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A LARGE FIRM MODEL OF THE LABOR MARKET WITH ENTRY, EXIT AND SEARCH FRICTIONS

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A large firm model of the labor market with entry, exit and search frictions *

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Abstract

I augment the standard large-firm matching model with a firm process of entry and exit. This extension requires the analysis of firm-level dynamics, which I present in this note. I also show the equivalence of the model with the one-worker firm model from Pissarides (2000).

JEL: J63.
Keywords: large firm, matching frictions, entry, exit, unemployment.

1 Introduction

The standard matching model of the labor market assumes that firms can hire at most one worker (Pissarides, 2000). This is a convenient assumption because it eases the analysis at the macro level. At the same time, this advantage may also be interpreted as a drawback of the model. For instance, allowing for more than one worker per firm has implications in terms of wage determination and hiring behavior in a Nash-bargaining environment; if there are decreasing returns to scale in production, firms may wish to overemploy in order to reduce marginal revenue and so wages[1]. As a consequence, Cahuc et al. (2007) have shown that allowing for intrafirm wage bargaining can reconcile the holdup literature with the fact that the capital-labor ratio is higher in Europe as compared to the USA.

Another limitation of the one-worker firm model is that one cannot separate policies that affect the size of firms with policies that influence the number of firms operating in the economy. Both phenomena affect

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[1] This has been largely discussed in the literature. See Cahuc et al. (2007), Cahuc and Wasmer (2001), Bertola and Garibaldi (2001), Smith (1999) and Bertola and Caballero (1994). If firms' production function is characterized with constant returns to scale, Cahuc and Wasmer (2001) argue that the model is essentially similar to the one-worker firm model. Cahuc et al. (2007) have also shown that if there are decreasing returns to scale and the production function is Cobb-Douglas and only depends on one type of labor, then the solution for employment corresponds to the standard case, which is multiplied by a fixed term that does not depend on any endogenous variable, meaning that the implications of intrafirm bargaining are marginal.

aggregate employment in the same way. A reason for disregarding these aspects is probably because this is potentially cumbersome in terms of attractiveness of the model. One would have to deal with firm-level dynamics at the same time as considering the firm decision to enter or exit the industry, implying firm heterogeneity and so requiring the model to include the evolution of the distribution of firms' vintages (Hopenhayn, 1992) together with the dynamics of unemployment.

The motivation to write this note is twofold. First, my aim is to address the second set of critiques. I extend the large-firm model with a process of firm entry and exit and analyze the implied firm-level dynamics. I show that the cost of such an extension is not high from a technical standpoint. In particular, firm-level employment follows a two-tier structure. Upon entry, firms post an infinite amount of vacancies so that employment immediately reaches its long-run target and then hire enough workers so as to keep the employment stock constant over time. Although firms post an infinite amount of vacancies at the very beginning of their lives, I show that this entry cost is finite, implying that the entire economy does not shut down if one augments the large-firm model with a process of firm entry and exit.

Second, I will derive the result by which firm entry and exit does not matter in a steady-state when returns to scale in production are constant. The intuition behind this finding is closely linked to that which explains why the large-firm model without entry is equivalent to the one-worker firm model of Pissarides (2000) under constant returns to scale (Cahuc and Wasmer, 2001): because firm size is undetermined in this case, the mass of firms cannot be identified either.

I list now in the next Section the assumptions characterizing my economy, before presenting the findings in Section 3.

2 Setup

Time is continuous. I consider an economy where firms are all characterized by the same production function, which displays constant returns to scale. The economy is similar to the one described in Cahuc et al. (2007) with one type of worker, but it is augmented with a process of firm entry and exit. In particular, firms die at an exogenous rate λ.

Only one good is produced in this economy, the market of which is competitive. I denote by M_t the mass of firms that operate on this market at time t and F is the constant-returns-to-scale production function, which depends on two factors of production: capital K_t and labor N_t. The capital market is perfect, but the labor market is characterized by matching frictions.

I denote by w_t the paid wage, which is determined under Nash bargaining with β denoting the bargaining power of workers. c is the cost of posting a vacancy, V_t the mass of posted vacancies (at the firm level), which are filled at a rate $h(\theta_t)$ that depends negatively on the labor market tightness $\theta_t = \frac{M_t V_t}{U_t}$, i.e. the vacancy-unemployment ratio. This rate is derived from a matching function $m(U_t, M_t V_t)$ with constant returns to scale, increasing in both its arguments, concave and satisfying the property $m(U_t, 0) = m(0, M_t V_t) = 0$, implying that $h(\theta_t) = \frac{m(U_t, M_t V_t)}{M_t V_t} = m(\theta_t^{-1}, 1)$. Jobs are destroyed at an exogenous rate s and I_t is investment in physical capital, which depreciates at a rate δ. Finally, r is the exogenous interest rate.

I will proceed recursively: consider first the situation of an incumbent firm, derive its optimal behavior and finally analyze the entry decision. For this purpose, I need to define the Bellman equations characterizing the behavior of firms and workers. Specifically, the value at time t of an incumbent firm with employment N_t and capital K_t is

$$\Pi(K_t, N_t) = \max_{V_t, I_t} \frac{1}{1 + (r + \lambda)dt} \{[F(K_t, N_t) - w_t N_t - cV_t - I_t]dt + \Pi(K_{t+dt}, N_{t+dt})\}$$

subject to the constraints

$$\begin{aligned} N_{t+dt} &= (1 - sdt)N_t + h(\theta_{t+dt})dtV_t, \\ K_{t+dt} &= (1 - \delta dt)K_t + I_t dt. \end{aligned}$$

The values of being unemployed and employed follow a standard formulation and write as

$$rU_t = b + \theta_t h(\theta_t)[W_t - U_t] + \dot{U}_t \tag{1}$$

and

$$rW_t = w_t + (s + \lambda)[U_t - W_t] + \dot{W}_t \tag{2}$$

with b the flow utility of being unemployed.

3 Equilibrium

3.1 First-order conditions, wage determination and steady-state profits

The first-order conditions are[2]

$$\begin{cases} \frac{\partial \Pi(K_{t+dt}, N_{t+dt})}{\partial N_{t+dt}} = \frac{c}{h(\theta_{t+dt})}, \\ \frac{\partial \Pi(K_{t+dt}, N_{t+dt})}{\partial K_{t+dt}} = 1, \\ \frac{\partial \Pi(K_t, N_t)}{\partial N_t} = \frac{1}{1+(r+\lambda)dt}\left([F_2(K_t, N_t) - w]\,dt + \frac{\partial \Pi(K_{t+dt}, N_{t+dt})}{\partial N_{t+dt}}(1 - sdt)\right), \\ \frac{\partial \Pi(K_t, N_t)}{\partial K_t} = \frac{1}{1+(r+\lambda)dt}\left(F_1(K_t, N_t)dt + \frac{\partial \Pi(K_{t+dt}, N_{t+dt})}{\partial K_{t+dt}}(1 - \delta dt)\right). \end{cases}$$

Hence,

$$F_1(k_t, 1) = r + \lambda + \delta \tag{3}$$

and

$$\frac{c}{h(\theta_t)} = \frac{1}{1 + (r + \lambda)dt}\left([F_2(k_t, 1) - w]\,dt + \frac{c}{h(\theta_{t+dt})}(1 - sdt)\right), \tag{4}$$

where k_t is the capital-employment ratio.

By letting $dt \longrightarrow 0$, equation (4) can be rewritten as

$$(r + \lambda + s)\frac{c}{h(\theta)} + \frac{h'(\theta)}{h(\theta)^2}c\dot{\theta} = F_2(k, 1) - w, \tag{5}$$

[2]Remember that, under constant returns to scale, we have in equilibrium that $\frac{dw}{dN} = 0$, implying that the firm cannot influence the wage by modifying its size (Cahuc and Wasmer 2001). Under an alternative framework, i.e. decreasing returns to scale, firms could use their monopsonistic power to overemploy and so extract larger rent from workers (see Cahuc et al., 2007, Stole and Zwiebel, 1996a and 1996b). In a previous version of this note I showed that considering a process of firm entry and exit in a context where returns to scale are decreasing does not affect the conclusion derived in Cahuc et al. (2007).

where the time subscript has been removed for notational convenience.

By combining (3), (5) and the fact that F is homogeneous of degree one, I have

$$(r+\lambda+s)\frac{c}{h(\theta)}+\frac{h'(\theta)}{h(\theta)^2}c\dot{\theta}=F(k,1)-(r+\lambda+\delta)k-w$$

Under Nash bargaining and from equations (1) and (2),

$$w=\beta\left[F(k,1)-k(r+\delta+\lambda)\right]+(1-\beta)b+\beta\theta c, \tag{6}$$

leading to

$$\left[r+\lambda+s+\beta\theta h(\theta)\right]\frac{c}{h(\theta)}=(1-\beta)\left[F(k,1)-(r+\lambda+\delta)k-b\right]. \tag{7}$$

In the above equation, I made use of the fact that k is at any time fixed according to (3) and implies with (5) and (6) that $\dot{\theta}=0$ along the unique stable path of the labor market tightness (Pissarides, 1985).

Since, $V^*=\frac{s}{h(\theta^*)}N^*$ and $I^*=\delta K^*$ in steady state[3], the steady-state present-discounted profits per employee π^* evaluated at k^* can be written as[4]

$$(r+\lambda)\pi^*=(1-\beta)\left[F(k^*,1)-b\right]-\frac{s+\beta\theta^* h(\theta^*)}{h(\theta^*)}c+\left[\beta(r+\lambda)-(1-\beta)\delta\right]k^* \tag{8}$$

Equation (8) corresponds to per employee profits of an *incumbent* firm, which has already reached its steady-state level of employment. But what about the convergence path at the firm level? I turn now to the analysis of firm-level dynamics, which will allow me to derive the entry behavior of firms.

3.2 Firm-level dynamics and implications for entry decisions

The conditions (3) and (7) imply that, after entry, adjustment in employment and capital at the firm-level follows a two-tier structure. At time of entry t_0, a firm posts an amount $V_0=\frac{N^*}{h(\theta^*)dt}$ of vacancies so in order to reach its steady-state level immediately, with $dt\to 0$, and invests a quantity $I_0=\frac{K^*}{dt}$ in physical capital. Obviously, those quantities are infinite. Then, at time $t>t_0$, it posts $V^*=\frac{sN^*}{h(\theta^*)}$ vacancies and invests $I^*=\delta K^*$ so that the employment and capital stocks keep constant over time.

This two-tier structure implies that upon entry firms pay a sunk cost C_0 so that their employment level reaches immediately its steady-state level, with C_0 defined as

$$C_0\equiv c_0N^*\equiv\lim_{\epsilon\to 0}\int_0^{\epsilon}e^{-(r+\lambda)x}\left(\frac{cN^*}{h(\theta^*)\epsilon}+\frac{K^*}{\epsilon}\right)dx \tag{9}$$

[3]Hereafter, variables evaluated at the steady state are followed by asterisks.

[4]As I will discuss in Section 3.2 below, firms adjust their level of employment and capital after entry so that they immediately reach their steady-state level. This also implies that employment and capital jump following an exogenous shock that pushes their value upward, suggesting that (8) may also refer to profits on the transition path. However, this is not necessarily true if the shock reduces firm-level employment, especially if s is low enough. In this case, firms simply freeze their hirings. See Garibaldi (2006).

The issue is to know whether the cost (9) converges to some finite value. If it is not the case, then no firm enters the market and the economy disappears. If the cost is finite, there remains the issue of whether it is low enough for firms to have incentive to enter.

After some calculations, one can rewrite the entry cost as

$$c_0 = \lim_{\epsilon \to 0} \frac{c/h(\theta^*) + k^*}{\epsilon} \frac{-e^{-(r+\lambda)\epsilon} + 1}{r + \lambda}$$

By applying a linear expansion to $e^{-(r+\lambda)\epsilon}$, one can approximate this value with $-(r+\lambda)\epsilon$. Hence,

$$c_0 = \lim_{\epsilon \to 0} \frac{c/h(\theta^*) + k^*}{\epsilon} \frac{1 - (r+\lambda)\epsilon - 1}{-(r+\lambda)},$$

which can be simplified as

$$c_0 = \frac{c}{h(\theta^*)} + k^*, \tag{10}$$

implying that the sunk cost is finite. I now turn to the entry decision of firms. After imputing the cost (10) into (8), one has

$$(r+\lambda)(\pi^* - c_0) = (1-\beta)\left[F(k^*, 1) - b - (r + \delta + \lambda)k^*\right] - (r+s+\lambda+\beta\theta^* h(\theta^*))\frac{c}{h(\theta^*)},$$

which, according to (7), is merely

$$\pi^* = c_0, \tag{11}$$

meaning that the sunk cost is exactly equal to discounted profits. Consequently, under constant returns to scale, adding a process of firm entry and exit to the large firm's matching model does alter the main features of the model: it is still equivalent to the one-worker firm framework of Pissarides (2000). Neither firm size nor the mass of firms are determined. Another implication of (11) is that any additional entry cost κ other than c_0 would make the economy shut down since $\pi^* < c_0 + \kappa$ in this case.

The model is solved recursively. First, equation (3) gives the capital-labor ratio. Second, (5) determines the labor market's tightness given the capital-labor ratio. Finally, unemployment is obtained thanks to its law of motion as in Pissarides (2000), which leads to the following steady-state formulation:

$$U^* = \frac{s + \lambda}{s + \lambda + \theta^* h(\theta^*)}.$$

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