soledad Arellano

196. Risk, Pay for Performance and Adverse Selection in a Competitive Labor Market
Felipe Balmaceda

195. Vertical Integration and Shared Facilities in Unregulated Industries
Felipe Balmaceda y Eduardo Saavedra

194. Detection of Breakpoints in Volatility
Viviana Fernández

193. Teachers' Salary Structure and Incentives in Chile
Alejandra Mizala y Pilar Romaguera

192. Estimando la demanda residencial por electricidad en Chile: a doña Juanita le importa el precio
José Miguel Benavente, Alexander Galetovic, Ricardo Sanhueza y Pablo Serra

191. Análisis y Recomendaciones para una Reforma de la Ley de Quiebras
Claudio Bonilla, Ronald Fischer, Rolf Lüders, Rafael Mery, José Tagle

190. Trade Liberalization in Latin America: The Case of Chile
Ronald Fischer

189. Time-Scale Decomposition of Price Transmission in International Markets
Viviana Fernández

188. Slow Recoveries. (Por aparecer en Journal of Development Economics)
Raphael Bergoeing, Norman Loayza y Andrea Repetto

187. Market Power in Mixed Hydro-Thermal Electric Systems
M. Soledad Arellano

186. Efectos de la privatización de servicios públicos en Chile: Casos sanitario, electricidad y telecomunicaciones
Ronald Fischer y Pablo Serra

185. A Hierarchical Model for Studying Equity and Achievement in the Chilean School Choice System
Alejandra Mizala, Pilar Romaguera y Carolina Ostoic

184. Innovaciones en Productividad y Dinámica de Plantas. (Revista de Análisis Económico, 18(2), pp. 3-32, 2003)
Raphael Bergoeing y Facundo Piguillem

183. The Dynamics of Earnings in Chile
Cristóbal Huneeus y Andrea Repetto

182. Monopoly Regulation, Chilean Style: The Efficient-Firm Standard in Theory and Practice
Álvaro Bustos y Alexander Galetovic

* Para ver listado de números anteriores ir a http://www.cea-uchile.cl/.