Implementation with Renegotiation When Preferences and Feasible Sets Are State Dependent*

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Abstract

In this paper, we present a model of implementation where infeasible allocations are converted into feasible ones through a process of renegotiation that is represented by a reversion function. We describe the maximal set of Social Choice Correspondences that can be implemented in Nash Equilibrium in a class of reversion functions that punish agents for infeasibilities. This is used to study the implementation of the Walrasian Correspondence and several axiomatic solutions to problems of bargaining and taxation.

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1 Introduction

Since Hurwicz's classic papers in the early 1970s, a great deal of attention has been devoted to the problem of implementing social choice rules when preferences are state dependent (see Jackson [12] for a survey). In contrast, very few contributions have dealt with the problem of implementing social choice rules when the set of feasible outcomes is state dependent. The problem is that, in this case, some messages yield infeasible allocations, which requires that we describe how to deal with them. The standard approach is to design a series of mechanisms in which the planner can ex-post verify whether or not players are exaggerating individual endowments or technological capabilities (i.e. by asking them to put endowments on the table).[1] If an infeasibility occurs, players expect serious punishment (Hurwicz et al., [11], Tian [24], Tian and Li [25], Hong [8, 9, 10], Serrano and Vohra [22] and Dagan et al.[5]).[2]

In this paper, we present a theory of how to deal with infeasibilities that is based on the idea that infeasible allocations are renegotiated. Consider the following example: The associate editor of a journal is in charge of a special issue for which she has selected 10 authors. She asks each author to submit a 20-page paper. One of the authors submits a 22-page paper and another author submits an 18-page one. In this case, it is likely that the editor will take no action. However, if both authors submit 22-page papers, she will have to deal with the infeasibility. It is likely that she will work with the authors to shorten the papers or with the managing editor in order to free up more pages, etc. In this case, feasibility is restored by means of negotiation. Another example is the legal system: once infeasibilities are detected, there are institutions designed to punish transgressors (if they

[1] This assumption is called the "no exaggeration assumption." It implies that agents play a different game at different states of the world, see Hong [10] p. 206, ll. 17-19.

[2] The work on manipulation via endowments (Postlewaite [18], Atlamaz and Klaus [2]) is also related to this literature.

can be identified) and to restore feasibility.[3] In this case, feasibility is restored automatically. Thus, the process used to deal with infeasibilities may reflect how agents renegotiate or how institutions operate. Furthermore, it is independent on the mechanism that created the infeasibility.

We model the social process that transforms infeasible allocations into feasible ones by means of a *reversion function*. This concept originates in Maskin and Moore [15] and has been developed by Jackson and Palfrey [13]. In these papers, the reversion function formalized the process of renegotiation through which agents trade goods allocated by the mechanism or veto some feasible allocations. In our case, the reversion function represents the way in which society reacts to infeasible allocations. Consequently, the properties that we impose on the reversion function are very different from those assumed by the earlier literature.

For the purposes of this paper, we assume complete information. This is a clean scenario which looks to be a good candidate for a first trial of our approach. We focus on Nash implementation and assume that agents know the reversion function. Therefore, the reversion function induces new preferences, which we will call *reverted preferences* (this is the "translation principle" in Maskin and Moore [15]). Reverted preferences are state dependent even if underlying preferences are not. Hence, when the feasible set is state dependent, implementation reduces to the case of implementation when only preferences are state dependent. However, as Maskin and Moore put it, "results from the standard literature are too abstract to give a clear indication of how serious a constraint renegotiation is. . . ."

We focus on a class of reversion function in which at least one agent is worse off if an infeasibility arises. For those authors, renegotiation comes from the mechanism's inability to stop agents from reaching mutually beneficial trades. In our case, renegotiation arises

[3]We can think of the feasible set including not only the properly feasible allocations, but also all punishments and additional devices that can be administered by the institutions designed, as well as the delays that may occur.

from the physical impossibility of carrying out the intended plans so that someone has to make a sacrifice in order to achieve feasibility. An extreme case of a non-rewarding reversion function is when all agents are punished when an infeasibility arises, so that they prefer any allocation without punishment to the situation in which they are punished. This strong form of punishment, which we will call *generalized severe,* resembles the one implicitly assumed in the previous literature. In our case, it serves an instrumental role: it provides necessary conditions for the implementation of social choice rules if the reversion function is non-rewarding (Proposition 1). A simple adaptation of the classic result shows that monotonicity is a necessary and almost sufficient condition for implementation in Nash equilibrium when reverted preferences are given by the generalized severe reversion function (Remark 1). Thus, our first task is to characterize monotonicity. We show that it is equivalent to a weak form of individual rationality and a generalized form of contraction consistency (Proposition 2). The former property is satisfied by most social choice rules and the latter is similar to Nash's independence of irrelevant alternatives.

Next, we apply the previous result to several frameworks and compare our findings with the earlier literature. We begin by considering exchange economies. Here, the non-rewarding condition may be violated unless renegotiation is sufficiently costly. In this case, the non-rewarding condition can be considered as a simplification that narrows down the class of renegotiation functions and thus is useful for obtaining analytical results. We prove that in these environments, weak unanimity is trivially satisfied by any individually rational social choice rule. But the individual rationality requirement, which is necessary and sufficient for feasible implementation in the set-up considered by Hurwicz et al. [11], is not necessary nor sufficient for implementation in our framework. This is due to the fact that in our case a generalized form of contraction consistency must be satisfied as well. We show that the Constrained Walrasian Rule satisfies generalized contraction consistency and is thus implementable if the reversion function is non-rewarding (Proposition 3). We turn our attention to bargaining problems. First, we notice that the non-rewarding condition always holds in this environment. We show that the Nash Bargaining solution can be implemented if the disagreement point is not state dependent (Proposition 4). This agrees with the findings of Serrano [21] and Naeve [17]. We also show that the Kalai-Smorodinski solution is not implementable. Finally, we consider the taxation problem in which the mechanism must collect a given amount of taxes. The non-rewarding condition also holds here. We describe the taxation rules that can be implemented and find that any taxation rule which continuously varies on incomes whenever incomes are larger than proposed taxes cannot be implemented in our framework. The difference between our approaches is that the no exaggeration assumption in their case (see Footnote 1) rules out many profitable deviations that are possible in our model.

In the sections that follow, we describe the model (Section 2), introduce reversion functions (Section 3) and study implementation under the assumption that the reversion function is non- rewarding (Sections 4 and 5) before presenting our conclusions (Section 6).

2 The Model

In this section we provide the main definitions. Let us first describe the environment.
Let $I = \{1, ..., n\}$ be the set of agents. Let ω_i denote agent i's type and let Ω_i denote agent i's type set. Let $\Omega \subseteq \prod_{i=1}^{n} \Omega_i$ be the set of all possible states of the world. Each $\omega \in \Omega$ is characterized by a feasible set $A(\omega)$ and a preference profile $R(\omega) = (R_1(\omega), ..., R_n(\omega))$. The feasible set $A(\omega)$ contains all feasible allocations including punishments that arise in state ω. Set $A \equiv \bigcup_{\omega \in \Omega} A(\omega)$. $R_i(\omega)$ is a preference relation, a complete, reflexive and transitive binary relation on $A(\omega)$. $P_i(\omega)$ denotes the corresponding strict preference relation. Let $L_i(a, \omega) = \{x \in A(\omega) : aR_i(\omega)x\}$ be agent i's lower contour set of a.
A correspondence $F : \Omega \twoheadrightarrow A$ such that $F(\omega) \subseteq A(\omega)$ for all $\omega \in \Omega$ will be called a *Social*

Choice Rule (*SCR* for brevity). A *mechanism* is a pair (M, g) where $M \equiv \prod_{i=1}^{n} M_i$ is the *message space* and $g : M \to A$ is the outcome function. M_i denotes agent i's *message space*. Let $m = (m_1, .., m_n) \in M$ be a list of messages, also written (m_i, m_{-i}). Given $\omega \in \Omega$, a mechanism (M, g) induces a game $(M, g, R(\omega))$. A message profile $m^* \in M$ is a *Nash equilibrium for* $(M, g, R(\omega))$ if, for all $i \in I$ $g(m^*)R_i(\omega)g(m_i, m^*_{-i})$ for all $m_i \in M_i$. $NE(M, g, R(\omega))$ will denote the set of Nash equilibrium outcomes of $(M, g, R(\omega))$. The mechanism (M, g) *implements* F *in Nash equilibrium* if $NE(M, g, R(\omega)) = F(\omega)$ for all $\omega \in \Omega$.

3 Reversion Functions

Since outcomes that are feasible in some states may be infeasible in others, we must describe how society deals with infeasible allocations. We assume that if an allocation is infeasible, it is transformed into a feasible one by a process that might involve delays (because renegotiation takes time), penalties to some individuals, etc. The systematic way in which the reallocation process takes place will be called a reversion function.[4] This reallocation may correspond to a "free-market renegotiation" or to a process where an institution applies some rule, i.e. a rationing scheme, a bankruptcy rule, etc. Formally:

Definition 1 *A reversion function is a map* $h : A \times \Omega \to A$ *such that, for each* $\omega \in \Omega$*: (i)* $h(a, \omega) \in A(\omega)$ *for all* $a \in A$ *and (ii)* $h(a, \omega) = a$ *for all* $a \in A(\omega)$.

A reversion function always yields feasible allocations (condition (i) above) and feasible allocations are not renegotiated (condition (ii) above). The latter condition is postulated in order to separate the issue of mutually advantageous renegotiation, which was the focus of the previous literature, from the issue of infeasibility, which is the focus of this paper.[5] In

[4] See Amorós [1] for a model with several reversion functions.

[5] A tautological interpretation is that $A(\omega)$ is the set of allocations that are not renegotiated at ω.

other words, this condition allows us to analyze renegotiation caused by infeasibility alone.

Under weak conditions, if the reversion function can be chosen by the planner, any single valued SCR can be implemented (a proof is available under request). But the designer, by assumption, cannot condition her actions on the state of the world.

To explain the next step, consider the simplest possible case: at states of the world ω and ω' the preference profiles are the same, say R. Let a, b and c be three allocations that are feasible at state ω. Assume that aP_ibP_ic for some agent i. Allocation a is not feasible at state ω' and is renegotiated to c, allocation b is feasible at ω'. So, even if the underlying preferences are the same in both states, player i prefers a to b at state ω and b to a at ω'. In order to formalize and extend this idea, we offer the following definition.

Definition 2 *Given $\omega \in \Omega$ and a reversion function h, the reversion of $R(\omega)$ on $A(\omega)$, denoted by $R^h(\omega)$ is defined by:*

$$aR_i^h(\omega)b \Longleftrightarrow h(a,\omega)R_i(\omega)h(b,\omega), \text{ for all } a,b \in A, \text{ for all } i \in I.$$

Then, when the reversion function is h, we can interpret that agents' preferences are the reverted preferences, i.e. they only care about reverted allocations. Let $L_i^h(a,\omega) = \{b \in A : h(a,\omega)R_i(\omega)h(b,\omega)\}$ be the lower contour set of a at ω with respect to $R^h(\omega)$.

The next definition is a straightforward adaptation of the standard notion of implementation in Nash equilibrium.

Definition 3 *A social choice rule F is h-implementable in Nash Equilibrium if there exists a game form (M, g) such that, for all $\omega \in \Omega$:*

$$F(\omega) = h(NE(M, g, R^h(\omega)), \omega)$$

In words, F is h-implementable in Nash equilibrium if and only if it is implementable in Nash equilibrium when, for each $\omega \in \Omega$, the correspondent preference profile is $R^h(\omega)$. Once we consider that agents' preferences are induced by the reversion function, we can deal with h-implementation exactly in the same way as in the classical implementation problem.

When considering the restrictions that a state dependent feasible set imposes on implementation, we concentrate on monotonicity (or Maskin-monotonicity). As observed by Jackson [12], monotonicity is the most important obstacle to implementation in Nash equilibrium. For instance, it is not generally satisfied by the Walrasian social choice rule. Monotonicity is a necessary and almost sufficient condition for a SCR to be implementable in Nash equilibrium (see Maskin [14] or Repullo [19]). It is therefore the first condition to be addressed. When a SCR satisfies monotonicity, if an alternative is implemented at one state of the world and rises in every agent's preference ranking at another state of the world, then it must be implemented also at the second state. Now we restate the definition of monotonicity in terms of reverted preferences. Let h be a reversion function.

Definition 4 *A social choice rule F is h-monotonic if, for all $\omega, \omega' \in \Omega$ and for every $a \in A$ such that $h(a, \omega) \in F(\omega)$*

$$L_i^h(a, \omega) \subseteq L_i^h(a, \omega') \text{ for all } i \Longrightarrow h(a, \omega') \in F(\omega')$$

Similarly, a social choice rule F that satisfies *h-no veto power* must select an allocation which is at the top of the reverted preference ranking of all agents except at most one. The importance of these concepts is highlighted by the following remark, whose proof is a straightforward adaptation of a standard result mentioned before and is therefore omitted:

Remark 1 *If a social choice rule is h-implementable in Nash equilibrium, it is h-monotonic. Moreover, in environments in which $\#I > 2$ if a social choice rule is h-monotonic and satisfies h-no veto power, it is h-implementable in Nash equilibrium.*

4 Non-Rewarding Reversion Functions: Basic Results

In this section, we focus our attention on a class of reversion functions where renegotiation is not advantageous for all players. We will call this class of functions *non-rewarding*. We will show that in this class, a particular reversion function, which we will call *generalized severe*, implements the maximal set of SCR. Then, we will characterize the SCR that can be implemented under generalized severe reversion functions. Let us start by defining the class of non-rewarding reversion functions:

Definition 5 *A reversion function is non-rewarding if, for all $\omega, \omega' \in \Omega$ and all $a \in A(\omega)$:*
(i) if $a \notin A(\omega')$ there exists $i \in I$ such $L_i^h(a, \omega) \nsubseteq L_i^h(a, \omega')$.
(ii) If there exist $i \in I$ and $b \in A$ with $aR_i(\omega)h(b, \omega)$ and $h(b, \omega')P_i(\omega')h(a, \omega')$ then there exist j and $c \in A(\omega')$ such that $aR_j(\omega)h(c, \omega)$ with $cP_j(\omega')h(a, \omega')$.

The idea behind non-rewarding reversion functions is that when agents renegotiate, something bad happens -delays, punishments engineered by the designer, etc.- The first condition asserts that if an allocation a passes from being feasible at state ω to being infeasible at state ω', at least one player must pay a price for the infeasibility so that it does not improve in everybody's ranking. The second condition implies that at least one agent suffers as a consequence of infeasibility in a way that could have been accomplished through a feasible allocation (for instance, the agent who is deemed responsible for the infeasibility is punished).

Consider now a specific reversion function which belongs to the class of non-rewarding ones. Suppose that, should an infeasibility arise, players are redirected to what they consider to be the worst possible allocation. This reversion function resembles the (implicit) assumption made in previous papers that agents do not choose infeasible messages because the planner detects infeasibility and imposes a punishment in such a way that agents prefer any other feasible allocation to this punishment. However, our interest in this particular reversion function arises from the fact that it allows us to find the maximal set of SCR that can be implemented under non-rewarding renegotiation (see Proposition 1 below).

Let $G \in A(\omega)$ for all $\omega \in \Omega$ be such that for all i, $aP_i(\omega)G$ with $a \neq G$ and $a \in A(\omega)$. G will be called the *generalized punishment point* because all agents are penalized. The reversion function with $s(a,\omega) = G$ if $a \notin A(\omega)$ will be called *generalized severe* and the induced preferences will be denoted by $R^s(\omega)$. They are characterized by the following three properties:

(1) If $a, b \in A(\omega)$ then $aR_i^s(\omega)b$ if and only if $aR_i(\omega)b$ for all $i \in I$.

(2) If $a \in A(\omega)$ with $a \neq G$ and $b \notin A(\omega)$ then $aP_i^s(\omega)b$ for all $i \in I$.

(3) If $a, b \notin A(\omega)$ then $aI_i^s(\omega)b$ for all $i \in I$.

We assume that the planner never wants to implement alternative G. Formally, we consider SCR F such that $F(\omega) \neq G$ for all $\omega \in \Omega$.

We show that generalized severe reversion implements the largest set of social choice rules among the class of non-rewarding reversion functions where h-no veto power holds.

Proposition 1 *Let F be an h-monotonic SCR. If h is non-rewarding, then F is monotonic with a generalized severe reversion function. Thus, if F satisfies h-no veto power, it is implementable in Nash Equilibrium with a generalized severe reversion function.*

Proof. In the proof s denotes the generalized severe reversion function. Let $a \in F(\omega)$ and assume that for some ω', $L_i^s(a,\omega) \subseteq L_i^s(a,\omega')$ for every $i \in I$. First, we show that $a \in A(\omega')$. We prove the claim by contradiction: assume that $a \notin A(\omega')$. In this case, $L_i^s(a,\omega') = \{G\} \cup A \backslash A(\omega')$, so $L_i^s(a,\omega) = (L_i(a,\omega) \cap A(\omega)) \cup A \backslash A(\omega) \subseteq \{G\} \cup A \backslash A(\omega')$. It follows that $A \backslash A(\omega) \subseteq A \backslash A(\omega')$ so $A(\omega') \subseteq A(\omega)$. The reversion function h is non-rewarding and $a \notin A(\omega')$, so $L_i^h(a,\omega) \nsubseteq L_i^h(a,\omega')$. Then, there exists b such that $aR_i(\omega)h(b,\omega)$ and $h(b,\omega')P(\omega')a$. From the definition of a non-rewarding reversion function there exist j and $c \in A(\omega')$ such that $aR_j(\omega)h(c,\omega)$ and $cP_j(\omega')h(a,\omega')$, but $h(c,\omega) = c$, because $A(\omega') \subseteq A(\omega)$, which yields a contradiction because $L_j^s(a,\omega) \subseteq L_j^s(a,\omega')$. Then it must be the case that $a \in A(\omega')$. In order to complete the proof it suffices to show that $L_i^h(a,\omega) \subseteq L_i^h(a,\omega')$ for every $i \in I$. We prove the claim by contradiction: assume that there exist $i \in I$ and $b \in A$ such that $aR_i(\omega)h(b,\omega)$ and $h(b,\omega)P_i(\omega')a$. From the definition of a non-rewarding reversion function there exist j and $c \in A(\omega')$ such that $aR_j(\omega)h(c,\omega)$ with $cP_j(\omega')h(a,\omega')$. If $c \in A(\omega)$ then $h(c,\omega) = c \in L_j^s(a,\omega) \backslash L_j^s(a,\omega')$, a contradiction. If $c \notin A(\omega)$ then $s(c,\omega) = G$ and $c \in L_j^s(a,\omega) \backslash L_j^s(a,\omega')$, a contradiction. The last claim follows because h-no veto power implies no veto power with a generalized severe reversion function. ■

The non-rewarding assumption is necessary for Proposition 1 to hold. Let $\Omega = \{\omega, \omega'\}$, $A(\omega) = \{a, b, c, G\}$ and $A(\omega') = \{a, b, G\}$. Let $n = 2$ and $R_i(\omega) = R_i(\omega') = R$ for $i = 1, 2$ where $bPaPc$. Let $F(\omega) = a$ and $F(\omega') = b$. Let $h(c,\omega') = b$. h does not satisfy the non-rewarding assumption at c. F is h-implementable in NE by the simple mechanism where agent 1 chooses among a and c, but it cannot be implemented by severe generalized punishment because F is not monotonic with respect to preferences R^s.

The rest of the section will be devoted to studying the implementability with non-rewarding reversion functions. According to Remark 1, this leads us to study h-monotonicity under preferences $R^h(\cdot)$.

We now introduce two properties that are necessary and sufficient for h-monotonicity under generalized severe reversion.

Definition 6 *A SCR F satisfies Weak Unanimity (WU) if, for all $\omega, \omega' \in \Omega$ such that $A(\omega') \subseteq A(\omega)$ and for all $a \in A(\omega) \backslash A(\omega')$ such that $L_i(a,\omega) \subseteq \{G\} \cup [A \backslash A(\omega')]$ for all $i \in I$, $a \notin F(\omega)$.*

Assume that all alternatives that are available at ω' are available at ω, too. Let a be available at ω but not at ω'. WU prescribes that if all alternatives that are available at ω' are strictly better than a for all agents, the planner should not choose to implement a at state ω. Breaking WU would create a problem of coordination at state ω': if such an a was chosen at state ω, any Nash equilibrium yielding a at ω would be a Nash equilibrium at ω' too, yielding $h(a, \omega') = G$.

Notice that WU is equivalent to the following condition: if $A(\omega') \subseteq A(\omega)$ and $a \in F(\omega) \backslash A(\omega')$ then there exists $b \in A(\omega')$, $b \neq G$ such that $aR_i(\omega)\, b$ for some $i \in I$. If WU holds, such a b can be used to prevent the implementation of G at state ω'. When the feasible set does not depend on the state of the world WU holds emptily.

Definition 7 *A SCR F satisfies Generalized Contraction Consistency (GCC) if, for all $\omega, \omega' \in \Omega$, and for all $a \in F(\omega) \cap A(\omega')$ such that $L_i(a, \omega) \cap A(\omega') \subseteq L_i(a, \omega')$ and $A(\omega') \backslash A(\omega) \subseteq L_i(a, \omega')$ for all $i \in I$, $a \in F(\omega')$.*

When preferences are fixed and $A(\omega') \subseteq A(\omega)$, $A(\omega') \backslash A(\omega) = \varnothing \subseteq L_i(a, \omega)$ for all i. In such a case, GCC prescribes choosing at state ω' any feasible allocation we have chosen at ω. Thus, GCC is a weak version of Nash Independence of Irrelevant Alternatives (see Roemer [20, p.55]). When the feasible set does not depend on the state of the world GCC coincides with Maskin Monotonicity. In the general case, GCC says that if a is selected at state ω, is feasible at ω', rises in everybody's preference ranking with respect to the alternatives that are feasible at ω' only and no better alternatives are available in $A(\omega') \backslash A(\omega)$, then a must be selected also at ω'.

Proposition 2 *A SCR is s-monotonic under generalized severe punishment if and only if it satisfies Generalized Contraction Consistency and Weak Unanimity.*

Proof. (Necessity) In the proof s denotes the generalized severe reversion function. We first show the necessity of WU. Let $A(\omega') \subseteq A(\omega)$ and $a \in F(\omega) \backslash A(\omega')$. The proof is by contradiction. Assume that $L_i(a, \omega) \subseteq \{G\} \cup [A \backslash A(\omega')]$ for all i. Observe that $L_i^s(a, \omega') = \{G\} \cup [A \backslash A(\omega')]$ for all i. We can write $L_i^s(a, \omega) = [L_i(a, \omega) \cap A(\omega)] \cup [A \backslash A(\omega)]$. Since $A(\omega') \subseteq A(\omega)$ then $A \backslash A(\omega) \subseteq A \backslash A(\omega')$. From the hypothesis of contradiction $L_i(a, \omega) \cap$

$A(\omega) \subseteq \{G\} \cup [A \backslash A(\omega')]$. It follows that $L_i^s(a,\omega) \subseteq \{G\} \cup A \backslash A(\omega') = L_i(a,\omega')$ for all i. As F satisfies s-monotonicity, $s(a,\omega') = G \in F(\omega')$, which yields a contradiction. Now consider GCC. Let $a \in F(\omega) \cap A(\omega')$ and assume that $L_i(a,\omega) \cap A(\omega') \subseteq L_i(a,\omega')$ and $A(\omega') \backslash A(\omega) \subseteq L_i(a,\omega')$ for all $i \in I$. We next prove that $L_i^s(a,\omega) \subseteq L_i^s(a,\omega')$ for all i. We can write $L_i^s(a,\omega) = [L_i(a,\omega) \cap A(\omega')] \cup \{[L_i(a,\omega) \cap A(\omega)] \backslash A(\omega')\} \cup A \backslash A(\omega)$. Observe that $A \setminus A(\omega) = [A(\omega') \setminus A(\omega)] \cup \{[A \setminus A(\omega')] \setminus A(\omega)\}$. It follows that $L_i^s(a,\omega) \subseteq L_i(a,\omega') \cap A(\omega') \cup [A \backslash A(\omega')] = L_i^s(a,\omega')$ for all i. By s-monotonicity $s(a,\omega') = a \in F(\omega')$.

(Sufficiency) Let F satisfy WU and GCC. Let $a \in F(\omega)$. Assume that $L_i^s(a,\omega) \subseteq L_i^s(a,\omega')$ for all i. We first prove by contradiction that $a \in A(\omega')$: assume $a \notin A(\omega')$. If $A(\omega') \subseteq A(\omega)$, by WU, there exist an agent i and $b \in A(\omega')$ such that $aR_i(\omega)b$. Then $aR_i^s(\omega)b$, and $bP_i^s(\omega')a$, because a is not feasible at ω', which yields a contradiction. Now let $A(\omega') \nsubseteq A(\omega)$. In this case, let i be any agent and let $b \in A(\omega') \backslash A(\omega)$. Then $aP_i^s(\omega)b$, and $bP_i^s(\omega')a$, because a is not feasible at ω', which yields a contradiction.

So far we have established that $a \in F(\omega) \cap A(\omega')$. We next prove by contradiction that $L_i(a,\omega) \cap A(\omega') \subseteq L_i(a,\omega')$ and $A(\omega') \backslash A(\omega) \subseteq L_i(a,\omega')$ for all $i \in I$. If this holds we can conclude by GCC. Assume first that $L_i(a,\omega) \cap A(\omega') \nsubseteq L_i(a,\omega')$ for some agent i. Let $b \in [L_i(a,\omega) \cap A(\omega')] \backslash L_i(a,\omega')$. We have $aR_i^s(\omega)b$ and $bP_i^s(\omega')a$, which yields a contradiction. Now assume that, for some agent i, $A(\omega') \backslash A(\omega) \nsubseteq L_i(a,\omega')$. Let $b \in [A(\omega') \backslash A(\omega)] \backslash L_i(a,\omega')$. We have $aP_i^s(\omega)b$ and $bP_i^s(\omega')a$, which yields a contradiction. ■

5 Non-Rewarding Reversion Functions: Applications

In this section, we apply the findings of the previous sections to withholding in exchange economies, bargaining and taxation methods.

5.1 Exchange Economies: Withholding

Consider an exchangeeconomy with n agents and K goods. We assume that agents' preferences do not vary but that endowments do. Let u_i be a utility function that represents agent i's preferences. Let $\Omega_i \subseteq \mathbf{R}_+^K$ be the set of agent i's possible endowments. We assume that the planner can only transfer goods among players. Then, the allocation set contains the set of the balanced net transfers and the generalized punishment point, $A = \{x \in R^{K \times n} : \sum_{s=1}^n x_s = 0\} \cup \{G\}$. For all $\omega \in \Omega$ the feasible set is $A(\omega) = \{x \in A : x_i + \omega_i \geq 0$ for $i = 1, ..., n\} \cup \{G\}$. Then $A(\omega') \subseteq A(\omega)$ if and only if $\omega' \leq \omega$. In order to describe preferences on net transfers, note that the utility agent i gets from transfer x_i when her endowment is ω_i is $u_i(x_i + \omega_i)$. Thus, utility functions are state dependent even if preferences are not. Let $u_i(G) = u_i(0) - \varepsilon$, $\varepsilon > 0$.

In this environment and with more than two agents, h-monotonicity is necessary and sufficient for F to be h-implementable in Nash equilibrium. It is easy to see that reversion functions might be *rewarding*. For instance, starting from an allocation in which agents are given goods that they scarcely care about, if the endowment of one good is reduced by, say, 1% and renegotiation does not entail any cost, it is possible to renegotiate this allocation in a way in which all of them are better off. The non-rewarding condition is plausible here if we assume that renegotiation is sufficiently costly -e.g. delay, transaction costs, etc.- for at least one agent.

Let us translate conditions WU and GCC into this framework. First consider WU. One can easily see that it suffices to consider only endowments ω, ω' such that $\omega' \leq \omega$. Then WU amounts to the following condition:

Condition α : For all $\omega, \omega' \in \Omega$ such that $\omega' \leq \omega$, if $a \in F(\omega) \setminus A(\omega')$ there exists i such that $u_i(\omega_i + a_i) \geq u_i(\omega_i - \omega_i')$.

Observe that if $(0, \omega_i) \subseteq \Omega_i$ for all i then Condition α only requires the SCR to be individually rational for at least one agent. It is a very weak requirement and it is obviously satisfied by many SCR, e.g., any Pareto efficient or any individually rational SCR.

Stronger requirements are imposed by GCC. In this case it also suffices to consider only endowments $\omega,\ \omega' \in \Omega$ such that $\omega' \leq \omega$. GCC is satisfied if and only if the following condition holds:

Condition β. For all $\omega, \omega' \in \Omega$ such that $\omega' \leq \omega$, if $a \in F(\omega) \cap A(\omega')$ and $a \notin F(\omega')$ there exists i and $x \in A(\omega')$ such that

$$\begin{aligned} u_i\left(\omega_i + a_i\right) &\geq u_i\left(\omega_i + x_i\right) \\ u_i\left(\omega_i' + a_i\right) &< u_i\left(\omega_i' + x_i\right) \end{aligned}$$

Let us compare our conditions with Hong [10], Corollary 1, p. 216. She showed that a SCR is implementable by a collection of state dependent mechanisms if and only if the following condition is satisfied

$$u_i(\omega_i + f_i(\omega)) \geq u_i(\omega_i - \omega_i') \text{ for all } i \tag{H}$$

Our Condition (α) is weaker than condition (H): If $x \in A(\omega')$, then $u_i(\omega_i + x_i) \geq u_i(\omega_i - \omega_i')$ for all i as all u_i are increasing. Then if $f(\omega) \in A(\omega')$, $u_i(\omega_i + f_i(\omega)) \geq u_i(\omega_i - \omega_i')$ for all i. So for all ω, ω' such that $f(\omega) \in A(\omega')$ condition (H) holds.

Note that our condition depends on the fact that each agent cannot simply retain part of her endowment. She has to make it compatible with other agents' messages. But our Condition (β) is not implied by Condition (H). Assume for instance that $f(\omega) \in A(\omega')$ then (H) imposes no restrictions on $f(\omega')$.

If the translations by $\omega - \omega'$ of all agents' indifference curves through $\omega' + f(\omega)$ are strictly above all agents' indifference curves through $\omega + f(\omega)$, then condition (β) implies $f(\omega) = f(\omega')$. Formally, if $\bigcap_{i=1}^{n} \{y : u_i(\omega_i' + f_i(\omega))) = u_i(y_i)\} \subseteq \bigcap_{i=1}^{n} \{y : u_i(y_i + \omega_i - \omega_i') > u_i(\omega_i + f_i(\omega))\}$

, condition (β) imposes that $f(\omega) = f(\omega')$.

The difference between our conditions and Hong's is explained by the fact that her goal is to design one feasible mechanism $(M(\omega), g(\omega))$ for each possible endowment ω, in a way such that the larger the feasible set, the larger the message space. Two of her assumptions make our approaches different: i) Hong assumes that players cannot exaggerate their endowment and that they can be punished for the message they send and not only for the allocation they intend to obtain if such an allocation is not feasible; and ii) Hong gives each player the power to retain part of her endowments. We assume that players can collectively cheat the planner through the mechanism by asking for a feasible allocation in which some agents retain a part of their endowment.

Let us first study the implementation of the Constrained Walrasian SCR.[6] The allocation $a \in A(\omega)$ is a *Constrained Walrasian Allocation (CWA) at* ω if there exists $p \in \mathbf{R}_+^K$ such that $a \in \arg\max \{u_i(\omega_i + x_i) : px_i \leq 0,\ x \in A(\omega)\}$ for all i. Such a p is said to be an equilibrium price supporting a at ω. Let $CW(\omega)$ denote the set of CWA at ω.

Proposition 3 *Let utility functions be increasing, continuous and quasi-concave. Let* $\Omega_i = (0, \overline{\omega}_i)$ *for all* i *for some* $\overline{\omega}_i \in (0, \infty)$*. Then the Constrained Walrasian SCR is implementable in Nash Equilibrium by generalized severe punishment.*

Proof. Under our assumptions, $CW(\omega)$ is not empty for all $\omega \in \Omega$. In order to prove the claim it suffices to show that CW satisfies Condition β. Let $\omega' \leq \omega$, $a \in CW(\omega) \cap A(\omega')$ and $a \notin CW(\omega')$. Let p be an equilibrium price at ω. Then there exist $x \in A(\omega')$ with $u_i(\omega' + x_i) > u_i(\omega' + a_i)$ and $px_i \leq 0$ for some i. $A(\omega') \subseteq A(\omega)$ so $x \in \{px_i \leq 0,\ x \in A(\omega)\}$.

[6] The Walrasian Correspondence defined by $WC(\omega) = \arg\max \{u_i(\omega_i + x_i) : px_i \leq 0\}$ is not implementable in Nash Equilibrium for the same reasons that prevent the standard Nash implementability of WC.

From the definition of CW it follows that $u_i(\omega_i + a_i) \geq u_i(\omega_i + x_i)$. Then CW satisfies Condition β. ■

A similar result holds for fixprice equilibria. Here we use the definition of Younès equilibrium (Younès [26]), though there are other concepts of fixprice equilibria, such as Drèze's equilibrium [6] (see also Grandmont and Laroque [7]), which includes restrictions on sales and Benassy's equilibrium [7]), which uses rationing schemes. Silvestre [23] proved that the three concepts are equivalent.

Definition 8 *Given a vector of prices p, an allocation x^* is a Constrained Younès Equilibrium at ω if it satisfies:*
(i) $\sum_{i=1}^{n} x_i^* = 0$
(ii) For all i $x_i^* + \omega_i$ maximizes u_i on the set
$\left\{z_i - \omega_i : z_i \in \mathbf{R}_+^K, \min\left\{x_{ij}^*, 0\right\} \leq z_{ij} - \omega_{ij} \leq \max\left\{x_{ij}^*, 0\right\}, p(z_i - \omega_i) \leq 0\right\} \cap A(\omega)$
(iii) There exist no pair of consumers (i, h), a commodity k and a real number $\varepsilon > 0$ such that $u_i(\omega_i + x_i^* + \varepsilon a^k) > u_i(\omega_i + x_i^*)$ and $u_h(\omega_h + x_h^* - \varepsilon a^k) > u_i(\omega_h + x_h^*)$, where a^k is defined by $a_1^k = -p_k$, $a_k^k = 1$ and $a_t^j = 0$ for $t \neq 1, k$.

We obtain the following result using the same argument as in Proposition 3:

Remark 2 *The constrained Younès Equilibrium correspondence is implementable in Nash Equilibrium by generalized severe punishment.*

5.2 Bargaining with Unknown Utility Possibility Set

Let us now consider non-cooperative implementation of cooperative solution concepts (Dagan and Serrano [4] and Naeve [17]). A bargaining problem is a pair (U, v) where $U \subseteq \mathbf{R}_+^n$ is the utility possibility set and $v \in U$ is the disagreement point. We assume that U is convex, closed, with a non-empty interior and comprehensive (i.e. $u \in U$ and $u' \leq u$, $u' \in \mathbf{R}_+^n$ implies $u' \in U$). For each bargaining problem (U, v), let $U_v = \{u \in U : u \geq v\}$ be bounded. The Nash Bargaining Solution (NBS) is defined as $NBS(U, v) = \arg\max_{u \in U_v} \prod_{i=1}^{n}(u_i - v_i)$. Let $NBS(U, v)_i$ be the utility awarded to i by the NBS. For further reference we notice that this solution is characterized by the following properties: strong efficiency, individual rationality, scale covariance, symmetry and independence of irrelevant alternatives.

We consider $U_v \cup \{G\}$ as the feasible set of (U, v). It is clear that in this environment the non-rewarding property holds because if a utility allocation is renegotiated, the utility of at least one agent must fall. Therefore, there is no loss of generality in considering only non-rewarding reversion functions. In fact, we consider a non-rewarding reversion function more suited to the situation and assume that all infeasible allocations are reverted to the disagreement point. Let h denote such reversion function, which we will call *non-severe* because $h(a, \omega) \neq G$ for all $a \in A$ and all $\omega \in \Omega$. Clearly, h is non-rewarding. Agent i's reverted preferences at (U, v) are described by

$$u_i^h(u, (U, v))) = u_i \text{ if } u \in U_v$$

$$u_i^h(u, (U, v)) = v_i, \text{ otherwise.}$$

We first notice that if the disagreement point is not known by the planner, NBS fails to satisfy GCC: Let $n = 2$ and let $U = \left\{x \in \mathbf{R}_+^2 : x_1^2 + x_2^2 \leq 1 \right\}$. Let $v = (0, 0)$ and let $v' = \left(\frac{\sqrt{2}}{2}, 0\right)$. Then $NBS(U, v) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) \in U_{v'} \subseteq U_v$ but $NBS(U, v) \neq NBS(U, v')$. Thus, according to Proposition 1, NBS is not implementable in NE by any non-rewarding reversion function.[7]

The Kalai-Smorodinski solution does not satisfy GCC even with a fixed disagreement point.

[7] This result agrees with the findings of Serrano [21]. A different interpretation of preferences on the utility possibility set may lead to more permissive results. One can interpret them as a measure of agents' satisfaction with respect to the disagreement point. Let $u_i(u, (U, v)) = u_i - v_i$. Then the preferences induced

Proposition 1 implies that it cannot be implemented in NE by any reversion function. Instead, when the disagreement point is known, the NBS satisfies both GCC and WU, as the reader can easily verify. However, Proposition 2 cannot be used to conclude that the NBS is implementable by generalized severe punishment because it does not satisfy no-veto and the Maskin Theorem requires at least three agents (see Remark 1).

We prove the result directly by using the characterizations developed by Moore and Repullo [16].

Proposition 4 *Let $n \geq 2$. The Nash Bargaining Solution is h-implementable in Nash Equilibrium with the non-severe reversion function if the disagreement point v is known.*

Proof. Let $x = NBS(U, v)$. Let $i \in I$ and let (U', v) be a bargaining problem. Let $u \in L_i^h(x, (U, v))$ such that, at (U', v) and with reverted preferences u is maximal for i in $L_i^h(x, (U', v))$ and u is maximal in $\mathbf{R}_+^n$ for all agents different from i. We prove that $u = NBS(U', v)$. Observe that it must be the case that u is feasible at U' otherwise all agents different from i would prefer some point in the interior of U'_v. Furthermore, $u_j = \max\left\{u'_j : u' = (u'_j, u'_{-j}) \in U'_v\right\}$ for all $j \neq i$. In particular u lies on the boundary of U'. We prove the claim by contradiction. Assume that $u \neq NBS(U', v)$. It must be the case that $NBS(U', v)_i > u_i$. If $NBS(U', v) \notin U_v$ then u is not maximal in $L_i^h(x, (U, v))$ for i when preferences are reverted at (U', v), a contradiction. So, it must be the case that

by h are

$$u_i^h(u, (U, v)) = u_i - v_i \text{ if } u \in U_v$$
$$u_i^h(u, (U, v)) = 0 \text{ otherwise}$$

Observe that $u_i^h(u, (U, v)) = u_i^h(u - v, (U - v, 0))$. The reader can easily check that from the translation invariance property of the NBS, the analysis of the problem with unknown disagreement point amounts to the previous situation with the disagreement point fixed and known at 0. In this case, applying Proposition 4 below yields a positive result.

$NBS(U',v) \in U_v$. Furthermore, $NBS(U',v) \neq NBS(U,v)$ and $NBS\ (U,v) \notin U'_{v'}$ otherwise u would not be maximal in $L_i^h(x,(U,v))$ for i under reverted preferences. Consider the segment joining $NBS(U',v)$ and u. Such a segment lies in U'_v because U'_v is convex and it intersects $\{u' \in U_v : NBS(U,v)_i \geq u'_i\ \}$ because U_v is convex and $NBS\ (U,v) \notin U'_v$. All along the segment, the coordinate i increases from u'_i to $NBS(U',v)_i$. Then there exists a point in $\{u' \in U_v : NBS(U,v)_i \geq u'_i\ \}$ which has the i^{th} coordinate strictly greater than u_i, a contradiction. Let u be maximal in R_+^n for all agents when preferences reverted at (U',v) then $u_j = \max\left\{u'_j : u' = (u'_j, u'_{-j}) \in U'_v\right\}$ for all j. From efficiency it follows that $u = NBS(U',v)$. NBS satisfies Individual Rationality, Pareto efficiency and GCC, too. Then, when $n \geq 3$, the family of sets $\left\{L^h(x,(U,v))\right\}_{x=NBS(U,v)}$ satisfies condition μ in Moore and Repullo [16]. When $n = 2$ it satisfies condition $\mu 1$ in the same paper because of the disagreement point. The application of Theorems 1 and 2 there, respectively, leads to the claim. ■

5.3 Taxation

A *taxation problem* is a pair $(x,T) \in R_+^n \times R_+$ where x is the vector of taxable incomes and T is the total amount to be collected such that $\sum_{i=1}^n x_i \geq T$ (Dagan et al. [5]). A *tax allocation* t for the taxation problem (x,T) is a vector in R_+^n such that $t \leq x$ and $\sum_{i=1}^n t_i = T$. A *taxation rule* is a function f which assigns a tax allocation to each taxation problem. We assume that the planner knows the amount to be collected, T, but does not know the taxable vector x. Let $S^n(T) = \left\{t \in R_+^n : \sum_{i=1}^n t_i = T\ \right\}$ be the set of tax allocations that collect T. Let $\Omega^n(T) = \left\{x \in R_+^n : \sum_{i=1}^n x_i \geq T\ \right\}$ be the set of the states of the world. Let $T^n(x) = T^n(x,T) = \left\{t \in R_+^n : 0 \leq t \leq x,\ \sum_{i=1}^n t_i = T\right\} \cup \{G\}$ be the set of feasible tax allocations at x. Each agent's preferences only depend on her after tax income and are strictly increasing. Then we can write $u_i(t,x) = x_i - t_i$ for each $x \in \Omega^n(T)$ and for each $t \in T^n(x,T) \setminus \{G\}$. Assume that only income exaggeration can be detected and punished. It is clear that in this environment, any reversion function must be non-rewarding so there is no loss of generality in considering only the latter. Therefore, the hypothesis of Proposition 1 is fulfilled.

We now characterize GCC in this environment.

Proposition 5 *A taxation rule satisfies GCC if and only if $f(x) = f(x')$ for all x, x' such that $f(x) \leq x' \leq x$.*

Proof. The necessity of the condition follows directly from the definition of GCC, because $T^n(x') \subseteq T^n(x)$ if and only if $x' \leq x$. We next prove the sufficiency of the condition. Let $t = f(x) \in T^n(x')$. First consider $x, x' \in \Omega^n(T)$ such that $x' \leq x$. Then $T^n(x') \subseteq T^n(x)$. This implies that $f(x) \leq x' \leq x$. It follows that $f(x) = f(x')$. So the condition required by GCC is satisfied. Now consider the case in which $x_j < x'_j$ for at least one j. The condition $T^n(x') \setminus T^n(x) \subseteq L_i(t, x')$ for all i is equivalent to $T^n(x') \setminus T^n(x) \subseteq \bigcap_{i=1}^n L_i(t, x') = \{t\} \cup \{G\}$. It is impossible in this case because the set $T^n(x') \setminus T^n(x) \supseteq \left\{t \in R_+^n : 0 \leq t \leq x', x_j < t \leq x'_j, \sum_{i=1}^n t_i = T\right\}$ is not empty and has the cardinality of the continuum. In this case, the condition required by GCC holds emptily. ■

Under the no-exaggeration assumption, any taxation rule can be implemented in NE (see Dagan et al. [5]). It is no longer true in our framework. Consider the proportional taxation rule, defined as

$$f_i(x) = \frac{x_i}{\sum_{i=1}^n x_i} T, \text{ for } i = 1, ..., n$$

Such a rule is not implementable in Nash equilibrium under any reversion function. We prove the claim by contradiction. Assume that f is implementable. Let $x^* \in \Omega^n(T)$. Observe that $0 < f_i(x^*) < x_i^*$ for all i. From the continuity of f, it follows that there is a neighborhood U of x^* that $0 < f_i(x) < x_i^*$ for $i = 1, ..., n$, if x belongs to U. Let $x \in U \cap \{x : x^* \leq x\}$ which is not empty. We have $0 < f(x) < x^* \leq x$ for all such x. As f is monotonic, Proposition 5 implies that $f(x) = f(x^*)$, which yields a contradiction.

More in general, let f be a continuous taxation rule such that for some x, $0 < f_i(x) < x_i$ for all i. If f is continuous but not constant whenever incomes are larger than proposed taxes for x, it cannot be implemented in our framework (the proof is available on request).

The difference between our result and the results obtained by Dagan et al. [5] stems from the fact that no exaggeration of incomes is possible in their approach, and, hence, many profitable deviations are ruled out.

6 Conclusions

This paper presents a new approach to dealing with the implementation problem when feasible sets are state dependent. It is based on the idea that feasibility is restored by a process that is independent of the mechanism and that reflects how agents renegotiate infeasible allocations into feasible ones or the working of institutions designed to cope with infeasibility. We have presented a class of reversion functions that are suited to our problem and we have found necessary and sufficient conditions for implementation when renegotiation takes this form. Finally, we have used our characterization results to study the implementation in Nash equilibrium of social choice rules in exchange economies, bargaining problems and taxation methods, and we have compared our results with those obtained in earlier literature.

A feature of the traditional approach of implementation when feasible sets are state dependent is that it requires a collection of state dependent mechanisms, which stands in contrast to the case when preferences are state dependent. This distinction differs vividly from our intuition on how markets cope with infeasible allocations, namely that the sign of excess demand entirely determines the adjustment irrespective of the cause of infeasibility.

Thus, our approach may offer a better understanding of market mechanisms than the traditional one, though the traditional approach is better suited to deal with topics such as withholding of endowments. Our approach can be generalized to deal with this case by introducing uncertainty in the renegotiation process or by writing the mechanism as an argument in the reversion function. These two extensions are easy to write, but require completely new analytical methods and therefore are left for future research.

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