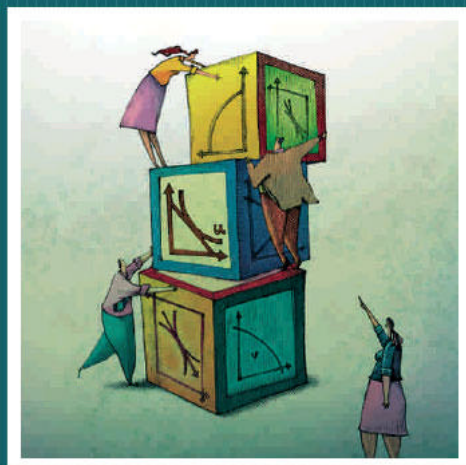


**DOCUMENTOS DE TRABAJO**  
**Serie Economía**



---

**Nº 279    JOB DESIGN AND INCENTIVES**

**FELIPE BALMACEDA**

---

# Job Design and Incentives

Felipe Balmaceda\*

Centro de Economía Aplicada (CEA)

Universidad de Chile<sup>†</sup>

February 18, 2011

## Abstract

This paper studies the problem of how to allocate  $n \geq 2$  independent tasks among an endogenously determined number of jobs in a setting with risk neutral workers subject to limited liability and ex-post asymmetric information. The main message is that firms narrow down the scope of their jobs to deal with workers' incentives to *game* the performance system (workers' incentives to work harder in tasks that are well rewarded ex-post and to underperform in tasks that are poorly rewarded). Firms' incentives to narrow job scopes are diminished when workers are intrinsically motivated by moral standards and, in contrast to Holmström and Milgrom (1991), when the degree to which tasks are substitutes increases.

**Keywords:** Job Design, Extrinsic and Intrinsic Motivation, Ex-post Private Information.

**JEL-Classification:** J41, J24, D21.

## 1 Introduction

Most economists assume that the main motivator for employees is incentive pay. However, studies point to the design of jobs as a major determinant of employee motivation, job satisfaction, productivity, commitment to an organization, absenteeism and turnover.

The question of how to design jobs so that employees are more productive was first answered in *The Wealth of Nations* by Adam Smith's well-known description of how pins should be manufactured. This idea paved the way for the "scientific management" philosophy set forth a century ago by Frederick W. Taylor (1911). The basic idea was to view job design as a scientific optimization problem, where industrial engineers study the production process and devise the most efficient way to break that process into individual, precisely defined tasks. Some of the known problems of the full specialization rationale are decreased employee satisfaction, increased turnover and absenteeism.<sup>1</sup> These observations led Herzberg to argue that in order

---

\*I would like to express my gratitude to participants in the regular seminar at CEA-University of Chile. I also want to acknowledge the assistance of Tibor Heumman. Financial support was provided by Fondecyt through research grant #1100267.

<sup>†</sup>República 701, Santiago 6521122, Chile. Email: fbalmace@dii.uchile.cl

<sup>1</sup>Smith explicitly recognized in *The Wealth of Nations* that specialization could lead to boredom and reduced productivity of workers.

to motivate workers to do good work, jobs should be enriched (i.e., add tasks, complexity and discretion) rather than simplified. Work should be designed and managed to foster responsibility, achievement, growth in competence, recognition, and advancement. This ideas were adopted by Hackman and Oldham (1976) in their development of their well-known Job Characteristics model.<sup>2</sup> This establishes that job enrichment generates intrinsic motivation and increases productivity. The most important element of their model is making work "meaningful" so that workers become more engaged to their jobs- are more interested, pay more attention, think more carefully and work more diligently.

In this paper I revisit the old question of how to design jobs. Namely, I ask how to split  $n \geq 2$  different tasks into an endogenously determined number of jobs and, how workers should be compensated in each of them. A job could have any number of tasks and tasks can be split in any number of jobs. The different approaches to the issue at hand are well summarized in Dewatripont et al. (2000). The approach pioneered by Holmström and Milgrom (1991) suggests that workers should be allocated tasks that have performance measures of similar quality. The conflicting-task approach proposed by Dewatripont and Tirole (1999) asserts in favor of avoiding job designs in which conflicting tasks are bundled into a job. The career concerns approach advanced by Dewatripont et al. (1999) argue in favor of focused jobs and bundling tasks that require similar skills so as to keep the inference process between aggregated performance and talent strong. A fourth approach, not discussed by Dewatripont et al. (2000), put forth by Laux (2001) shows that combining tasks lowers the limited-liability rent that must be given to a worker so that he works hard in each task and thus it is optimal to bundle more than one task into each job. These approaches have in common that there are spillovers across tasks, some of them assume it and in others it is a consequence of optimal contracting.

In section 2, a principal-agent model based on Baker (1992) is proposed. There are  $n \geq 2$  different tasks that *must* be performed and allocated to an endogenous number of jobs each requiring one worker, and the firm can compensate each worker according to: (i) an aggregated performance measure that is positively correlated with the worker's contribution to firm value (performance-based pay);<sup>3</sup> and (ii) his total time worked (time-based pay). Tasks are completely independent in the production function, the marginal cost of the time allocated to any task is independent of the time allocated to any other task, and it is strictly increasing with the amount of time allocated to the corresponding task. There is no time limit.<sup>4</sup> Furthermore, I assume away the incentives-in-teams problem by posing that for each possible job there is a performance measure that depends only on the tasks bundled into the job. After contracting, but before choosing their time allocation, workers receive with positive probability private information about the marginal performance sensitivity of time in each task (that is, the marginal impact that the time allocated to a given task has on the performance measure).<sup>5</sup> This information is soft and thus it cannot be communicated to the firm. The firm

---

<sup>2</sup>This is the most widely cited model in the job design literature.

<sup>3</sup>Tasks are independent in the performance measure in the sense that the marginal sensitivity of time in one task is independent of the time allocated to other tasks.

<sup>4</sup>Assuming that there is a time limit is as if tasks were substitutes and this may lead to specialization even in the absence of incentive problems. Thus, biasing the result in favor of specialization.

<sup>5</sup>Workers could also receive information about the the marginal product of time in each task, but this is of no use in this setting.

as well as workers are risk-neutral and workers face a limited-liability constraint equal to  $L \geq 0$ . Because the firm is ex-post uninformed, forcing contracts are never optimal since the firm would not know what is the ex-post optimal time allocation even if it were able to observe it.

In Section 3, the case of contractible output is analyzed and only monetary rewards matter. The optimal job design is to allocate all tasks to one job. The reason is twofold: Tasks are neither complements nor substitutes, and splitting a multi-task job entailing  $K \leq n$  tasks into  $K$  single-task jobs results in higher total compensation costs, since more workers must be hired and each must be  $L$  plus half of the expected output. Thus, I deliberately consider a framework in which under output contractibility, the firm would prefer a multiple-task job entailing the  $n$  tasks to any other task allocation, so that I can fully isolate incentive considerations from others in the literature such as technological and learning effects, conflicting tasks, effort substitution and non-linear incentive contracts that might make independent tasks to be treated as complements.

In Section 4, I consider the case in which output is non-contractible, the firm neither observes the marginal product of time nor the marginal performance sensitivity of time and workers are motivated only by monetary rewards. I show that despite task independence, the incentive problem considered here results in strong dis-economies of scope in firms that lead them to bundle fewer tasks into each job. More specifically, there exists a strictly positive threshold for the limited liability  $L$  such that for  $L$  below this threshold, the optimal job design entails  $n$  single-task jobs, while for  $L$  greater than the threshold, the optimal job design entails at least one multi-task job.

The optimal pay-for-performance sensitivity in any job bundling  $K \geq 2$  tasks is a weighted mean of the optimal performance pay sensitivity for each single task-job entailing one of the  $K$  tasks. This implies that a worker assigned to a multi-task job disregards valuable information for some tasks and places too much weight on less valuable information for some tasks. By narrowing the scope of each job to full specialization and properly adjusting performance-based pay, the firm makes workers more responsive to their private information in tasks with a performance measure highly correlated to workers' contribution to firm value and less responsive to that in tasks with low correlation, and by properly adjusting time-based pay, the firm makes each worker to spend an amount of time, that is on average, equal to the expected marginal product of time. However, there is a cost from splitting a multi-task job with  $K$  tasks into  $K$  single-task jobs, which is that more workers are hired and each of them must be paid a positive limited-liability rent. In short, when choosing a job design, the firm trades-off the benefits of better suited incentives that result in higher total output against higher total compensation costs of hiring more workers.<sup>6</sup>

So far, the model predicts that firms deal with workers' incentives to *game* the system (workers' incentives to work harder in tasks that are well rewarded ex-post and to underperform in tasks that are poorly

---

<sup>6</sup>In the appendix, I show that the result is robust to the use of either task-specific performance measures or task-specific time measures. In fact, as long as either the time or the performance measure is aggregated at the job level, the benefit from splitting the  $n$  tasks among  $n$  jobs exists.

rewarded) by narrowing down the scope of the jobs. However, social psychologists have argued for a long time that non-monetary rewards play an important role in aligning workers' incentives with those of their firms. In Section 5, I examine the role of moral standards as a source of non-monetary incentives or intrinsic motivation. By moral standards I mean workers' desire to do good work and this translates in that there is an *ideal* time allocation. Workers' utility falls as they deviate from this *ideal* time allocation and thus moral standards work as an intrinsic motivator. The extent to which deviations from the ideal time allocation harm a worker depends on his degree of adherence to the standards, denoted by  $m$ . Smith argues in *Theory of Moral Sentiments* that moral standards arise on the basis of a market-like process (see, Ottensson (2002)) in a similar way that self-interest gives rise to trade of services and goods in the *Wealth of Nations*. Following this ideas, the standard in each job here arises from the interaction between the management team and workers and it is assumed to be equal to the optimal time allocation when output is contractible.<sup>7</sup>

As in the case in which moral standards play no role, there exists a threshold for the limited liability  $L$  such that for  $L$  below this threshold the optimal job design entails only fully specialized jobs, while for  $L$  greater than the threshold, the optimal job design entails at least one multi-task job. This threshold falls as  $m$  rises. Thus, firms and workers able to agree on standards are more likely to choose a job design entailing at least one multi-task job. The reason is simple: moral standards do exactly what we expect them to do, decrease workers' incentives to *game* the performance measure; that is, workers with standards focus more on tasks with high ex-post marginal product of time and less on tasks with low ex-post marginal product of time. This shows that there is a causal relationship between motivation and enriched jobs or multi-tasking, however, the causality is reversed with respect to the one proposed by Hackman and Oldham (1976) since it goes from intrinsically motivated workers to multi-tasking and not the other way around.

In Section 6, I enriched the model by allowing for two different types of spillovers across tasks: (i) when the time spent in one task impacts the marginal cost of time spent in other tasks (i.e., the effort substitution effect introduced in the literature by Holmström and Milgrom (1991)); and (ii) when the time spent in one task changes the marginal product of time in other tasks. I show that in both cases strong substitution hinders full specialization and facilitates multi-tasking. This is in stark contrast with the prediction arising from different versions of the Holmström and Milgrom (1991)'s multi-task agency model.

When spillovers across tasks are introduced in the cost of time function and tasks are substitutes, workers care less about in which tasks they spend more time working and thus they become more responsive to their private information. This increases firm value since workers' time allocation is guided more by the marginal product of time than their preferences over tasks. In a sense, substitution across tasks transforms the worker's multi-tasking problem in a sort of single-tasking problem since the worker focuses more on the task with greater observed performance sensitivity. As degree to which tasks are substitutes rises, the threshold for the

---

<sup>7</sup>The results are robust to any standard that is positively correlated to the time allocation when output is contractible. Yet, the algebra is much more cumbersome and there is no further gain in intuition. The results for imperfect correlation are available upon request.

limited liability  $L$  above which the optimal job design entails at least one multi-task job falls, which makes multi-tasking jobs more likely to be part of an optimal job design.

When spillovers are introduced into the production function. The degree of substitution does not directly impact worker's time allocation since they are not compensated according to output. From the firm's point of view, however, substitution across tasks makes the firm to care less in an ex-ante sense about which tasks get assigned more time and thus it becomes profitable to offer high power incentives so that workers focus more on tasks with greater performance sensitivity ex-post. Thus, as the degree to which tasks are substitutes increases, the firm is more likely to adopt an optimal job design that admits at least one multi-task job. The intuition is similar to the previous one. From the firm's point of view substitution makes a multi-task job to look more like a single-task job, since it is better to induce workers to focus more on their private information.

Finally, in Section 7, I provide some empirical predictions concerning the relationship between uncertainty and incentives and also the relationship between job design, productivity and job satisfactions. I finish by offering concluding remarks.

**Literature Review.** The papers closest to this in terms of modeling assumptions is Baker (1992). Baker (1992) argues that often the measured outcomes are only partially related to the principal's goal and the agent may be asymmetrically informed about the effects of his actions. As a result the principal may choose to leave the decisions to the agent and incentivize him with an output-based contract. My model builds on Baker (1992) by considering the same informational problem but extended to several heterogeneous and independent tasks. The difference in terms of the behavior of the optimal contract are discussed in detail after this is derived and here I focus on optimal job design and also consider non-monetary incentives, which are not considered his paper.

In terms of this paper's goal, the seminal paper by Holmström and Milgrom (1991) is the closest to this one. They consider a principal-agent model with a risk-averse agent, imperfect observability of output for each possible task, and the agent's cost of effort function depends on the total effort the agent devotes to all tasks. Their model's key insight is that low-powered incentives arise as the optimal contracting solution when there are differences in measurement accuracy across tasks. The reason is that providing incentives in one activity decreases with the difficulty of measuring performance in other activities when both activities make competing demands on the agent's time (i.e., tasks are substitutes). Holmström and Milgrom also find: (i) that it is never optimal for two agents to be jointly responsible for any given task. This is due to fact that with risk aversion there is a fixed cost of engaging in another task that results from the risk associated to any task; and (ii) when several tasks must be allocated to workers, it is optimal to assign hard-to-measure tasks to one worker and easy-to-measure tasks to the other. The reason stands for the fact that this separation allows the principal to provide strong incentives for tasks that are easy to measure without fearing that the agent will substitute effort away from other, harder-to-measure tasks. Thus, the separation of tasks among

agents in Holmström and Milgrom's setting arises because there are negative spillovers across tasks that are built in the time of cost function and, thus it might well be that specialization is optimal even in the absence of any agency problem.

Itoh (1994) asks the same question that I ask here but in the Holmström and Milgrom (1991)'s setting. Mainly, he asks, in a two tasks setting in which there is only an aggregated performance measure, whether it is optimal to bundle the two tasks in one job or separate them into single-task jobs. He shows that when tasks are weak substitutes, incentive consideration causes the principal to bundle the two tasks into one job rather than hire two agents and make each of them specialize in just one task; while when tasks are strong substitutes specialization is optimal. This is driven by the fact that a joint performance is available and tasks are substitutes and thus workers tend to focus in one task only. Thus, he shows that the Holmström and Milgrom (1991)'s model yields the opposite result than mine.

There are several differences with the predictions arising from the Holmström and Milgrom (1991)'s setting. My rationale for low-powered incentives is somewhat different from theirs. Because they assume the principal can contract in each task separately, under the assumption of independent tasks, their model's predictions are identical to the single-task agency model. When I allow the firm to contract separately on each task, I still obtain the result that firms treat tasks as interwind since it is optimal for firms to an aggregated time measure. So my result arises from aggregated performance measures together with task heterogeneity, and not from substitution across tasks. Furthermore, when substitution is included in my model, the effect on the power of incentives is exactly the opposite of that made by the models using Holmström and Milgrom (1991)'s model. The reason is that the optimal time allocation in my model depends on workers' ex-post information, which makes the result in positive return from inducing workers to focus in one task ex-post. Finally, the difference in modeling assumptions provides different predictions concerning how to structure jobs and they do not consider the role of non-monetary incentives.

Dewatripont et al. (1999) ask how many tasks among  $n \geq 2$  it is optimal to allocate to a worker in a setting where incentives come from career concerns and the market observes an aggregated performance measure. Thus, in their setting a worker's only reason to exert effort is to influence the market's perception about his ability. They show that specialization arises as the optimal job design whenever increasing the number of tasks the agent has to perform reduces the link between performance and perceived ability, because performance becomes noisier. Their results suggest that agents should specialize in tasks that require similar talents, so as to keep the inference process about talent as strong as possible. This paper is somewhat related to mine in the sense the focus result arises from an interaction between the number of tasks and the equilibrium power of incentives. In their model incentives refer to implicit incentives and the mechanism is different from the one proposed here, yet both point to the beneficial incentive effect of specialization. However, the fact that their model is concerned with career concerns only make their rationale more suited to explain focused job designs on public agencies and ONGs, while mine is more suited to explain that in

private and for-profit firms.

Raith (2008) also considers a model similar to Baker (1992) and thus to mine. He assumes as Baker does that contracts can be based on a performance measure and time worked. The main difference stands for the fact that he allows to contract on measure of true, but noisy, output and the agent has private information about his contribution to output. He shows that the use of a noisy output performance measure to provide effort incentives is costly to the principal. More importantly, Raith argues, as Baker (1992) did, that the principal may want to use both input and output performance measures. The former in order to induce the agent to work harder and the latter in order to encourage the agent to properly utilize his specific knowledge about the consequences of his actions on output. I see at least three main difference with Raith (2008). First, the reason for the use of time-based pay here is different from that in Raith. In the absence of risk in his model, optimal time-based pay is zero and the optimal performance-based pay is equal to  $1/2$ . Thus, compensation is independent of the mean productivity and uncertainty and the difference between them across tasks. While I assume no risk and this provides a more richer environment that speaks more directly to issue of effort miss-allocation across tasks, which is the key concern in the multi-task literature and, therefore, on the job design literature. Second, the trade-off between total effort and effort mis-allocation across tasks is absent in Raith (2008) since in his model the worker's compensation is based on a noisy measure of true output and not on a potentially distorted performance measure as it is here. The existence of this trade-off leads the principal to choose an optimal contract that induces the worker to disregard his private information more than efficiency would require. This has important consequences on the choice of the optimal contract and job design. Third, the optimal contract for any given worker not only depends on the main parameters as in Raith (2008), some of which are intended to capture similar dimensions of the optimal contracting problem, but also on the task allocation or job design, which is endogenous. In fact, if Raith (2008) were to assume that tasks, as I do here, are technologically independent and there is no time cost spillovers, then his two tasks model would collapse to his one task model. This implies that his model is not suited to study job design in a setting with independent tasks, and thus job design issues in his model would be driven by the similar trade-offs to the ones considered by Holmström and Milgrom (1991).

Laux (2001) analyzes a multi-task agent model in which the agent's effort choice on each task is binary, effort costs are linear and the production function is linear and thus tasks are independent from each other. Yet, the optimal contract depends on the number of success and the optimal contract pays a positive bonus only when the maximum number of successes is realized. This makes effort in one task complementary to effort in other tasks. He shows then that incentive problems are a natural source of economies of scope in the sense that allocating multiple tasks to a single agent relaxes the agent's limited-liability constraint. The main consequence of this is that it might be optimal to increase the scope of the job with the natural consequence that the agent exerts an inefficiently high amount of total effort. Thus, he finds the opposite result than mine and the rationale for his result is of a different nature. Mainly, multi-tasking arises as a mechanism to lower



the agent's limited-liability rent.

Dewatripont and Tirole (1999) assume that there are direct conflicts between tasks. They show that it is always better to split the task of finding evidence in favor and against a decision between two agents. The reason stands for the fact that the optimal compensation can be based only on an aggregated measure of the task and this is increasing in the outcome of one task and decreasing in the outcome of the other task. This implies that it is impossible to induce one agent to exert more effort in both tasks and thus it is optimal to split the task between two different agents to avoid conflicts of interest in job design. In this paper separation also arises because there is technological link between tasks.

Bond and Gomes (2009) generalize the model in Laux (2001) by considering contracts in which an upper bound on payments and monotonicity constraint are imposed, the agent's effort choice on each task is non-binary, and the production function is non-linear. They identify a different source of allocational inefficiency across tasks, which is that under the optimal contract, it could be optimal for the agent to focus only on a sub-set of tasks, and its consequence is that there is insufficient total effort. Furthermore, they show that small changes in fundamentals can cause the agent's effort to collapse. MacDonald and Marx (2001) analyze a two-task principal-agent model where activities are substitutable for the agent and complementarity for the principal, and the agent prefers (or has lower cost) working on one of the tasks. Their main result is that the difference in the cost of exerting effort in each task creates a distortion in the allocation of effort. The optimal compensation alleviates this problem by leading the agent to view the activities as complements, and this complementarity is typically achieved using a contract that is non-monotone. However, they do not focus on job design issues.

These papers find different sources of effort miss-allocation than the one I provide in this paper. Laux (2001) finds that the agent may be overloaded and exerts excessive effort while Bond and Gomes (2009) and MacDonald and Marx (2001) find the opposite: excessive focus. None of these papers focus on the interaction of job design and incentive contracting when there is an aggregated distorted performance measure, and a worker's compensation is based on a performance and time measure. Thus, the allocative inefficiency I discover is of a different nature. Mainly, excessive focus in my model arises as an optimal response to the poor quality of the performance measure and not as the agent's optimal response to incentives.

Finally, this paper is also related to the literature on intrinsic motivation and moral hazard. For a single-task setting with risk-averse workers, Casadesus-Masanell (2004) considers three different types of non-monetary preferences among which moral standards is one of them. For this case, he shows that contracts that account for the agent's concern with moral standards generally rely on weaker incentives and higher fixed pay as compared to those from the standard agency model. There are several other papers studying moral hazard with different behavioral preferences such as Murdock (2002), Besley and Ghatak (2005), Prendergast (2008) and Delfgaauw and Dur (2008) who are concerned with mission-oriented agents (i.e., agents that derive utility from the firm's outcome) and show that incentives are weaker when agents exhibit

this kind of preferences and that the matching of workers and firms is crucial in terms of efficiency; and Akerlof and Kranton (2008) who study social identity and show that compensation costs fall and monitoring takes place less often when agents are concerned with their identity. Yet, none of these papers is concerned with the the relationship between behavioral preferences and job design in a multi-task setting.

## 2 The Model

Lets consider a model with risk-neutral workers (agents), indexed by  $i = \{1, \dots, N\}$ , and a firm (principal). Total output is the sum of the outputs on each job and each job requires one worker. Thus, there is no incentive-on-teams problem. Jobs can be set up to be accomplished with a variable number of tasks. The output of a job is a function of the tasks involved in the job in a manner that will be explained below.

**Tasks and Technology.** There is a finite number of tasks, indexed by  $j = \{1, \dots, n\}$  with  $n \leq N$ , that *must* be performed and firms can control them by inclusion in each job. When task  $j$  is bundled into the job performed by worker  $i$ , the variable  $x_{ij}$  takes the value 1; else  $x_{ij} = 0$ . Each task is indivisible due, for instance, to the fact that each task requires the use of assets that cannot be operated by several individuals at the same time. Thus, the possibility of task sharing is excluded. Furthermore, the principal does not perform any task.

Let  $x_i$  be the binary column vector  $(x_{i1}, \dots, x_{in})$  and  $x$  be the matrix  $(x_1, \dots, x_n)$ . There are at most  $2^n$  different task assignments or binary vectors that a firm may choose for any given job. Because each task can be assigned to one job only, the job design  $x$  is subject to the restriction  $\sum_i x_{ij} = 1$ . If  $x_i = (0, \dots, 0)$ , then worker  $i$  gets assigned no tasks and therefore he is not hired by the firm. Job design is non-contractible since this is a complex decision that it is hard to verify in court. Let  $X \equiv \{x_i \in \{0, 1\}^n, i = 1, \dots, N \mid \sum_{i=1}^N x_{ij} = 1, \forall j = 1, \dots, n\}$  be the set of all possible job designs; that is, matrices whose columns are binary vectors of dimension  $n$  and the sum of the elements in each row sum up to 1.

The production function is

$$Y(t, \omega) = \sum_{j=1}^n \omega_j t_j,$$

where  $t_j$  is the time allocated to task  $j$ ,  $t \equiv (t_1, \dots, t_n)$  is the time allocation and  $\omega_j$  is the  $j$ -task's marginal product of time.<sup>8</sup> In addition, this assumption ensures that tasks are independent from each other.

Observe that the technology is such that tasks are neither complements nor substitutes and thus there are no technological spillovers across tasks. In Section 6, I discuss how the results change when there are technological spillovers across tasks. In addition, for any given time allocation, total output is independent of the job design. Thus, job design matters only to the extent that it affects workers' incentives and compensation.

---

<sup>8</sup>The linearity with respect to each  $t_j$  is not needed and it is assumed for the sake of simplicity. Any concave function of  $t_j$  will give the same results.

The task-specific marginal productivity shocks  $\omega_j$ 's are independently distributed across tasks. Mainly, each  $\omega_j$  is distributed with mean  $\mathbb{E}(\omega_j)$ , variance  $\sigma_{\omega_j}^2$  and positive support. From here onwards, I call  $\sigma_{\omega_j}^2$  the technological uncertainty in task  $j$  and  $\omega_j$  the marginal product of time in task  $j$  and define  $\omega \equiv (\omega_1, \dots, \omega_n)$  as the marginal product of time.

**Workers' Preferences.** Workers supply labor inelastically and labor is the only input. They are risk neutral, time averse and time costs are measured in dollars. Namely, worker  $i$ 's preferences are captured by the following utility function:

$$U(w_i, t_i, x_i) \equiv w_i - \frac{1}{2} \sum_{j=1}^n x_{ij} t_j^2,$$

where  $w_i$  is his total compensation (described below),  $t_i \equiv (x_{i1}t_1, \dots, x_{in}t_n)$  is worker  $i$ 's time allocation and  $\frac{1}{2} \sum_{j=1}^n x_{ij} t_j^2$  is his dis-utility (measured in money) from working.<sup>9</sup> The cost of all other uses of time is normalized to zero. In section 6, I discuss how the results change when there are time cost spillovers across tasks.

This formulation assumes that from workers' viewpoint tasks are neither complements nor substitutes with regard to the time allocation and there is no overall time constraint. An important consequence of this is that there is no effort substitution problem. Thus, the nature of the job design problem is different from that studied by the literature spawn by the Holmström and Milgrom (1991)'s multi-tasking model. Furthermore, risk neutrality is key since this implies that there is no implicit fixed-cost of providing incentive for effort, which is the case when workers are risk averse (see, also Holmström and Milgrom (1991) and Itoh (1992)).

Workers are wealth constrained and therefore unable to make arbitrarily large transfers to the firm in order to increase the efficiency of their relationship. Thus, a limited-liability constraint arises, which implies that each worker has to be given a minimum consumption level  $L \geq 0$  every period, regardless of performance. Workers' outside option is normalized to zero.

**Contracts.** As in Baker (1992), I assume that output is non-contractible and, therefore, cannot be used to provide a worker with incentives. There is, however, a contractible performance measure that depends on the job design and takes the form

$$P(t_i, x_i, \mu) = \sum_{j=1}^n x_{ij} \mu_j t_j,$$

where  $\mu_j$  is task  $j$ 's marginal performance sensitivity of time.

The performance sensitivity shocks  $\mu_j$ 's are independently distributed across tasks. Mainly,  $\mu_j$  is distributed with mean  $\mathbb{E}(\mu_j)$ , variance  $\sigma_{\mu_j}^2$  and positive support.  $\sigma_{\mu_j}^2$  is a measure of the amount of valuable information that the worker in charge of task  $j$  may have about the performance measure. From here onwards, I call  $\mu_j$  the marginal sensitivity of time in task  $j$ ,  $\sigma_{\mu_j}^2$  the performance uncertainty in task  $j$ , and

<sup>9</sup>The quadratic assumption is meant to simplify the algebra, what is needed for the results to hold is the strict convexity of the time costs function.

define  $\mu \equiv (\mu_1, \dots, \mu_n)$  as the sensitivity of time.

The firm can also rely on another contractible measure that captures a worker's total working time, but not how he allocates his time across tasks.<sup>10</sup> The time performance measure is given by

$$T(t_i, x_i) = \sum_{j=1}^n x_{ij} t_j.$$

I believe that this performance measure will most adequately represent reality, because most workers are not likely to inform their supervisors of the exact time they spend in different tasks, but they do report the total time they have worked during the day (e.g. time sheets).

Since compensation is based only on variables that depend only on the tasks assigned to a worker, there is no incentives-on-teams problem. Clearly, there are some types of tasks that are inappropriate for teams, such as those that require sophisticated use of highly specialized individual knowledge or expertise, or creative composition that requires surfacing, organizing, and combining into an original whole ideas or images that initially are only partially formulated. Such work is inherently more suitable for individual than for collective performance and thus one can think in these types of tasks here. In particular, because of the tasks here require specific knowledge.

In Appendix D, I will analyze the case in which there are task-specific time measures, and then the case in which there are task-specific performance measures.

The firm uses the performance measure ( $P$ ) as well as the aggregated time measure ( $T$ ) in a linear incentive contract of the form

$$w_i = \alpha_i + \beta_i P(t_i, x_i, \mu) + \gamma_i T(t_i, x_i).$$

A contract  $C_i$  therefore consists of a fixed wage  $\alpha_i \in \mathfrak{R}_+$ , a pay-for-performance sensitivity  $\beta_i \in \mathfrak{R}_+$  and a time-wage  $\gamma_i \in \mathfrak{R}_+$ . The use of linear incentive contracts greatly simplifies the analysis, facilitates the interpretation of the results, allows simple comparative statics with respect to the main parameters and facilitates the comparison with competing theories.<sup>11</sup>

Observe that the performance and time measure are such that the marginal impact of the time allocated by any given worker to any given task is independent of the time allocated by the same or other worker to a different task. Note also that the performance and time measure are job dependant, while output is not. This is done to avoid the incentives-in-teams problem and to focus only individual incentives.

Furthermore, as in Baker (1992) and Baker et al. (1994), I assume that  $\mathbb{E}(\mu_j) = \mathbb{E}(\omega_j) \forall j$ . This implies that the marginal performance sensitivity of time in task  $j$  is an unbiased estimator of the marginal product

<sup>10</sup>Baker (1992) also allows the firm to contract on effort, but he focuses on the one task case and thus the non-contractibility of effort allocation across tasks is not an issue in his paper.

<sup>11</sup>The linear contract is optimal if  $T$  is not available and  $P$  is interpreted as the probability of success, and the agent is paid a positive bonus when success takes place and nothing otherwise.

of time in task  $j$ . This is required because the piece rate is paid for something other than what the principal cares about. Thus, to interpret the magnitude of  $\beta$ , it is useful to scale  $P$  so that the piece rate can be compared to one. Such scaling involves multiplying the performance measure by the average dollar value of an incremental unit of this performance measure. In this way, the average value of an additional unit of the scaled performance measure is 1.

It will prove useful in the forthcoming analysis to define the following known statistics: the average of the expected performance sensitivities in job  $x_i$  as  $\mathbb{E}\mu(x_i) \equiv \sum_j x_{ij} \mathbb{E}(\mu_j) / \sum_j x_{ij}$ , and the variance of the expected performance sensitivities in job  $x_i$  as  $\sigma_\mu^2(x_i) \equiv \sum_j x_{ij} (\mathbb{E}(\mu_j)^2 - \mathbb{E}(\mu(x_i))^2) / \sum_j x_{ij}$ .

Let  $\rho_j$  be the correlation between  $\omega_j$  and  $\mu_j$ . Then, I will assume the following

**A1.**  $\rho_j \sigma_{\omega_j} / \sigma_{\mu_j} \leq 1, \forall j = 1, \dots, n$ .

This ensures that regardless of the job design chosen, performance- and time-based pay are non-negative.

**Workers' Information.** After workers have signed contracts, but before they choose their time allocation, they learn  $\mu_j$  with probability  $p$  and learn nothing with probability  $1 - p$ .<sup>12</sup> Thus,  $p$  captures the quality of the information technology available to the firm. The principal knows the distribution of  $\omega_j$  and  $\mu_j$ , but not their realizations. Thus, the firm does not know what the worker is going to do when faced with a particular incentive contract; nor would it know whether a worker's time allocation were optimal if it could observe it. Thus, forcing contracts in this setting are not efficient.

**Timing.** The precise timing is as follows. Stage 1, the firm chooses a job design  $x_i \in X$  and then offers each worker  $i$  a contract  $C_i$ . Then, each worker accepts or rejects. At Stage 2, each worker privately learns  $\mu$  with probability  $p$  and learns nothing otherwise. Then each worker chooses his time allocation. After that, the performance measure  $P$ , the time measure  $T$  and the output  $Y$  are realized, and compensation takes place according to contracts  $C$ .

### 3 Contractible Output

As a benchmark, let's consider the case in which output is contractible and each worker learns the marginal product of time  $\omega$  with probability  $p$ . Yet, they cannot communicate that information to the firm.

#### 3.1 Time Allocations

Given an arbitrary contract  $C_i$ , worker  $i$  earns an expected utility

$$\max_{t_i \in \mathbb{R}_+^n} \left\{ \alpha_i + \beta_i \sum_{j=1}^n \tilde{\omega}_j x_{ij} t_j + \gamma_i \sum_{j=1}^n x_{ij} t_j - \frac{1}{2} \sum_{j=1}^n x_{ij} t_j^2 \right\},$$

<sup>12</sup>I could also assume that worker  $i$  learns  $\omega_j$  with probability  $p$  and nothing with probability  $1 - p$  and the results will remain identical.

where  $\tilde{\omega}_j = \omega_j$  when the worker is informed and  $\tilde{\mu}_j = \mathbb{E}(\omega_j)$  otherwise.

The first-order conditions are:

$$t_j : x_{ij}(\beta_i \tilde{\omega}_j + \gamma_i - t_j) \leq 0, \quad x_{ij} t_j (\beta_i \tilde{\omega}_j + \gamma_i - t_j) = 0 \text{ and } x_{ij} t_j \geq 0.$$

It is easy to check that the objective function is strictly concave in  $t_i$  and thus the first-order conditions are necessary and sufficient. Thus, a standard analysis of the first-order conditions result in that worker  $i$ 's time allocation in task  $j$  is given by:

$$t_j^*(C_i, x_i) = x_{ij}(\beta_i \tilde{\omega}_j + \gamma_i).$$

Thus, when output is contractible the time allocation requires worker  $i$  to assign more time to tasks with greater realized and average marginal product of time, and the time allocated to any given task is independent of the marginal product of time and the marginal cost of time in other tasks. This is due to the fact that tasks are technologically and informationally independent and workers see them also as independents. Furthermore, the optimal time in each task rises as time- and performance-based pay rises

### 3.2 Optimal Contracts

Given a job design  $x \in X$ , the firm chooses compensation contracts  $C_i$  for  $i = 1, \dots, N$  to solve the following problem

$$\begin{aligned} & \max_{C_i \in \mathfrak{R}, t \in \mathfrak{R}_+^n} \mathbb{E} \sum_{i=1}^N \left\{ \sum_{j=1}^n \omega_j x_{ij} t_j - \alpha_i - \beta_i \sum_{j=1}^n \omega_j x_{ij} t_j - \gamma_i \sum_{j=1}^n x_{ij} t_j \right\} \\ & \text{subject to} \\ & t_i \in \arg \max_{t_i \in \mathfrak{R}_+^{x_i}} \left\{ \alpha_i + \beta_i \sum_{j=1}^n \omega_j x_{ij} t_j + \gamma_i \sum_{j=1}^n x_{ij} t_j - \frac{1}{2} \sum_{j=1}^n x_{ij} t_j^2 \right\}, \quad \forall i, \\ & \mathbb{E} \left\{ \alpha_i + \beta_i \sum_{j=1}^n \omega_j x_{ij} t_j + \gamma_i \sum_{j=1}^n x_{ij} t_j - \frac{1}{2} \sum_{j=1}^n x_{ij} t_j^2 \right\} \geq 0, \quad \forall i, \\ & \alpha_i \geq L, \beta_i \geq 0 \text{ and } \gamma_i \geq 0, \quad \forall i, \end{aligned}$$

where the first constraint is worker  $i$ 's incentive compatibility constraint and the second is his participation constraint. For each worker, the incentive-compatibility constraint can be simplified to:  $t_j^*(C_i, x_i) = x_{ij}(\beta_i \tilde{\omega}_j + \gamma_i)$ , and the limited-liability constraint implies that the participation constraint is satisfied and thus ignored.

Because the problem is strictly convex, the first-order conditions are necessary and sufficient and as

follows

$$\beta_i : \sum_{j=1}^n x_{ij} \left( p\mathbb{E}(\omega_j^2) + (1-p)\mathbb{E}(\omega_j)^2 \right) (1-2\beta_i) - 2\gamma_i \sum_{j=1}^n x_{ij} \mathbb{E}(\omega_j) \leq 0,$$

and

$$\gamma_i : \sum_{j=1}^n x_{ij} \mathbb{E}(\omega_j) - 2 \sum_{j=1}^n x_{ij} (\beta_i \mathbb{E}(\omega_j) + \gamma_i) \leq 0.$$

Then, solving the first-order conditions for  $\beta_i$  and  $\gamma_i$  leads to the following result.

**Lemma 1.** *Suppose job design  $x \in X$  is chosen. Then: (i) worker  $i$ 's optimal contract entails a fixed wage  $\alpha^*(x_i) = L$ , a pay-for-performance sensitivity  $\beta^*(x_i) = \frac{1}{2}$  and a time wage  $\gamma^*(x_i) = 0$ . Worker  $i$ 's expected compensation is*

$$w^*(x_i) \equiv I(x_i)L + \frac{1}{4} \sum_{j=1}^n x_{ij} \left( p\sigma_{\omega_j}^2 + \mathbb{E}(\omega_j)^2 \right), \quad (1)$$

where  $I(x_i)$  is an indicator function equal to 1 when  $\sum_{j=1}^n x_{ij} \geq 1$  and equal to zero otherwise. (ii) Profits are

$$\Pi^*(x, p) \equiv \frac{1}{4} \sum_{j=1}^n \left( p\sigma_{\omega_j}^2 + \mathbb{E}(\omega_j)^2 \right) - \sum_{i=1}^N I(x_i)L. \quad (2)$$

Observe that total compensation and profits rise with technological uncertainty in each task (i.e., with the variance of  $\omega_j$ ). As in Baker (1992),  $\sigma_{\omega_j}^2$  is a measure of the amount of valuable information that the worker in charge of task  $j$  possesses when informed. When  $\sigma_{\omega_j}^2$  is low, the marginal product of the worker's time does not vary much in different states of the world. This means that the worker has little information that the principal does not have that would allow him to choose a more adequate time allocation. Alternatively, when  $\sigma_{\omega_j}^2$  is high, the worker is able to modify his time allocation significantly in response to his private information and thus producing more valuable outcomes. Because the worker is not always informed and information is valuable, profits also increase with the probability to be informed.

### 3.3 Job Design

The optimal job design entails to maximize profits by allocating the  $n$  tasks across different jobs. The optimal job design solves the following problem

$$\max_{x \in X} \Pi^*(x, p). \quad (3)$$

Let  $x^*$  be the job design when output is contractible. Then, the following readily follows from equations (2) and (3).

**Proposition 1.** *The optimal job design when output is contractible entails  $x_{ij}^* = 1, \forall j = 1, \dots, n$  and some  $i = 1 \dots, N$  and  $x_{i'j}^* = 0, \forall i' \neq i, \forall j = 1, \dots, n$ .<sup>13</sup>*

This says when output is contractible and there are neither technological nor time cost spillovers across tasks, the optimal job design allocates all tasks to one job. The reason is that under limited liability and no risk, the profit-maximizing time allocation can be implemented at no extra cost and wage costs are minimized since the full multi-tasking job saves the fixed cost per worker arising from the limited liability  $L$ . Hence, if the firm has strict preferences for a job design different from the full multi-tasking job when output is non-contractible, then the result must be due to incentive considerations only.

## 4 Non-contractible Output

In this section, I derive the optimal contract as well as job design when neither output nor the time allocation can be contracted on.

### 4.1 The Optimal Time Allocation

Given a contract  $C_i$  and a job  $x_i$ , worker  $i$  chooses the time allocation that solves the following problem

$$\max_{t_i \in \mathbb{R}_+^n} U(C_i, t_i, x_i),$$

where

$$U(C_i, t_i, x_i) \equiv \alpha_i + \beta_i \sum_{j=1}^n \tilde{\mu}_j x_{ij} t_j + \gamma_i \sum_{j=1}^n x_{ij} t_j - \frac{1}{2} \sum_{j=1}^n x_{ij} t_j^2,$$

$\tilde{\mu}_j = \mu_j$  when worker  $i$  is informed and  $\tilde{\mu}_j = \mathbb{E}(\mu_j)$  otherwise.

The first-order conditions are:

$$t_j : x_j(\beta_i \tilde{\mu}_j + \gamma_i - t_j) \leq 0, \quad x_{ij} t_j (\beta_i \tilde{\mu}_j + \gamma_i - t_j) = 0 \text{ and } x_{ij} t_j \geq 0.$$

It is easy to check that the objective function is strictly concave in  $t_i$  and thus the first-order conditions are necessary and sufficient. Thus, a standard analysis of the first-order conditions result in that worker  $i$ 's time allocation in task  $j$  is given by:

$$t_j(C_i, x_i) = x_{ij}(\beta_i \tilde{\mu}_j + \gamma_i).$$

Worker  $i$  allocates more time to tasks with greater realized and average task sensitivity (i.e., tasks where  $\tilde{\mu}_j$  is high), and the amount of time allocated to any given task rises with both time- and performance-based

<sup>13</sup>I am assuming that when the firm is indifferent between splitting tasks among different jobs and bundling all of them together in one job, the firm chooses the latter. This occurs only when  $L = 0$ .



pay.

As performance-based pay rises, workers become more responsive to their private information, while as time-based pay increases, workers become less responsive to it. This suggests that when the performance measure  $P$  is highly correlated with true output, increasing performance-based pay induces workers to allocate, on average, more time to the critical tasks (i.e., tasks with greater realized marginal product of time).

## 4.2 Optimal Contracts

Given a job design  $x \in X$ , the firm chooses compensation contracts  $C_i$  for  $i = 1, \dots, N$  to solve the following problem

$$\begin{aligned} & \max_{C_i, t_i \in \mathfrak{R}_+^n} \mathbb{E} \sum_{i=1}^N \left\{ \sum_{j=1}^n \omega_j x_{ij} t_j - \alpha_i - \beta_i \sum_{j=1}^n \mu_j x_{ij} t_j - \gamma_i \sum_{j=1}^n x_{ij} t_j \right\} \\ & \text{subject to} \\ & t_i \in \arg \max_{t_i \in \mathfrak{R}_+^n} \left\{ \alpha_i + \beta_i \sum_{j=1}^n \mu_j x_{ij} t_j + \gamma_i \sum_{j=1}^n x_{ij} t_j - \frac{1}{2} \sum_{j=1}^n x_{ij} t_j^2 \right\}, \quad \forall i, \\ & \mathbb{E} \left\{ \alpha_i + \beta_i \sum_{j=1}^n \mu_j x_{ij} t_j + \gamma_i \sum_{j=1}^n x_{ij} t_j - \frac{1}{2} \sum_{j=1}^n x_{ij} t_j^2 \right\} \geq 0, \quad \forall i, \\ & \alpha_i \geq L, \beta_i \geq 0 \text{ and } \gamma_i \geq 0, \quad \forall i, \end{aligned}$$

where the first constraint is worker  $i$ 's incentive compatibility constraint and the second is his participation constraint. For each worker, the incentive-compatibility constraint can be simplified to:  $t_j(C_i, x) = x_{ij}(\beta_i \tilde{\mu}_j + \gamma_i)$  and the limited-liability constraint ensures that the participation constraint is satisfied and therefore ignored.

Because, the problem is strictly convex, the first-order conditions are necessary and sufficient and as follows

$$\begin{aligned} \beta_i : & \sum_{j=1}^n x_{ij} \left( p \mathbb{E}(\omega_j \mu_j) + (1-p) \mathbb{E}(\omega_j) \mathbb{E}(\mu_j) \right) - \\ & 2 \sum_{j=1}^n x_{ij} \left( \beta_i (p \mathbb{E}(\mu_j^2) + (1-p) \mathbb{E}(\mu_j)^2) + \gamma_i \mathbb{E}(\mu_j) \right) \leq 0, \end{aligned}$$

and

$$\gamma_i : \sum_{j=1}^n x_{ij} \mathbb{E}(\omega_j) - 2 \sum_{j=1}^n x_{ij} (\beta_i \mathbb{E}(\mu_j) + \gamma_i) \leq 0.$$

Then, using the fact that  $\mathbb{E}(\mu_j) = \mathbb{E}(\omega_j)$  and solving the first-order conditions for  $\beta_i$  and  $\gamma_i$ , the follow-

ing result is obtained.

**Lemma 2.** *Suppose job design  $x \in X$  is chosen. Then: (i) Worker  $i$ 's optimal contract entails a fixed wage  $\alpha(x_i) = L$ , a pay-for-performance sensitivity*

$$\beta(x_i) = \frac{1}{2} \frac{\sum_{j=1}^n x_{ij} (p\sigma_{\mu_j} \omega_j + \sigma_{\mu}^2(x_i))}{\sum_{j=1}^n x_{ij} (p\sigma_{\mu_j}^2 + \sigma_{\mu}^2(x_i))}, \quad (4)$$

and a time wage

$$\gamma(x_i) = \frac{1}{2} (1 - 2\beta(x_i)) \mathbb{E}\mu(x_i). \quad (5)$$

Worker  $i$ 's expected compensation is

$$w(x_i) \equiv I(x_i)L + \frac{1}{4} (\mathbb{E}\mu(x_i))^2 \sum_{j=1}^n x_{ij} + \beta(x_i)^2 \sum_{j=1}^n x_{ij} (p\sigma_{\mu_j}^2 + \sigma_{\mu}^2(x_i)). \quad (6)$$

(ii) Worker  $i$ 's total expected time is

$$\mathbb{E} \sum_{j=1}^n x_{ij} t_j(C_i, x_i) = \frac{1}{2} \sum_{j=1}^n x_{ij} \mathbb{E}(\mu_j).$$

(iii) Profits are

$$\Pi(x, p) \equiv \frac{1}{4} \sum_{i=1}^N (\mathbb{E}\mu(x_i))^2 \sum_{j=1}^n x_{ij} + \sum_{i=1}^N \beta(x_i)^2 \sum_{j=1}^n x_{ij} (p\sigma_{\mu_j}^2 + \sigma_{\mu}^2(x_i)) - \sum_{i=1}^N I(x_i)L. \quad (7)$$

When a worker is assigned to a single-task job, the optimal pay-for-performance sensitivity is  $\beta(x_i) = \rho_j \sigma_{\omega_j} / 2\sigma_{\mu_j}$ , which is identical to that derived by Baker (1992) when the firm can contract on time at no cost. As in Baker (1992), the firm wants the worker to use his private information, but not as much as it would be optimal when the worker can be compensated according to true output ( $\beta^*(x_i) = 1/2$ ). The reason is that there could be states in which the marginal product of time is very different from the marginal performance sensitivity of time in the corresponding task and therefore the worker will allocate too much or too little time to the task relative to the efficient amount. Because the worker's dis-utility of time is strictly convex, choosing the wrong amount of time is costly for the firm. In response to this cost, the firm lowers the power of performance pay and rises that of time-based pay. The former reduces workers' incentives to use their information and to allocate time to tasks, and the latter provides workers with incentives to allocate time to tasks.

When a job entails more than one task, performance-based pay performs two functions: (i) as in the one task case, it provides workers with incentives to use their private information with respect to  $\mu$ ; and (ii) it induces workers to take into account the fact that there is ex-ante heterogeneity on the expected marginal product of time across tasks. In a single-task job, performance-based pay does not perform the second

function since this is done by mean of properly adjusting time-based pay. This explains the term  $\sigma_\mu^2(x_i)$  in the optimal pay-for-performance sensitivity.<sup>14</sup> Thus, the role of time-based pay is to ensure that workers allocate positive time to tasks and that of performance-based pay is to customize workers' time allocation to the ex-ante and ex-post heterogeneity in the marginal product of time.

Because on average tasks with higher ex-post performance sensitivity are not necessarily tasks with higher ex-post productivity, workers use their private information to allocate their time in order to increase their payoffs from performance-based pay without increasing total output as much. This induces the firm to provide incentives using both performance- and time-based pay: time-based pay avoids highly unequal (across tasks) time allocations, but fails to induce workers to take into account task heterogeneity, while performance-based pay induces workers to consider ex- and ex-post task heterogeneity, but it results, on average, on costlier time allocations due to cost convexity.

Finally, observe that bundling more tasks into a job not only affect the time allocation across tasks, but also the total time. In particular, bundling more tasks into a job not only results in a change in the allocation of time to tasks, but also in an increase in the total expected time that a worker devotes to his job. However, the total expected time that worker  $i$  spends on a job with  $K \geq 2$  tasks is the same as the total expected time that  $K$  identical workers spend on  $K$  single-task jobs each entailing one of the  $K$  tasks bundled into worker  $i$ 's job. Thus, the choice of a given job design will be determined by how the number of tasks bundled into a job changes the optimal time allocation across tasks rather than how that affects the total time allocated to same tasks.

### 4.3 The Second-Best Job Design

The firm chooses job design  $x \in X$  that maximizes profits given in equation (7). The first term in this equation can be written as the return to the workers' time when they ignore their private information (i.e.,  $\frac{1}{4} \sum_{j=1}^n x_{ij} \mathbb{E}(\omega_j)^2$ ) minus the extra compensation that must be given to the worker, due to cost convexity, in order to induce them to customize their time allocation to the ex-ante task heterogeneity (i.e.,  $\frac{1}{4} \sum_{j=1}^n x_{ij} \sigma_\mu^2(x_i)$ ). The second term is the return from inducing workers to use their private information. Observe that if  $\mu$  and  $\omega$  are perfectly correlated, the optimal performance pay sensitivity is equal to 1/2 and thus profits are identical to that when output is contractible. Thus, key to understand the benefits from bundling heterogeneous tasks into a job is to understand the behavior of  $\beta(x_i)$  with respect to the number of tasks.

In order to do so, it is useful to re-write the optimal pay-for-performance sensitivity as a linear combination of the the performance-pay sensitivity in each single-task job entailing one of the task in the multi-task

---

<sup>14</sup>The optimal contract derived in Baker (1992) under the assumption that time is contractible corresponds to a special case of the more general model proposed here: either corresponds to the single-task job case or to the case in which there are  $n$  identical tasks.

job and the optimal pay-for-performance when output is contractible; that is,

$$\beta(x_i) = \sum_{j=1}^n \psi_j(x_i) \beta(x_{ij}) + \psi_{n+1}(x_i) \beta^*(x_i),$$

where  $\beta(x_{ij}) \equiv \frac{1}{2} \rho_j \frac{\sigma_{\omega_j}}{\sigma_{\mu_j}}$  is the performance-pay sensitivity in the single-task job entailing task  $j$ ,  $\psi_j(x_i) \equiv \frac{x_{ij} p \sigma_{\mu_j}^2}{\sum_{j=1}^n x_{ij} (p \sigma_{\mu_j}^2 + \sigma_{\mu}^2(x_i))} \in [0, 1]$  and  $\psi_{n+1}(x_i) \equiv \frac{\sum_{j=1}^n x_{ij} \sigma_{\mu}^2(x_i)}{\sum_{j=1}^n x_{ij} (p \sigma_{\mu_j}^2 + \sigma_{\mu}^2(x_i))} \in [0, 1]$ .<sup>15</sup>

Holding  $\sigma_{\mu}^2(x_i)$  constant,  $\beta(x_i)$  decreases when a task with lower-than-average  $\beta(x_{ij})$  is bundled into the job and rises when a task with higher-than-average  $\beta(x_{ij})$  is bundled into the job. This implies that as tasks with lower-than-average performance-pay sensitivity are bundled into the job, workers' time allocation, on average, is more evenly distributed across tasks. On the one hand, this implies lower profits from worker  $i$  since there are more states in which worker  $i$ 's time allocation differs from the time allocation that maximizes firm value. On the other hand, due to cost convexity, total expected time costs per-task are lower.

When tasks differ not only on their performance uncertainty, but also in terms of their expected performance sensitivity there is a reinforcing effect at play. The firm wants to induce workers to take into account not only their private information, but also the difference in expected marginal product of time across tasks. In a multi-tasking job, this cannot be done by adjusting time-based pay, since this treats all tasks alike, and thus it has to be done by increasing performance-based pay. Because of a convex time cost function, this will be costly and thus will result in a higher expected compensation. However, from the firm's point of view what matters is how big is the limited-liability rent that must be given to a worker in a multi-task job entailing  $K \geq 2$  tasks relative to the sum of the limited-liability rents that must be given to  $K$  different workers assigned to the same  $K$  tasks. Observe that

**Lemma 3.** *For any  $L \geq 0$ , the limited-liability rent of allocating any  $K$  tasks to worker  $i$  is lower than the sum of the limited-liability rents of allocating the  $K$  tasks to  $K$  workers.*

Thus, similarly to Laux (2001), a multi-task job with  $K$  tasks results in a lower limited-liability rent that having  $K$  different specialized jobs each one entailing one of the  $K$  tasks. In this sense incentives problems are a natural source of economies of scope in terms of wage costs. The reason is different from that in Laux. Here this is due to the fact that costs are convex and tasks are heterogeneous, while in Laux, this is due to the fact that the optimal contract makes workers to behave as if tasks were complements.<sup>16</sup> However, in contrast to Laux (2001), different task allocations result in different levels of output.<sup>17</sup> In particular, one can show the following.

<sup>15</sup>Note that  $\beta(x_{ij})$  is the coefficient that arises from regressing  $\mu_j$  with  $\omega_j$  with an intercept. The reason for the existence of the intercept is that the firm can "mean adjust" the regression line since when the mean is the same in each task, contracting on total effort is as if the firm knows the expected value of  $P$  for a given time allocation.

<sup>16</sup>The optimal contract pays a bonus only when the  $n$  tasks are successful and nothing otherwise.

<sup>17</sup>In Laux (2001), the output does not change since effort is a 0-1 decision and he focuses on the case in which the firm chooses to implement an effort equal to 1 in each task regardless of the task allocation chosen.

**Lemma 4.** *The output of a multi-task job with  $K$  tasks is lower than the sum of the outputs of  $K$  different jobs each comprising one of the  $K$  tasks.*

It readily follows from lemmas 3 and 4 that when choosing how many tasks to bundle into each job, the firm trades-off a lower limited-liability rent against a lower output. When the latter dominates the former, full specialization is preferred to multi-tasking jobs. This is shown in the proposition.

**Proposition 2.** *Suppose workers are compensated on total time and on an aggregated performance measure. Then (i) there exists a limited liability threshold, denoted by  $\hat{L} > 0$ , such that the second-best optimal job design entails only full specialized jobs for all  $L \leq \hat{L}$  and it entails at least one multi-tasking job otherwise; and (ii) the second-best total surplus is lower than the first-best total surplus as long as  $\rho_j < 1$  for some  $j = 1, \dots, n$ .*

In order to maximize profits the firm has to design jobs so that workers' time allocation becomes, on average, as close as possible to their time allocation when output is contractible. This can be done either by bundling many tasks into a job and lowering the limited-liability rent and output or by splitting tasks in different jobs and achieving exactly the opposite. This result shows that narrowing down the scope of the job is the most profitable way when the limited liability is sufficiently small. The reason is that this allows the firm to better exploit workers' private information without incurring in much higher compensation costs. More specifically, the firm makes workers more responsive to their private information in those tasks in which the performance measure is highly correlated to the worker's contribution to firm value and less responsive to that in tasks with low correlation. When the limited liability  $L$  is large, the gain due to the fact that specialization improves the time allocation across tasks cannot compensate for the higher compensation costs that result from splitting a job between several workers. Thus, job design entails multi-tasking when  $L$  is large. However, there would be less multi-tasking than it is optimal when output is contractible. Thus, regardless of the size of  $L$ , incentives are a natural source of dis-economies of scope in firms. In fact, when  $L = 0$ , the firm chooses a job design entailing  $n$  single-task jobs.

There are several remarks worth doing here.

First, this result highlights the benefits of specialization without resorting to task substitutability (complementarity), risk-aversion, conflicting tasks or career concerns, which are the frictions that have led other researches to get rationales for specialization in multi-tasking frameworks. Specialization is mainly the outcome of three things: task heterogeneity, non-contractibility of output and cost convexity.

Second, I conjecture that if there are less workers than tasks  $N < n$ , then the optimal job design would entail to group tasks according to their similarity with respect to the  $\beta(x_i)$ s and expected marginal performance sensitivity. Due to cost convexity, this will minimize time costs in each job. In fact, it is simple to show that if there are two types of tasks and two workers, the optimal is to allocate all tasks of one kind to one worker and all the rest to the other. Furthermore, when  $L$  is sufficiently large, the optimal job design is a combinatorial problem that it is hard to solve in a closed form solution. I conjecture that the optimal job

design is to bundle the tasks that have the  $\beta(x_i)$ s closer to each other, since this will minimize the variance of the time allocation, while the marginal cost of saving one worker remains constant. This can be easily proven for the case in which tasks are identical in every dimension, but the correlation coefficient.

Third, it is easy to show that when contracting on total time is not possible, adding tasks to the job not only changes the time allocation across tasks, but also changes the total time worked. In particular, adding a task with an above-average correlation coefficient rises total effort, while adding a task with a below-average coefficient reduces it. Thus, the consequence of adding more tasks to a job would be more drastic here than in the standard multi-task problem studied by Holmström and Milgrom (1991) since it is not only a matter of effort substitution, but also of a reduction in total effort relative to the case in which the job is split in single-task jobs each embodying one of the task previously bundled into the original job. In addition, the result in the proposition above holds as it is.<sup>18</sup>

Fourth, in the appendix I show that this result is robust to the use of either task-specific performance measures or task-specific time measures. In fact, as long as either the time or the performance measure is aggregated at the job level, the firm is always better-off choosing more specialization than it is optimal when output is contractible.

## 5 Intrinsic Motivation: Moral Standards

The idea that monetary rewards induce workers to work more and harder is uncontroversial among economists, and economic theory has been able to provide powerful theoretical tools that help us to predict, in a manner consistent with reality, how people respond to monetary rewards. However, we have neglected some other important aspects of human behavior and human interaction and this impairs our ability to understand how firms change when these human dimensions are considered. It is unquestionable that workers act in the interests of their employers for a myriad of reasons, some different from monetary incentives, such as the fact that workers care about what they do. The economics literature has focused largely on the efficiency gains that arise from workers sharing the preferences of their employers.<sup>19</sup> Here I consider one type of behavioral preferences that capture the idea that workers care about what they do, which is the existence of *moral standards* about doing good work and study how firms modify optimal contracts and job design when workers are concerned with them.

Smith argues in the *Theory of Moral Sentiments* that common moral standards arise on the basis of a market-like process (see, Ottensson (2002)) in a similar way that self-interest gives rise to trade of services and goods in the *Wealth of Nations*. He holds that individuals are motivated by the desire for mutual sympathy and it is desire that induce people to adopt general rules and to develop a conscience of what is right and what is wrong. Furthermore, he believes that this process is somewhat rational in the following sense:

<sup>18</sup>To prove this one needs to substitute the term  $\sigma_\mu(x_i)$  for  $\mathbb{E}(\omega_j)^2$  and repeat step-by-step the proofs of the lemmas above.

<sup>19</sup>See, for instance, Murdock (2002), Besley and Ghatak (2005), Prendergast (2007), Casadesus-Masanell (2004) and Delfgaauw and Dur (2008).

people, in order to facilitate their ability to predict what their own behavior should be (that is, what would enjoy mutual sympathy with others), learn to adopt an objective standpoint from which to judge their own behavior. Smith calls that standpoint the *impartial spectator*, and he thinks that people take the impartial spectator's judgements as the standard of morality. He asserts that it is the desire for mutual sympathy, together with the daily interactions people have with one another, what motivate people to find out and adopt common rules of behavior and judgement. It is then the desire for mutual sympathy among people belonging to an organization that I assume here it takes place and therefore gives rise to moral standards about what is good work.

In order to model moral standards in a parsimonious way, I assume that each worker is endowed with a behavioral preference function of the following form:

$$M(t_i, x_i) \equiv m \left( S - \frac{1}{2} \sum_{j=1}^n x_{ij} (\omega_j - t_j)^2 \right),$$

where  $S$  is the level of satisfaction achieved when complete mutual sympathy is achieved and  $m \geq 0$  is a parameter capturing the degree to which worker  $i$  cares about not achieving mutual sympathy and this is determined by an un-modeled human interaction process between the principal and workers. Thus, when  $m = 0$ , the principal-worker relationship is not guided by the desire of mutual sympathy and thus it is identical to the one studied in the previous section, and as  $m \rightarrow \infty$ , mutual sympathy becomes so important so that the worker will prefer to choose actions that ensure full sympathy.<sup>20</sup>

Observe that the existence of moral standards results, from workers' point of view, in an ideal allocation of time across tasks, and workers' utility falls as they deviate from this ideal time allocation. The extent to which this deviation harms a worker depends on the degree to which he cares about mutual sympathy  $m$ . As  $m$  rises, worker  $i$  responds more to variations in  $\omega_j$  and thus it seems natural to parameterize his concern for mutual sympathy by the intensity  $m$  because worker  $i$  tends either to care highly about the standard or to not be very responsive to it. Furthermore,  $m$  is a personal trait that cannot arise from, for instance, peer sanctioning or monitoring since workers' actions are unobservable. This parameter captures more closely that workers have moral sentiments such as guilt or shame or religious beliefs such as the Protestant work ethic. Furthermore, the extent to which workers engage on behaviors that are deemed morally incorrect (i.e., time allocations different from the ideal one) is determined in part by the degree people are bind by their moral sentiments.

It is also worthwhile to note I could also model the behavioral function with standards that are not perfectly correlated with the true state  $\omega_j$ . For instance, I could assume that the standard in each task  $j$  is given by a random variable  $\psi_j$  that workers learn with probability  $p$ . The results will be very similar to the

---

<sup>20</sup>I could have model standards as a feature of each job rather than each task; that is, the standard is concerned with the total time worked. However, this will link the marginal return to one task to the marginal return to other tasks. This relationship would be similar in spirit to assume spillovers across tasks, which is undesirable since the result would not be driven only by incentive considerations.

ones derived below as long as the correlation between  $\omega_j$  and  $\psi_j$  is positive, yet the mathematical expressions will be much more cumbersome and the main intuition for the results will be less straightforward.<sup>21</sup> Thus, for the sake of brevity and clarity, I have chosen to study the perfect correlation case (i.e.,  $\psi_j = \omega_j$ ).

## 5.1 Time Allocation

Given a contract  $C_i$  and a job  $x_i$ , worker  $i$  chooses the time allocation that solves the following problem

$$\max_{t_i \in \mathfrak{R}_+^n} U(w_i, t_i, m, x_i),$$

where

$$U(w_i, t_i, m, x_i) \equiv \alpha_i + \beta_i P(t_i, x_i, \tilde{\mu}) + \gamma_i T(t_i, x_i) + M(t_i, x_i) - \frac{1}{2} \sum_{j=1}^n x_{ij} t_j^2.$$

It is simple to show that worker  $i$  chooses a time allocation consistent with

$$t_j(x_i, m) = \frac{x_{ij}}{1+m} (\beta_i \tilde{\mu}_j + \gamma_i + m \tilde{\omega}_j).$$

A worker's benefit from increasing the time allocated to a task results from an increase in extrinsic rewards and, for  $t_j(x_i, m) < \tilde{\omega}_j$ , from a decrease in the utility loss because his time gets closer to the ideal. Hence, a worker is motivated by both, extrinsic rewards ( $\beta_i \tilde{\mu}_j + \gamma_i$ ) and intrinsic rewards ( $m \tilde{\omega}_j$ ).

For a given contract, the effect of increases in  $m$  on the time allocated to task  $j$  depends on the monetary incentives. When monetary incentives are strong relative to non-monetary incentives (i.e.,  $t_j(x_i, m) > \tilde{\omega}_j$ ), more morally inclined workers allocate less time to task  $j$  to avoid the utility loss from not meeting the standard, while the opposite happens when extrinsic incentives are such that  $t_j(x_i, m) < \tilde{\omega}_j$ . Thus, an increase in the degree to which a worker is concerned with moral standards creates a trade-off between extrinsic and intrinsic incentives. The consequences are not only that the total time worked changes, but also the allocation of time across tasks changes.

Observe that worker  $i$ 's participation constraint is given by:

$$U(w_i, t_i, m, x_i) \geq 0.$$

This suggests that having moral sentiments for not satisfying the standard, it is something that morally-driven workers might not like and therefore must be compensated for. In fact, the worker's participation constraint could be binding when  $S$  is sufficiently small. To avoid this then I will assume that  $S \geq \frac{1}{2} \frac{1}{1+m} \sum_{j=1}^n \left( (1+m(1-p)) \sigma_{\mu_j}^2 + \mathbb{E}(\omega_j^2) \right)$ . This ensures that workers' participation constraint is never binding, since  $S$  is greater than the maximum utility loss from choosing at time allocation different from the ideal one.

<sup>21</sup>The results for this case are available upon request.



## 5.2 The Optimal Contracts

The firm then chooses contracts so as to maximize

$$\max_{\{C_i\}_{i=1}^n \in \mathbb{R}_+^3} \mathbb{E} \sum_{i=1}^N \left( \sum_{j=1}^n \omega_j t_j(x_i, m) - \alpha_i - \beta_i \sum_{j=1}^n \mu_j t_j(x_i, m) - \gamma_i \sum_{j=1}^n t_j(x_i, m) \right).$$

Then, simple calculations lead to the following first-order conditions,

$$\begin{aligned} \beta_i : \quad & \frac{1}{1+m} \sum_{j=1}^n x_{ij} \left( p \mathbb{E}(\omega_j \mu_j) + (1-p) \mathbb{E}(\mu_j) \mathbb{E}(\omega_j) \right) - \\ & \frac{m}{1+m} \sum_{j=1}^n x_{ij} \left( p \mathbb{E}(\omega_j \mu_j) + (1-p) \mathbb{E}(\mu_j) \mathbb{E}(\omega_j) \right) - \\ & \frac{2}{1+m} \sum_{j=1}^n x_{ij} \left( \beta_i (p \mathbb{E}(\mu_j^2) + (1-p) \mathbb{E}(\mu_j)^2) + \gamma_i \mathbb{E}(\mu_j) \right) \leq 0, \end{aligned} \quad (8)$$

and

$$\gamma_i : \frac{1}{1+m} \sum_{j=1}^n x_{ij} \mathbb{E}(\omega_j) - \frac{m}{1+m} \sum_{j=1}^n x_{ij} \mathbb{E}(\mu_j) - \frac{2}{1+m} \sum_{j=1}^n x_{ij} (\beta_i \mathbb{E}(\mu_j) + \gamma_i) \leq 0. \quad (9)$$

The following result follows from limited liability, the first-order conditions in equations (8) and (9) and the fact that  $\mathbb{E}(\omega_j) = \mathbb{E}(\mu_j)$ .

**Proposition 3.** (i) if  $m \geq 1$ , then  $\alpha(x_i, m) = L$ ,  $\gamma(x_i, m) = 0$  and  $\beta(x_i, m) = 0$ ; and (ii) if  $m < 1$ , then  $\alpha(x_i, m) = L$ ,  $\beta(x_i, m) = (1-m)\beta(x_i)$  and  $\gamma(x_i, m) = (1-m)\gamma(x_i)$ ,

When the degree to which a worker is concerned with moral standards is low (i.e.,  $m < 1$ ), performance- and time-based pay are positive, while when that is high ( $m \geq 1$ ), time- and performance-based are set to zero. A worker extremely concerned with the standards allocates ex-post, from the firm's perspective, more time to tasks than it is optimal (i.e.,  $t_j > (1/2)\tilde{\omega}_j$ ). The firm would like to counterweigh the worker's incentive to overexert himself by offering negative extrinsic incentives, but limited liability precludes that. In contrast, when  $m$  is low, the firm still needs to provide the worker with extrinsic incentives in order to allocate more time to tasks and to do so in a way that considers the ex-ante and ex-post heterogeneity of the marginal product of time.

## 5.3 The Second-Best Job Design

Worker  $i$ 's total expected time is given by

$$\mathbb{E} \left( \sum_{j=1}^n x_{ij} t_j(x_i, m) \right) = \begin{cases} \frac{1}{2} \left( 1 + \frac{2m}{1+m} \beta(x_i, m) \right) \sum_{j=1}^n x_{ij} \mathbb{E}(\mu_j), & \text{if } m < 1 \\ \frac{m}{1+m} \sum_{j=1}^n x_{ij} \mathbb{E}(\mu_j), & \text{if } m \geq 1. \end{cases}$$

As in the previous case bundling more tasks into a job not only affects the time allocation across tasks, but also total time. In fact, a worker who is intrinsically motivated is induced to increase his total expected time compared to previous case in which intrinsic rewards play no role. Thus, it is no longer the case that the total time that worker  $i$  spends on a job with  $K \geq 2$  tasks is the same as the total time that  $K$  identical workers spend on  $K$  single-task jobs each comprising one of the  $K$  task bundled into worker  $i$ 's job. The reason is that  $\beta(x_i, m)$  varies in a non-linear way with the number of tasks.

Substituting the optimal contract into the objective function one gets that profits are

$$\Pi(x_i, m) \equiv \begin{cases} \frac{(1-m)^2}{1+m} \Pi(x_i) + \frac{m}{1+m} \sum_{j=1}^n x_{ij} (p\sigma_{\omega_j}^2 + \mathbb{E}(\omega_j)^2) & \text{if } m < 1, \\ \frac{m}{1+m} \sum_{j=1}^n x_{ij} (p\sigma_{\omega_j}^2 + \mathbb{E}(\omega_j)^2) & \text{if } m \geq 1. \end{cases}$$

It readily follows from the envelope theorem that

$$\frac{\partial \Pi(x_i, m)}{\partial m} = \frac{1}{(1+m)^2} \begin{cases} \sum_{j=1}^n x_{ij} (p\sigma_{\omega_j}^2 + \mathbb{E}(\omega_j)^2) - (1-m)(3+m)\Pi(x_i) & \text{if } m < 1, \\ \sum_{j=1}^n x_{ij} (p\sigma_{\omega_j}^2 + \mathbb{E}(\omega_j)^2) & \text{if } m \geq 1. \end{cases} \quad (10)$$

It follows from this that the effect of an increase in  $m$  on firm's profits depends mainly on the impact of that on time worked. The first-term arises because an increase in  $m$ , holding constant extrinsic incentives, induces workers to allocate more time to tasks since they place more value on the standards. This term is positive since it is the sum across tasks of the expected marginal return to time. The second term, which corresponds to firm's profits, captures the fact that an increase in  $m$ , holding constant the return to time from being concerned with the standards, results in a decrease in total time because of the worker's utility loss from choosing a time allocation different from the ideal time allocation. However, since  $\sum_{j=1}^n x_{ij} (p\sigma_{\omega_j}^2 + \mathbb{E}(\omega_j)^2) > 2\Pi(x_i)$ , it is very easy to check the following.

**Proposition 4.** *Profits rise with  $m$ .*

This shows that firms will benefit from developing a culture inside the firm to adhere to moral standards about doing good work. This suggests that firms will have incentives to spend time and money creating a corporate culture about doing work. By culture I mean shared assumptions and values that are the result of ongoing process of creating a common view of what good works means and that it is the result of frequent interactions between the management team and workers. Van den Steen (2010b) and Van den Steen (2010a) provides nice theoretical foundations for the emergence of corporate culture. He shows that culture is the result of two forces, shared belief that come from (often unintentionally) screening, self-sorting, and manager-directed joint learning and shared experience. This ideas provides a nice microfoundation for the parameter  $m$  here. For evidence in favor of a positive relationship between  $m$  and profits or other outcome measures correlated to profits see Podsakoff et al. (2009) and the discussion in section 7.

This leads to the following result

**Proposition 5.** *The threshold  $L(m)$  decreases with  $m$  for all  $m < 1$  and  $L(m)$  is independent of  $m$  for all  $m \geq 1$ .*

This proposition shows that having workers concerned with moral standards about doing work favors job designs that entail multi-tasking jobs. In fact, when workers' concern with mutual sympathy is very strong ( $m \geq 1$ ), the firm prefers a full multi-tasking to any other job design. The reason is that the firm sets extrinsic rewards equal to zero and thus all workers choose the same time allocation regardless of the job design. Since hiring more workers is costlier, the firm prefers to bundle all tasks into one job. While when workers' concern is lower than 1, multi-tasking becomes more attractive as  $m$  rises. The reason is that in a multi-task job an increase in  $m$  induces workers, on average, to increase the time allocated to tasks with greater expected product of time and to decrease the time allocated to tasks with lower expected product of time. This reallocation of time due to an increase in  $m$ , which does not take place in a single-task job, decrease the time allocation distortion or *gaming*. This implies that if there is a profit gain from an increase in  $m$ , it is greater for a multi-task job than a single-task job, while if there is profit loss from this, it is smaller in multi-task job than in a single-task job. Thus, moral standards do exactly what we expect them to do, decrease workers incentives to game the performance measure.

Note that  $m$  plays no role in terms of job design when output is contractible. The reason is that the firm will always choose a full multi-task job and an increase in  $m$  raises the profitability of a multi-task job relative to single-task jobs entailing the same tasks. Thus, the novelty of this paper is to show how intrinsic motivation changes optimal job designs when the firm's ability to contract is limited. In doing so, it highlights the fact that firms respond to non-contractibility restrictions by not only modifying optimal contracts, but also changing the bundling of tasks into jobs.

This suggests that firms that develop a culture of adherence to standards not only are more profitable, but also different from those that do not develop such culture. The former are more multi-tasking and offer less strong incentives than than the latter (see Helper et al. (2010)). In short, they look more like the so called modern firms.

## 6 Spillovers Across Tasks

Thus far, I have assumed that tasks are neither complements nor substitutes. This has been done to focus on the case in which a-priori there is no reason for the firm to choose either a fully specialized job design or a multi-task job design and to emphasize that neither spillovers nor learning nor conflicting tasks are needed in order for task-splitting to be optimal.

In this section, I will introduce spillovers across tasks in two different ways. First, by building them into the cost of time function to study the role that effort substitution plays here and how it compares to that in the Holmström and Milgrom (1991)'s linear agency model and, second by modifying the production technology.

## 6.1 The Return to Effort Substitution

Worker  $i$ 's cost of time function is now given by

$$\frac{1}{2} \frac{1}{1 + \eta(\sum_{j=1}^n x_{ij} - 1)} \sum_{j=1}^n \left( x_{ij} t_j^2 + \eta x_{ij} t_j \sum_{j' \neq j}^n x_{ij'} t_{j'} \right)$$

where  $\eta \in (-1, 1)$  and captures the extent to which the time spent in task  $j$  affects the marginal cost of the time spent in task  $j'$ . Scaling the dis-utility by  $1/(1 + \eta(\sum_{j=1}^n x_{ij} - 1))$  ensures that changes in  $\eta$  affect the time allocation across tasks, but not the level of dis-utility for an even allocation of time across tasks. When  $\eta > 0$ , the greater the time allocated to any given task, the greater the marginal cost of the time allocated to the remaining tasks, while when  $\eta \leq 0$ , the opposite holds. Furthermore, in order to ensure a positive time allocation for any job design, I will assume that  $\hat{\eta}_i \equiv 1 + \eta(\sum_{j=1}^n x_{ij} - 1) > 0$ .

Simple calculations lead to that the worker  $i$ 's time allocation in task  $j$  is given by

$$t_j(C_i, x_i, \eta) = x_{ij} \left( \beta_i \tilde{\mu}_j + \gamma_i + \beta_i \frac{\eta}{1 - \eta} \sum_{h=1}^n x_{ih} (\tilde{\mu}_j - \tilde{\mu}_h) \right).$$

Notice that when tasks are substitutes (i.e.,  $\eta > 0$ ), tasks with a greater-than-average observed performance sensitivity ( $\tilde{\mu}_j$ ) gets relatively more time allocated to them as  $\beta_i$  rises, while when they are complements, an increase in  $\beta_i$  induces worker  $i$  to allocate relatively less time to tasks with greater-than-average performance sensitivity. Thus, substitutability across tasks and performance-based pay are complementary instruments in terms of inducing a worker to focus more on tasks with greater performance sensitivity. Furthermore, as  $\eta$  rises, ceteris-paribus, workers allocate relatively more time to tasks with greater-than-average performance sensitivity and relatively less to tasks with lower-than-average performance sensitivity. When tasks are substitutes, ultimately workers do not care much about how they allocate their time and thus they focus more value on tasks with greater performance sensitivity, while when tasks are complements allocating too much time to one task and too little to the rest increases total time costs more than income.

In order to ensure that the firm's problem is strictly concave, I assume that the parameters are such that the following holds

- Assumption 2.  $(\hat{\eta}_i - \eta) \sum_{j=1}^n x_{ij} p \sigma_{\mu_j}^2 \geq \hat{\eta}_i \left( (1 - \eta) \mathbb{E} \mu(x_i) - \sigma_{\mu}^2(x_i) \right) \sum_{j=1}^n x_{ij}$

Given job design  $x \in X$ , the firm chooses compensation contracts to solve the following problem

$$\max_{\{C_i\}_{i=1}^n} \mathbb{E} \sum_{i=1}^n \left\{ \sum_{j=1}^n \omega_j t_j(C_i, x_i, \eta) - \alpha_i - \beta_i \sum_{j=1}^n \mu_j t_j(C_i, x_i, \eta) - \gamma_i \sum_{j=1}^n t_j(C_i, x_i, \eta) \right\}.$$

Because, the focus is on interior time allocations and the problem is strictly convex, the first-order

conditions are necessary and sufficient and as follows

$$\beta_i : \mathbb{E} \left\{ \sum_{j=1}^n x_{ij} (\omega_j - \beta_i \mu_j - \gamma_i) \frac{1}{1-\eta} (\hat{\eta}_i \tilde{\mu}_j - \eta \sum_{h=1}^n x_{ih} \tilde{\mu}_h) - \sum_{j=1}^n \mu_j t_j(C_i, x_i, \eta) \right\} \leq 0$$

and

$$\gamma_i : \sum_{j=1}^n x_{ij} (\mathbb{E}(\omega_j) - 2\beta_i \mathbb{E}(\mu_j) - 2\gamma_i) \leq 0.$$

Solving for  $\beta_i$  and  $\gamma_i$  from the first-order conditions, making use of the fact that random variables are independent across tasks (i.e.,  $\mathbb{E}(\mu_j \mu_h) = \mathbb{E}(\mu_j) \mathbb{E}(\mu_h)$  and  $\mathbb{E}(\omega_j \mu_h) = \mathbb{E}(\omega_j) \mathbb{E}(\mu_h)$ ),  $\mathbb{E}(\omega_j) = \mathbb{E}(\mu_j)$  and after a few steps of simple, but lengthy algebra, one can show that the optimal contracts entail a performance-based pay equal to

$$\beta(x_i, \eta) = \frac{1}{2} \frac{\hat{\eta}_i \sum_{j=1}^n x_{ij} (p \rho_j \sigma_{\mu_j} \sigma_{\omega_j} + \sigma_{\mu}^2(x_i)) - \eta \sum_{j=1}^n x_{ij} p \rho_j \sigma_{\mu_j} \sigma_{\omega_j}}{\hat{\eta}_i \sum_{j=1}^n x_{ij} (p \sigma_{\mu_j}^2 + \sigma_{\mu}^2(x_i)) - \eta \sum_{j=1}^n x_{ij} p \sigma_{\mu_j}^2} \quad (11)$$

and a time-based wage equal to

$$\gamma(x_i, \eta) = \frac{1}{2} (1 - 2\beta(x_i, \eta) \mathbb{E} \mu(x_i)). \quad (12)$$

The intuition for this result is the same as the previous one: the power of  $\beta(x_i, \eta)$  and  $\gamma(x_i, \eta)$  jointly determine worker  $i$ 's time allocation, whereas the relative weights on  $\beta(x_i, \eta)$  and  $\gamma(x_i, \eta)$  balance the dual goal of getting the worker to use his information and to work harder at the lowest cost possible. Thus, the main trade-offs in terms of incentives remain unchanged when the time allocated to any given task impacts the marginal cost of the time allocated to any other task. Nonetheless the impact of effort substitution on job design is not straightforward. In order to understand the role of  $\eta$ , it is necessary first to study the behavior of the optimal contract with respect to the degree to which tasks are substitutes ( $\eta$ ).

In order to do so, it is instructive to compare  $\beta(x_i, \eta)$  with  $\beta(x_i)$ . It is easy to show after a few steps of simple algebra that  $\beta(x_i, \eta) > \beta(x_i)$  if and only if  $\eta > 0$  and  $\beta(x_i, \eta) \leq \beta(x_i)$  otherwise. Thus, when task complementarity weakens performance-based incentives and strengthens time-based incentives, while task substitutability strengthens the first and weakens the second.

When tasks are substitutes (i.e.,  $\eta > 0$ ), a worker focuses relatively more on tasks with greater-than-average performance sensitivity and relatively less on tasks with lower-than-average performance sensitivity, while the opposite happens when tasks are complements. The fact that substitution across tasks induces workers to pay more attention to tasks with greater marginal performance sensitivity, increases the firm's marginal return to use performance-based pay (i.e., higher  $\beta_i$ ) since this reinforces the worker's focus on the more productive tasks and decreases the marginal return to use time-based pay.

Partially differentiating  $\beta(x_i, \eta)$  with respect to  $\eta$ , it is easy to show that performance-based incentives increase while time-based incentives decrease as  $\eta$  increases, and that  $\beta(x_i, \eta)$  is bounded above. This prediction is different from that arising from the effort-substitution model proposed by Holmström and Milgrom (1991). In their model when tasks are substitutes in the agent's utility, the interaction makes it necessary for the principal to reduce the power of performance pay on all outcomes, because sharpening the incentives on any one of them causes the agent to divert his time away from the others. Conversely, when tasks are complements, stronger incentives in one task cause the agent to increase the time in each task, so more powerful incentives on each tasks become optimal for the principal. This provides the basis for the prediction that the optimal job design should entail the bundling of mutually complementarity and similar (in terms of the quality of the performance measure) tasks from a worker's point of view. The difference stands for the fact that in the linear agency model at the time workers choose their time allocation, they have no extra information and they focus more on the tasks that ex-ante have a higher pay-for-performance sensitivity. In contrast, here workers time allocation depends not only on the ex-ante performance sensitivity, but also on the ex-post performance sensitivity. Since the firm does not have this information when choosing the optimal contract, the contract must be designed to take into account workers' incentives to disregard productive tasks and consider unproductive ones ex-post. When tasks are substitutes, ex-post the worker will focus more on tasks with greater marginal performance sensitivity regardless of the ex-ante performance sensitivity, and to the extent this is correlated with the marginal product of time, it is profitable to increase the power of performance pay.

In order to discuss the implications that the existence of spillovers across tasks have for job design, it is worthwhile to notice that the expected total surplus is

$$\begin{aligned} \Pi(x, p, \eta) &\equiv \frac{1}{4} \sum_{i=1}^N (\mathbb{E}\mu(x_i))^2 \sum_{j=1}^n x_{ij} + \\ &\sum_{i=1}^N \beta(x_i, \eta)^2 \left( \frac{\hat{\eta}_i}{1-\eta} \sum_{j=1}^n x_{ij} (p\sigma_{\mu_j}^2 + \sigma_{\mu}^2(x_i)) - \frac{\eta}{1-\eta} \sum_{j=1}^n x_{ij} p\sigma_{\mu_j}^2 \right). \end{aligned} \quad (13)$$

Because  $\eta$  has no impact on fully specialized jobs, complementarity will favor multi-tasking jobs when profits rise with the degree of substitution  $\eta$ . It readily follows from the envelope theorem that

$$\begin{aligned} \frac{\partial \Pi(x, p, \eta)}{\partial \eta} &= \frac{\beta(x_i, \eta)}{(1-\eta)^2} \left( \left( \sum_{j=1}^n x_{ij} - 1 \right) \sum_{j=1}^n x_{ij} (p\sigma_{\omega_j \mu_j} + \sigma_{\mu}^2(x_i)) - \sum_{j=1}^n x_{ij} p\sigma_{\omega_j \mu_j} - \right. \\ &\left. \beta(x_i, \eta) \left( \left( \sum_{j=1}^n x_{ij} - 1 \right) \sum_{j=1}^n x_{ij} (p\sigma_{\mu_j}^2 + \sigma_{\mu}^2(x_i)) - \sum_{j=1}^n x_{ij} p\sigma_{\mu_j}^2 \right) \right) \geq 0 \quad \forall \eta. \end{aligned}$$

To show that this is positive it is sufficient to note that  $\sigma_{\mu_j}^2 \geq \sigma_{\omega_j \mu_j}$ ,  $\forall j = 1, \dots, n$  and  $\hat{\eta}_i \geq \eta$ ,  $\forall \eta$ . This leads to the following result.

**Proposition 6.** *Suppose workers are compensated on total time and on an aggregated performance measure and time measure. Then (i) there exists a limited liability threshold, denoted by  $L(\eta) \geq 0$ , such that the second-best optimal job design entails only full specialized jobs for all  $L \leq L(\eta)$  and it entails multi-tasking jobs otherwise; (ii)  $L(\eta)$  falls monotonically with  $\eta$ ; and (iii) the second-best total surplus is lower than the first-best total surplus as long as  $\rho_j < 1$  for some  $j = 1, \dots, n$ .*

The predication here is exactly the opposite of that emanating from the linear agency model proposed by Holmström and Milgrom (1991). The intuition is as follows. Complementarities across tasks provide workers with incentives to disregard their private information, to the extent that workers' information is valuable for the firm, this decreases workers' contribution to firm value. Thus, the firm's expected return is lower when tasks are complements. When tasks are substitutes workers care less about in which tasks they spend more time working and thus they become more responsive to their private information. This increases firm value since workers' time allocation is guided more by the marginal product of time than their preferences over tasks. In a sense, substitution across tasks transforms the worker's multi-tasking problem in a sort of single-tasking problem since the worker focuses more on the task with greater observed performance sensitivity.

It is worthwhile to finish this sub-section by noticing that Holmström and Milgrom (1991) see Baker's model as a special case of their model (see, pag. 31, footnote 11) in the sense that for the one task case studied by Baker, the state-contingent strategy for the agent is equivalent to a vector effort strategy. For the multi-task case this mapping is not feasible. This explains, together with other things, why the prediction here is rather different from that in Holmström and Milgrom (1991)'s paper.

## 6.2 Technological Spillovers

Thus far I have looked at a production technology in which there are no spillovers across tasks. In the last sub-section, I consider the case in which spillovers are brought into the model through workers' cost of time function, while here I assume this effect away and bring that tension by mean of modifying the production function in the simplest way possible.

Let the technology be:

$$Y(t, \omega, \theta) = \frac{1}{1 + \theta(\sum_{j=1}^n -1)} \sum_{j=1}^n \left( \omega_j t_j + \theta t_j \sum_{h \neq j}^n t_h \right),$$

where  $\theta \in (-1, 1)$  and captures the extent to which the time spent in task  $j$  affects the marginal product of time of the time spent in task  $j'$ . Scaling the production technology by  $1/(1 + \theta(\sum_{j=1}^n -1))$  again ensures that changes in  $\theta$  affect the time allocation across tasks, but not the level of output for an even allocation of time across them. Furthermore, I assume that  $\hat{\theta}_i \equiv 1 + \theta(\sum_{j=1}^n x_{ij} - 1) > 0$ .

Because the performance measure is not affected by  $\theta$ , worker  $i$ 's effort is the same as in section 4. Thus,

when faced with contract  $C_i$  and allocated to job  $x_i$ , worker  $i$  chooses his time allocation according to:

$$t_j(C_i, x_i) = x_{ij}(\beta_i \tilde{\mu}_j + \gamma_i).$$

In order to ensure that the firm's problem is strictly concave, I assume that the parameters are such that the following holds

- Assumption 3.  $\left(1 + \frac{\theta}{\hat{\theta}_i}\right) \sum_{j=1}^n x_{ij} (p\sigma_{\mu_j}^2 + \mathbb{E}(\omega_j)^2) \sum_{j=1}^n x_{ij} \geq \left(\sum_{j=1}^n x_{ij} \mathbb{E}(\omega_j)\right)^2$ .

Given job design  $x \in X$ , the firm chooses compensation contracts to solve the following problem

$$\begin{aligned} \max_{\{C_i\}_{i=1}^n} \mathbb{E} \sum_{i=1}^n \left\{ \frac{1}{\hat{\theta}_i} \sum_{j=1}^n \left( \omega_j t_j(C_i, x_i) + \theta t_j(C_i, x_i) \sum_{h \neq j}^n t_h(C_i, x_i) \right) \right. \\ \left. - \alpha_i - \beta_i \sum_{j=1}^n \tilde{\mu}_j t_j(C_i, x_i) - \gamma_i \sum_{j=1}^n t_j(C_i, x_i) \right\}. \end{aligned}$$

The first-order conditions are as follows

$$\begin{aligned} \beta_i : \frac{1}{\hat{\theta}_i} \sum_{j=1}^n x_{ij} \left( p \mathbb{E}(\omega_j \mu_j) + (1-p) \mathbb{E}(\omega_j) \mathbb{E}(\mu_j) \right) + 2 \frac{\theta}{\hat{\theta}_i} \mathbb{E} \left( \sum_{j=1}^n x_{ij} (\beta_i \tilde{\mu}_j + \gamma_i) \sum_{h=1}^n x_{ih} \mu_h \right) - \\ 2 \left( 1 + \frac{\theta}{\hat{\theta}_i} \right) \sum_{j=1}^n x_{ij} \left( \beta_i (p \mathbb{E}(\mu_j^2) + (1-p) \mathbb{E}(\mu_j)^2) + \gamma_i \mathbb{E}(\mu_j) \right) \leq 0, \end{aligned}$$

and

$$\gamma_i : \sum_{j=1}^n x_{ij} \mathbb{E}(\omega_j) - 2 \sum_{j=1}^n x_{ij} (\beta_i \mathbb{E}(\mu_j) + \gamma_i) \leq 0.$$

Solving for  $\beta_i$  and  $\gamma_i$  from the first-order conditions, making use of the fact that random variables are independent across tasks (i.e.,  $\mathbb{E}(\mu_j \mu_h) = \mathbb{E}(\mu_j) \mathbb{E}(\mu_h)$  and  $\mathbb{E}(\omega_j \mu_h) = \mathbb{E}(\omega_j) \mathbb{E}(\mu_h)$ ),  $\mathbb{E}(\omega_j) = \mathbb{E}(\mu_j)$  and after a few steps of simple, but lengthy algebra, one can show that the optimal performance-based pay is equal to:

$$\beta(x_i, \theta) = \frac{1}{2 \hat{\theta}_i} \frac{\sum_{j=1}^n x_{ij} (p \rho_j \sigma_{\mu_j} \sigma_{\omega_j} + \sigma_{\mu}^2(x_i))}{\sum_{j=1}^n x_{ij} (p \sigma_{\mu_j}^2 + \sigma_{\mu}^2(x_i)) + \theta \sum_{j=1}^n x_{ij} \sigma_{\mu}^2(x_i)} \quad (14)$$

and a time-based wage

$$\gamma(x_i, \theta) = \frac{1}{2} (1 - 2\beta(x_i, \theta)) \mathbb{E}\mu(x_i). \quad (15)$$

The intuition for this result is the same as for the one in which tasks are independent with regard to the time costs: the power of  $\beta(x_i, \theta)$  and  $\gamma(x_i, \theta)$  jointly determine worker  $i$ 's time allocation, whereas the relative weights on  $\beta(x_i, \theta)$  and  $\gamma(x_i, \theta)$  balance the dual objectives of getting the worker to use his private information and work harder at the possible cost.



The more novel part arises from comparing comparing  $\beta(x_i, \theta)$  with  $\beta(x_i)$ . It is easy to see that  $\beta(x_i, \theta) > \beta(x_i)$  if and only if  $\theta < 0$  and  $\beta(x_i, \theta) \leq \beta(x_i)$  otherwise. Thus, when tasks are substitutes in the production function (i.e.,  $\theta < 0$ ), the power of performance-based pay is higher than that when tasks are independent, while when tasks are complements (i.e.,  $\theta > 0$ ), the opposite happens. Thus, task substitutability strengthens performance-based incentives and weakens time-based incentives, while task complementarity weakens the first and strengthens the second.

As the degree to which tasks are substitutes falls (i.e.,  $\theta$  rises), the marginal return to time in task  $j$  that comes from the direct marginal product of time  $\omega_j$  falls, while the marginal return to time that comes from the impact that time has on the other tasks rises. This implies that the firm's return, ceteris-paribus, from inducing the worker to focus more on tasks with greater ex-post performance sensitivity is smaller, while the return from choosing a more evenly (across tasks) distributed time allocation rises. In a sense, complementarity makes the firm to value less the worker's private information. Thus, as  $\theta$  rises, it is profitable for the firm to lower performance-based pay and rise time-based pay, since the former induces workers to disregard their private information and the latter induces workers to allocate more time to tasks regardless of the expected marginal product of time ( $\mathbb{E}(\omega_j)$ ).

In order to discuss the implications that the existence of spillovers across tasks have for job design, it is worthwhile to notice that the expected total surplus is

$$\Pi(x, p, \theta) \equiv \frac{1}{4\hat{\theta}_i} \sum_{i=1}^N (\mathbb{E}\mu(x_i))^2 \sum_{j=1}^n x_{ij} + \sum_{i=1}^N \beta(x_i, \theta)^2 \left( \sum_{j=1}^n x_{ij} (p\sigma_{\mu_j}^2 + \sigma_{\mu}^2(x_i)) + \frac{\theta}{\hat{\theta}_i} \sum_{j=1}^n \sigma_{\mu}^2(x_i) \right). \quad (16)$$

Because  $\theta$  has no impact on fully specialized jobs, task complementarity will favor multi-tasking jobs when profits rise with the degree of complementarity  $\theta$ . It readily follows from the envelope theorem that

$$\begin{aligned} \frac{\partial \Pi(x, p, \theta)}{\partial \theta} &= - \frac{\beta(x_i, \theta)}{\hat{\theta}_i^2} \left( \beta(x_i, \theta) \sum_{j=1}^n x_{ij} \sigma_{\mu}^2(x_i) + \sum_{j=1}^n x_{ij} (p\sigma_{\omega_j \mu_j} + \sigma_{\mu}^2(x_i)) \left( \sum_{j=1}^n x_{ij} - 1 \right) \right) - \\ &\quad \frac{1}{\hat{\theta}_i^2} (\mathbb{E}\mu(x_i))^2 \sum_{j=1}^n x_{ij} \leq 0, \quad \forall \theta_i. \end{aligned}$$

This leads to the following result.

**Proposition 7.** *Suppose workers are compensated on total time and on an aggregated performance and time measure. Then (i) there exists a limited liability threshold, denoted by  $L(\theta) \geq 0$ , such that the second-best optimal job design entails only full specialized jobs for all  $L \leq L(\theta)$  and it entails at least one multi-tasking job otherwise; (ii)  $L(\theta)$  rises monotonically with  $\theta$ ; and (iii) the second-best total surplus is lower than the first-best total surplus as long as  $\rho_j < 1$  for some  $j = 1, \dots, n$ .*

This result is consistent with the one find in the previous sub-section; that is, as the degree to which

tasks are substitutes increases, the firm is more likely to adopt an optimal job design that admits at least one multi-task job. The intuition is similar to the previous one. From the firm's point of view substitution makes a multi-task job to look more like a single-task job since it is better to induce workers to focus more on their private information.

Again this results is in stark contrast with one derived under the rationale of the linear agency-model with multiple tasks.

## 7 Predictions and Final Remarks

### 7.1 Predictions

**The Risk and Incentives Trade-off.** Since most of the debate in the incentive contracting literature revolves around the relationship between risk and incentives, it is worthwhile to study how these two things are related in the context of my model. The standard rationale for linking pay to performance is that it helps to align workers' incentives with those of the firm—the well-known incentive effect. The main prediction of the single-task agency theory is the existence of a negative trade-off between risk and incentives. Pay for performance imposes risk on risk-averse workers that result in higher wage costs. The risk increases with the uncertainty of the environment, thereby giving rise to a negative trade-off. While appealing, this prediction is not borne out by the data. For many occupations, the evidence suggests that pay for performance is more prevalent the more uncertain the environment (see, Prendergast (2002)).

There are two different rationales for this evidence. One advanced by Prendergast (2002) arguing that workers have knowledge about uncertain events and must be given incentives to make a correct use of it. An alternative advanced by Balmaceda (2009) arguing that in the absence of incentives to exert effort issues, but in the presence of incomplete information about workers' ability, the prevalence of pay-for-performance rises with environmental uncertainty, but pay-for-performance sensitivity falls with it.

The predicted relationship between performance pay and uncertainty and other parameters here is as follows

**Proposition 8.**  $\beta(x_i, m_i)$  increases with  $\rho_j$ ,  $\sigma_\mu^2(x_i)$  and  $\sigma_{\omega_j}$ , decreases with  $p$  and increases with  $\sigma_{\mu_j}^2$  if and only if  $\rho_j \sigma_{\omega_j} / \sigma_{\mu_j} > 4\beta(x_i)$ .

Performance-based incentives are high power when the correlation coefficient is high and low power otherwise. The reason is that a greater correlation implies, ceteris-paribus, that the worker's information is more valuable; the worker's time allocation matches the firm's desired time allocation more often. Performance-based incentives are more powerful as task-specific's productivity uncertainty rises. The reason is that when the variance of  $\omega_j$  is large, the marginal product of the worker's time on output varies greatly in different states of the world. This means that the worker has much better information than the firm has and that would allow him to modify his time allocation significantly in response to his information, which, in turn produces more

valuable outcomes. The rationale for why increases in the heterogeneity on expected marginal productivity across tasks, as measured by  $\sigma_{\mu}^2(x_i)$ , increase the power of performance-based incentives is similar. The reason is that on average the expected marginal product of time vary greatly across tasks. This means that it is optimal to provide workers with more powerful incentives so that workers' time allocation become more responsive to the increased task heterogeneity.

The impact of the worker's knowledge about the performance measure as measured by  $\sigma_{\mu_j}^2$  is ambiguous. When the variance of  $\mu_j$  is large, the marginal product of the worker's action on  $P$  is a noisy reflection of the marginal product of the worker's action on  $Y$ . A greater variance implies that the optimal time allocation will vary wildly with  $\mu$ . Given the convex cost function, the worker's expected cost will be high, and the firm will have to compensate the worker for this expected cost by settling for weak incentives rather than strong but frequently dysfunctional incentives.

In the model here then, whether one should expect to see a positive or a negative relationship between incentives and uncertainty depends on which type of uncertainty is being considered. According to Proposition 8, an increase in technological uncertainty in task  $j$ , measured by  $\sigma_{\omega_j}$ , results in an increase of the intensity of performance-based compensation, while an increase in performance uncertainty in task  $j$ , measured by  $\sigma_{\mu_j}$ , results in a decrease of performance-based pay incentives when  $\rho_j$  is small and in an increase of it otherwise.<sup>22</sup> However, in the single-task case, an increase in performance uncertainty in task  $j$  always results in a decrease of performance-based pay and in an increase in time wages.

This result depends crucially on the fact that performance sensitivity uncertainty affects workers' marginal return to time, and that workers receive information before choosing their time allocation. This uncertainty should not be interpreted as the environmental uncertainty or risk in the linear agency model. Rather, it is valuable information from workers' viewpoint about what they ought to do and to the extent that  $\rho_j > 0$ , this is also valuable from firms' viewpoint. Thus, the relationship between the power of incentives and uncertainty seems more subtle than what most empirical studies so far have assumed. A proper identification of the empirical relationship between the power of incentives and uncertainty requires to distinguish between technological, performance and environmental uncertainty since their impact on pay-for-performance is different. Furthermore, it also requires to control for job design, since the model predicts a non-monotone relationship between performance uncertainty and performance-based incentives that depends on the quality of information possessed by workers and the structure of the job.

A paper that attempts to distinguish between different types of uncertainty and deals with workers' knowledge is Erkens et al. (2006). They find, among 2,200 U.S. manufacturing plants, that approximately 49% compensate production employees based on their outputs and that there is a lot of heterogeneity in

---

<sup>22</sup>This rationale is different from that in Prendergast (2002) since he derives a prediction of a positive relation between risk and incentives prevalence by assuming that increased volatility leads to increased delegation, while here the prediction regards the strength of incentives. Mainly, his prediction says the piece rate is more likely to be found when there is more environmental uncertainty. Thus, in contrast with Prendergast, the model here predicts that, holding delegation constant, the power of incentives, measured by the pay-for-performance intensity, should be higher in less uncertain environments.

compensation across workers. They argue that a large portion of the heterogeneity in compensation can be explained by workers' specific knowledge. Furthermore, they find that the noisiness of output performance measures is negatively associated with their use. However, this last piece of evidence is concerned with pay-for-performance prevalence and not intensity and thus the model here has nothing to say about this.

**Performance, Satisfaction and Job Design.** The seminal paper by Hackman and Oldham (1976) argue that job complexity impacts behavioral and attitudinal outcomes through their influence on three critical psychological states: experienced meaningfulness (i.e., the degree to which an employee feels the job has value and importance), experienced responsibility (i.e., the degree to which an employee feels liable and accountable for job results), and knowledge of results (i.e., the degree to which the employee is aware of his or her level of performance). Specifically, they argue that skill variety, task identity and task significance are thought to be the main job characteristics that affect experienced meaningfulness, autonomy is thought to impact experienced responsibility, and feedback from the job is thought to impact knowledge of results. In short, this literature argue that motivational work characteristics increase job satisfaction.

DeVaro et al. (2007) evaluate the empirical relevance of the Job Characteristics Model using a unique, nationally representative data from a survey of British establishments. The results generally support the predictions that task variety and worker autonomy are positively associated with labor productivity and that autonomy is positively associated with worker satisfaction. In contrast to previous studies, they find that the impact of task variety on performance-related outcomes is stronger than that on workers' satisfaction.

Humphrey et al. (2007) study the relationship between motivation broadly defined and job design by mean of conducting a meta-analysis of 259 studies including 219,625 participants. They find that job enrichment (skill variety, task identity, and task significance) explains 25% of the variance in subjective performance and 34% in job satisfaction, while social characteristics (interdependence among jobs, feedback from others, social support, and interaction outside the organization) explain incremental variances of 9% of the variance in subjective performance and 17% in job satisfaction. Judge et al. (2001) conducts a meta-analysis on 312 samples with a combined number of observations equal to 54,417. They find that mean correlation between overall job satisfaction and job performance was estimated to be 30%.<sup>23</sup> They also report that the satisfaction-performance correlation is substantially stronger in high-complexity jobs than low-complexity jobs. Though job satisfaction and job performance were correlated for jobs with medium and low complexity (correlation 29%), this value is significantly lower than the average correlation for high complexity jobs (correlation 52%). Judge et al. (2010) conducts a meta-analysis across 115 correlations from 92 independent samples, and find that pay level was correlated 15% with job satisfaction and 23% with pay satisfaction. The results suggest that, within-studies, level of pay bears a positive, but small, relationship to job and pay satisfaction. Between studies, there is little relationship between average pay in a sample and the average

---

<sup>23</sup>This correlation did not vary significantly according to the measure of performance, although it should be noted that the vast majority of the studies measured job performance with supervisory ratings

level of job or pay satisfaction.

Freeman and Kleiner (2000) estimate the effects of job enrichment programs on productivity using panel data on firms and the effects of these on workers using a survey of employees and find that job enrichment barely affects firm productivity, but substantially improves worker satisfaction. Cappelli and Neumark (2001) confirms this result in larger sample and find that these human resource practices increase compensation. Thus, one can be fairly sure that there exists a positive and strong relationship between job complexity and job satisfaction and a positive, but less strong, relationship between job complexity and productivity. Furthermore, the evidence points to a weak relationship between pay and job and pay satisfaction.

The model predicts the following.

**Proposition 9.** *(i) A worker's total time worked rises with  $m$ ; (ii) holding monetary rewards constant, a worker's non-monetary satisfaction rises with  $m$ , while keeping non-monetary satisfaction constant, monetary satisfaction falls with  $m$ ; and (iii) suppose that  $\sigma_{\omega_j} \geq \sigma_{\omega_j \mu_j}$ , then a worker's total productivity rises with  $m$ .*

This result combined with that in proposition 5 imply that there is a positive relationship between total-time worked, non-monetary satisfaction and productivity and job complexity, and a negative relationship between monetary satisfaction and job complexity. Furthermore, compensation and job satisfaction are negatively related.

The results here not only suggests that there is a relationship between non-monetary satisfaction and job complexity, but a causal one. Mainly, the causality goes from non-monetary rewards to job complexity and not the other way around. That is, when workers are strongly adhere to moral standards about doing good work, firms are more likely to assign workers to multi-task jobs. Furthermore, the model also predicts that workers in more complex jobs not only should declare themselves as more satisfied, but also their pay should be based more on input (time) and less on output than workers performing in specialized jobs entailing the same tasks.<sup>24</sup> The result in proposition 9 also suggests that workers more concerned with moral standards should work harder and be more productive, be happier with their job and less happy with their total compensation than workers less concerned with moral standards.

Podsakoff et al. (2009) conducts a meta-analytic examination of the relationships between organizational citizenship behaviors (OCBs) and a variety of individual-and organizational-level outcomes. OCBs are classified into two groups: one that includes any behavior that involves helping others within the unit of analysis such as altruism and interpersonal helping and another that includes any behavior that goes directly toward the benefit of the unit of analysis such as compliance and endorsing, supporting, and defending

---

<sup>24</sup>Helper et al. (2010) provide evidence that suggests that for firms with production processes with a high return to multi-tasking and producing exact levels of output, time rate pay or time rates with low-powered incentives are the optimal form of compensation. They argue that the adoption of flexible manufacturing led to multi-tasking and to changes in compensation contracts and an important reason for that was that under flexible manufacturing workers' specific knowledge became crucial.

organizational objectives. Results, based 51,235 individuals, indicated that OCBs are positively related to employee performance and rewards, and results based on 3,611 units show that a positive relationship between OCBs and productivity, efficiency, cost reductions and profitability. Furthermore, they find stronger relationships between OCBs and unit-level performance measures in longitudinal studies than in cross-sectional studies, providing some evidence that OCBs are causally related to these variables as the model here predicts.

## **7.2 Final Remarks**

In this paper, I extended Baker (1992)'s model to a multi-task setting with independent tasks to study job design. The relationship between tasks arises as the result of compensation being based on aggregated measures and ex-post asymmetric information and not as the result of the effort substitution effect proposed by Holmström and Milgrom (1991). The main insight of the paper is that incentives problems of the type considered here are a natural source of dis-economies of scope in firms despite the fact that there is no spillovers across tasks. Thus, the benefits of specialization goes beyond the learning-by-doing effect identified by Smith. Furthermore, the paper highlights the importance of job design rather than incentive contracts as a major determinant of the incentives provided by firms, and workers' preferences as major determinant of job design. Mainly, it shows that intrinsic motivation arising from workers' adherence of moral standards favors job designs in which multi-tasking jobs are adopted.

The results here also provide a novel rationale for the relationship between job design and spillovers across tasks that is of a different nature of that suggested by the Holmström and Milgrom (1991)'s model. Mainly, the model shows that as the degree of substitution rises, firms are more likely to adopt multi-task jobs.

Finally, the tractability of the model proposed here lends itself to study other issues such as the effort-in-teams problem and the communication of information problem. I have assumed that workers' cannot credible communicate their private information to the principal and assumed away the incentives-in-teams problem. I hope to undertake these questions in future research.

## References

- Akerlof, George A. and Rachel E. Kranton**, “Identity, Supervision, and Work Groups,” *American Economic Review*, May 2008, 98 (2), 212–17.
- Baker, George P.**, “Incentive Contracts and Performance Measurement,” *Journal of Political Economy*, June 1992, 100 (3), 598–614.
- Baker, George, Robert Gibbons, and Kevin J Murphy**, “Subjective Performance Measures in Optimal Incentive Contracts,” *The Quarterly Journal of Economics*, November 1994, 109 (4), 1125–56.
- Balmaceda, Felipe**, “Uncertainty, Pay for Performance, and Asymmetric Information,” *Journal of Law, Economics and Organization*, 2009, 58 (1), 211–236.
- Besley, Timothy and Maitreesh Ghatak**, “Competition and Incentives with Motivated Agents,” *American Economic Review*, June 2005, 95 (3), 616–636.
- Bond, Philip and Armando Gomes**, “Multitask principal-agent problems: Optimal contracts, fragility, and effort misallocation,” *Journal of Economic Theory*, January 2009, 144 (1), 175–211.
- Cappelli, Peter and David Neumark**, “Do “High-Performance” Work Practices Improve Establishment-Level Outcomes?,” *Industrial and Labor Relations Review*, 2001, 54(4), 737–775.
- Casadesus-Masanell, Ramon**, “Trust in Agency,” *Journal of Economics & Management Strategy*, 09 2004, 13 (3), 375–404.
- Delfgaauw, Josse and Robert Dur**, “Incentives and Workers’ Motivation in the Public Sector,” *Economic Journal*, 01 2008, 118 (525), 171–191.
- den Steen, Eric Van**, “Culture Clash: The Costs and Benefits of Homogeneity,” *Management Science*, 2010, 56 (10), 1718–1738.
- , “On the origin of shared beliefs (and corporate culture),” *RAND Journal of Economics*, 2010, 41 (4), 617–648.
- DeVaro, Jed, Robert Li, and Dana Brookshire**, “Analysing the job characteristics model: new support from a cross-section of establishments.” *International Journal of Human Resource Management*, 2007, 18 (6), 986 – 1003.
- Dewatripont, Mathias and Jean Tirole**, “Advocates,” *Journal of Political Economy*, February 1999, 107 (1), 1–39.
- , **Ian Jewitt, and Jean Tirole**, “The Economics of Career Concerns, Part II: Application to Missions and Accountability of Government Agencies,” *The Review of Economic Studies*, 1999, 66 (1), 199–217.

- , —, and —, “Multitask agency problems: Focus and task clustering,” *European Economic Review*, May 2000, 44 (4-6), 869–877.
- Erkens, David, John H. Evans, and Yuhchang Hwang**, “Knowledge Sharing and Incentive Design in Production Environments: Theory and Evidence,” Technical Report 210, W. P. Carey School of Business, School of Accountancy Arizona State University Novemebr 2006.
- Freeman, Richard B. and Morris M. Kleiner**, “Who Benefits Most from Employee Involvement: Firms or Workers?,” *American Economic Review*, 2000, 90 (2), 219–223.
- Hackman, Richard J. and Greg R. Oldham**, “Motivation through the design of work: test of a theory,” *Organizational Behavior and Human Performance*, 1976, 16 (2), 250 – 279.
- Helper, Susan, Morris M. Kleiner, and Yingchun Wang**, “Analyzing Compensation Methods in Manufacturing: Piece Rates, Time Rates, or Gain-Sharing?,” Working Paper 16540, National Bureau of Economic Research November 2010.
- Holmström, B. and P. Milgrom**, “Multi-task Principal-Agent Analyses: Incentive Contracts, Asset Ownership and Job Desing,” *Journal of Law, Economics and Organization*, 1991, 7, 24–52.
- Humphrey, Stephen E., Jennifer D. Nahrgang, and Frederick P. Morg**, “Integrating Motivational, Social, and Contextual Work Design Features: A Meta-Analytic Summary and Theoretical Extension of the Work Design Literature,” *Journal of Applied Psychology*, December 2007, 92 (5), 1332–1356.
- Itoh, H.**, “Cooperation in Hierarchical Organizations: An Incentive Perspective,” *Journal of Law, Economics and Organization*, 1992, 8, 321–345.
- , “Job Design, Delegation and Cooperation: A Principal-Agent Analysis.,” *European Economic Review*, 1994, 38, 691–700.
- Judge, Timothy A., Carl J. Thoresen, Joyce E. Bono, and Gregory K. Patton**, “The job satisfaction–job performance relationship: A qualitative and quantitative review.,” *Psychological Bulletin*, may 2001, 127.
- , **Ronald F. Piccolo, Nathan P. Podsakoff, John C. Shaw, and Bruce L. Rich**, “The relationship between pay and job satisfaction: A meta-analysis of the literature,” *Journal of Vocational Behavior*, 2010, 77 (2), 157 – 167.
- Laux, C.**, “Limited Liability and Incentive Contracting with Multiple Projects.,” *RAND Journal of Economics.*, 2001, 32, 514–526.
- MacDonald, G. and L. Marx**, “Adverse Specialization,” *Journal of Political Economy*, 2001, 109 (4), 864–899.



- Murdock, Kevin**, “Intrinsic Motivation and Optimal Incentive Contracts,” *RAND Journal of Economics*, Winter 2002, 33 (4), 650–671.
- Ottenson, James R.**, *Adam Smith’s Marketplace of Life*, Cambridge, MA.: Cambridge University Press, 2002.
- Podsakoff, Nathan P., Steven W. Whiting, Philip M. Podsakoff, and Brian D. Blume**, “Individual- and Organizational-Level Consequences of Organizational Citizenship Behaviors: A Meta-Analysis,” *Journal of Applied Psychology*, 2009, 94 (1), 122 – 141.
- Prendergast, C.**, “The Tenuous Trade-off between Risk and Incentives,” *Journal of Political Economy*, 2002, 110, 1071–1102.
- Prendergast, Canice**, “The Motivation and Bias of Bureaucrats,” *American Economic Review*, March 2007, 97 (1), 180–196.
- , “Intrinsic Motivation and Incentives,” *American Economic Review*, May 2008, 98 (2), 201–05.
- Raith, Michael**, “Specific knowledge and performance measurement,” *RAND Journal of Economics*, 2008, 39 (4), 1059–1079.

## A Proof of Results in Section 4

*Proof of lemma 3.* Note that worker  $i$ 's total time costs are given by

$$\frac{1}{8}(\mathbb{E}\mu(x_i))^2 \sum_{j=1}^n x_{ij} + \frac{1}{2}\beta(x_i)^2 \sum_{j=1}^n x_{ij}(p\sigma_{\mu_j}^2 + \sigma_{\mu}^2(x_i)).$$

Suppose first that the expected marginal product of time is the same across all tasks; that is,  $\mathbb{E}(\mu_j) = \mathbb{E}(\mu_j)$ ,  $\forall j = 1, \dots, n$ . Consider any worker, say worker  $i$ , who is assigned to a job with  $K > 1$  tasks. Because  $\mathbb{E}(\omega_j)$  is the same for all  $j$ , the limited liability rent from assigning worker  $i$  to a multi-task job with  $K$  tasks is given by

$$L + \frac{1}{8} \left( \sum_{j=1}^K \mathbb{E}(\omega_j)^2 + \frac{\left( \sum_{j=1}^K p\rho_j \sigma_{\omega_j} \sigma_{\mu_j} \right)^2}{\sum_{j=1}^K p\sigma_{\mu_j}^2} \right). \quad (\text{A1})$$

Suppose now that worker  $i$ 's job is split in  $K$  different jobs each having one task only. Then the sum of the limited liability rents of the workers allocated to these jobs is

$$\sum_{i=1}^K L + \frac{1}{8} \sum_{j=1}^K \left( \mathbb{E}(\omega_j)^2 + p\rho_j^2 \sigma_{\omega_j}^2 \right). \quad (\text{A2})$$

Assuming that  $L = 0$  and subtracting the limited liability rent in equation (A1) from the sum of the limited liability rents in equation (A2), one can show that this difference is positive if and only if

$$\sum_{j=1}^K p\sigma_{\mu_j}^2 \sum_{j=1}^K p\rho_j^2 \sigma_{\omega_j}^2 \geq \left( \sum_{j=1}^K p\rho_j \frac{\sigma_{\omega_j}}{\sigma_{\mu_j}} \sigma_{\mu_j}^2 \right)^2,$$

which holds true by the Cauchy-Schwartz inequality.

This says that the limited liability rent from a worker assigned to a job with more than 1 task is lower than the sum of the limited liability rents from the equivalent number of single-task jobs when the limited liability constraint is set to zero. Observe that this is even more so when  $L > 0$ , since this must be paid to each worker who is assigned to at least one task.

Suppose now that  $\mathbb{E}(\mu_j) \neq \mathbb{E}(\mu_{j'})$  for some  $j, j' = 1, \dots, n$ ,  $j \neq j'$  and  $L = 0$ . The sum of the limited liability rents from job design in which there are  $K$  single-task jobs and these are assigned to  $K$  different workers is

$$\frac{1}{8} \sum_{j=1}^K \left( \mathbb{E}(\omega_j)^2 + p\rho_j^2 \sigma_{\omega_j}^2 \right),$$

while the limited liability rent from bundling the same  $K$  tasks into one job and assigning this job to worker

$i$  is given by

$$\frac{1}{8} \sum_{j=1}^K \left( \mathbb{E}(\omega_j)^2 - \sigma_\mu(x_i) \right) + \frac{1}{2} \beta(x_i)^2 \sum_{j=1}^K \left( p\sigma_{\mu_j}^2 + \sigma_\mu(x_i) \right).$$

After a few steps of simple algebra, it is easy to show that the limited liability from the sum of  $K$  single-tasks job is greater than that from one job with the same  $K$  tasks if and only if

$$\left( \sum_{j=1}^K \rho_j \sigma_{\omega_j} p (\sigma_{\mu_j} - \rho_j \sigma_{\omega_j}) + \sum_{j=1}^K p \sigma_{\mu_j} (\rho_j \sigma_{\omega_j} - \sigma_{\mu_j}) \right) K \sigma_\mu^2(x_i) \geq \quad (A3)$$

$$\sum_{j=1}^K p \sigma_{\mu_j}^2 \sum_{j=1}^K p \rho_j^2 \sigma_{\omega_j}^2 - \left( \sum_{j=1}^K p \rho_j \sigma_{\omega_j} \sigma_{\mu_j} \right)^2.$$

It readily follows from Cauchy-Schwartz inequality that the right-hand side of this equation is non-negative for any  $K \geq 2$ , while the sign of left-hand side is ambiguous. On the one hand, assumption **A2** ensures that  $\sigma_{\mu_j} \geq \rho_j \sigma_{\omega_j}$  which implies that the second term on the LHS is non-positive, while the first-term is non-negative. Adding these two terms it is easy to see that the LHS is given by

$$- \sum_{j=1}^K p (\sigma_{\mu_j} - \rho_j \sigma_{\omega_j})^2 < 0.$$

Thus, the LHS is lower than the RHS and therefore full specialization leads to higher limited liability rents. Again this is even more so when  $L > 0$ .  $\square$

*Proof of lemma 4.* The output of a job consisting of  $K$  tasks is given by

$$\frac{1}{2} \left( \mathbb{E}\mu(x_i) \right)^2 \sum_{j=1}^n x_{ij} + \beta(x_i)^2 \sum_{j=1}^n x_{ij} (p\sigma_{\mu_j}^2 + \sigma_\mu^2(x_i)).$$

while the sum of the outputs from  $K$  jobs each comprising one of the  $K$  tasks is given by:

$$\sum_{j=1}^K \left( \frac{1}{2} \mathbb{E}(\omega_j)^2 + p\rho_j^2 \sigma_{\omega_j}^2 \right).$$

The rest of the proof is identical to the one in lemma 3.  $\square$

*Proof of Proposition 2.* For  $L = 0$  this readily follows from lemma 3 and equation (7), since this implies that the profits from a job comprising  $K$  tasks is given by

$$\frac{1}{4} \sum_{j=1}^K \left( \mathbb{E}(\omega_j)^2 - \sigma_\mu(x_i) \right) + \beta(x_i)^2 \sum_{j=1}^K \left( p\sigma_{\mu_j}^2 + \sigma_\mu(x_i) \right).$$

while the sum of the profits from  $K$  jobs each comprising one of the  $K$  tasks is given by:

$$\frac{1}{4} \sum_{j=1}^K \left( \mathbb{E}(\omega_j)^2 + p\rho_j^2 \sigma_{\omega_j}^2 \right).$$

For  $L > 0$  it is sufficient to notice that profits in job comprising  $K$  tasks falls at rate of 1 with  $L$ , while the sum of the profits from  $K$  single-task jobs falls at rate of  $K$  with  $L$ . Thus the difference between the profit from a multi-task job with  $K$  tasks and that for the sum of profits from  $K$  single-task jobs rises with  $L$  at rate  $K - 1$  for all  $K \geq 2$  and thus by continuity with respect to  $L$  there exist an  $L$ , denoted by  $\hat{L}$ , such that the profit from a multi-task job with at least 2 tasks is greater than the sum of the profits of two single-task jobs comprising these two tasks.  $\square$

## B Proof of Results in Section 5

*Proof of proposition 5.* Let  $L(m)$  be the limited liability level that leaves the firm indifferent between  $K$  single-task jobs and the most profitable job design entailing multi-tasking. That is,  $L(m)$  solves the following  $\Pi(x_i, m) = \sum_{j=1}^K \Pi(x_{ij}, m)$ . Thus, it readily follows from proposition 3 and equation (10) that for all  $m < 1$  and any  $K > 2$

$$(K - 1) \frac{\partial L(m)}{\partial m} = \frac{(1 - m)(3 + m)}{(1 + m)^2} \left( \Pi(x_i) - \sum_{j=1}^n \Pi(x_{ij}) - L(K - 1) \right)$$

Because proposition 2 ensures that RHS is negative and  $K > 1$  ensures that LHS is positive,  $L(m)$  falls with  $m$  for all  $m < 1$ .

It readily follows from the proposition 3 that for all  $m \geq 1$  and any  $K > 2$

$$(K - 1) \frac{\partial L(m)}{\partial m} = 0$$

since

$$\Pi(x_i, m) \equiv \frac{m}{1 + m} \sum_{j=1}^n x_{ij} \left( p\sigma_{\omega_j}^2 + \mathbb{E}(\omega_j)^2 \right) - L.$$

$\square$

## C Contractible Time

### D Task-specific Contractible Measures

Here, I study the robustness of the result to the availability of different performance and time measures. First, I consider the case in which the principal can contract on task-specific time measures in each task and the aggregated output performance measure  $P$ . Second, I assume that the principal can contract on task-specific performance measures in each task and the aggregated time measure  $T$ . I do not consider the case in which the firm can contract in both dimensions at the task level since as become obvious job design become irrelevant due to the lack of spillovers effects.

#### D.1 Task-Specific Time Measures

So far I have assumed that workers workers have full discretion on how to allocate their time across tasks. Here, I assume that firms can contract on workers' time allocation to each task as they see it fit. If firms were to be informed about the realized marginal product of time in each task, this will lead to the efficient time allocation. However, firms do not know them and thus it is not obvious how the optimal contract will be modified to deal with extra information.

Let  $\gamma_{ij}$  be the time-based pay in task  $j$ . Then, given a contract  $C_i \equiv (\alpha_i, \beta_i, \gamma_{ij})$ , it is straightforward to show that the worker chooses a time allocation consistent with

$$t_j(C_i, x_i) = x_{ij}(\beta_i \tilde{\mu}_j + \gamma_{ij}). \quad (\text{D1})$$

Given job design  $x \in X$ , the firm chooses compensation contracts to solve the following problem

$$\max_{\{C_i\}_{i=1}^n} \mathbb{E} \sum_{i=1}^n \left\{ \sum_{j=1}^n \omega_j t_j(C_i, x_i) - \alpha_i - \beta_i \sum_{j=1}^n \mu_j t_j(C_i, x_i) - \sum_{j=1}^n \gamma_{ij} t_j(C_i, x_i) \right\}.$$

The first-order conditions are as follows

$$\begin{aligned} \beta_i : \sum_{j=1}^n x_{ij} \left( p \mathbb{E}(\omega_j \mu_j) + (1-p) \mathbb{E}(\omega_j) \mathbb{E}(\mu_j) \right) - \\ 2 \sum_{j=1}^n x_{ij} \left( \beta_i (p \mathbb{E}(\mu_j^2) + (1-p) \mathbb{E}(\mu_j)^2) + \gamma_{ij} \mathbb{E}(\mu_j) \right) = 0, \end{aligned} \quad (\text{D2})$$

and

$$\gamma_{ij} : x_{ij} \mathbb{E}(\omega_j) - 2x_{ij}(\beta_i \mathbb{E}(\mu_j) + \gamma_{ij}) = 0. \quad (\text{D3})$$

Thus, solving for each  $\gamma_{ij}$ , using the fact that  $\mathbb{E}(\omega_j) = \mathbb{E}(\mu_j)$  and substituting this back into the first-order

condition for  $\beta_i$ , one gets that the optimal contract is given by a performance-based performance sensitivity

$$\beta(x_i) = \frac{1}{2} \sum_{j=1}^n \theta_j(x_i) \beta(x_{ij}), \quad (\text{D4})$$

and a time wage in task  $j$  equals to

$$\gamma_{ij}(x_i) = x_{ij} \frac{1}{2} \mathbb{E}(\mu_j) (1 - 2\beta(x_i)). \quad (\text{D5})$$

After a few steps of simple algebra, one can show that the total surplus when the firm can contract on effort in each task and thus it limits the worker's discretion is given by

$$\Pi(x, p) \equiv \frac{1}{4} \sum_{j=1}^n \mathbb{E}(\omega_j)^2 + \frac{1}{4} \sum_{i=1}^n \frac{\left( \sum_{j=1}^n x_{ij} p \rho_j \sigma_{\omega_j} \sigma_{\mu_j} \right)^2}{\sum_{j=1}^n x_{ij} p \sigma_{\mu_j}^2}.$$

When the firm can monitor the time allocated to each task, the firm chooses a task-specific time wage so that workers adjust their time to the expected marginal product of time in each task and a positive performance-based pay to induce workers to use their private information. By contracting on the time allocated to each task, the firm can tune better the contract to deal with difference in expected marginal product of time across tasks. However, since the performance-base pay relies on a measure that is aggregated (at the job level), workers continue to have incentives to game the performance measure by allocating their time across tasks in non-optimal from the firm's viewpoint. In fact, the optimal performance-based pay sensitivity is equal to the one that arises when all tasks have the same expected marginal product of time and thus the worker's incentives to game the output performance measure remain intact. This together with the result in proposition 2 lead to the following

**Proposition 10.** *Suppose the firm has access to a task-specific time measure for each task. Then (i) there exists a limited liability threshold, denoted by  $\hat{L} > 0$ , such that the second-best optimal job design entails only full specialized jobs for all  $L \leq \hat{L}$  and it entails multi-tasking jobs otherwise; and (ii) the second-best total surplus is lower than the first-best total surplus as long as  $\rho_j < 1$  for some  $j = 1, \dots, n$ .*

Thus, the full specialization result is robust to the existence of a task-specific time measure for each task. Furthermore, having a more rich set of time measures is of no value from the firm's point of view since the result will be full specialization anyway.

## D.2 Task-Specific Performance Measures

So far I have assumed that there is only one aggregated output-performance measure available and thus it seems reasonable to think that the full specialization result is driven by this assumption. Here, I assume that

the firm has access to one performance measure per task together with the total time input measure.

Let  $\beta_{ij}$  be the pay-for-performance sensitivity paid in task  $j$ . Then, it is easy to show that when faced with contract  $C_i \equiv (\alpha_i, \beta_{ij}, \gamma_i)$  and allocated to job  $x_i$ , worker  $i$  chooses his time allocation according to

$$t_j(C_i, x_i) = x_{ij}(\beta_{ij}\tilde{\mu}_j + \gamma_i).$$

Given job design  $x \in X$ , the firm chooses compensation contracts to solve the following problem

$$\max_{\{C_i\}_{i=1}^n} \mathbb{E} \sum_{i=1}^n \left\{ \sum_{j=1}^n \omega_j t_j(C_i, x_i) - \alpha_i - \sum_{j=1}^n \beta_{ij} \mu_j t_j(C_i, x_i) - \gamma_i \sum_{j=1}^n t_j(C_i, x_i) \right\}.$$

The first-order conditions are as follows

$$\beta_{ij} : x_{ij} \left( p \mathbb{E}(\omega_j \mu_j) + (1-p) \mathbb{E}(\omega_j) \mathbb{E}(\mu_j) \right) - 2x_{ij} \left( \beta_{ij} (p \mathbb{E}(\mu_j^2) + (1-p) \mathbb{E}(\mu_j)^2) + \gamma_i \mathbb{E}(\mu_j) \right) = 0,$$

and

$$\gamma_i : \sum_{j=1}^n x_{ij} \mathbb{E}(\omega_j) - 2 \sum_{j=1}^n x_{ij} (\beta_{ij} \mathbb{E}(\mu_j) + \gamma_i) = 0.$$

Thus, summing over the first-order conditions for  $\beta_{ij}$  in order to solve for  $\sum_{j=1}^n \beta_{ij} \mathbb{E}(\omega_j)$ , substituting this into the FOC for  $\gamma_i$ , using the fact that  $\mathbb{E}(\omega_j) = \mathbb{E}(\mu_j)$  and substituting this back into the first-order condition for  $\beta_{ij}$ , one gets that the optimal contract is given by

$$\beta_j(x_i) = 1 - \frac{1}{p\sigma_{\mu_j}^2 + \mathbb{E}(\mu_j)^2} \left( p\sigma_{\mu_j}^2(1 - \ell_j) + \mathbb{E}(\mu_j) \sum_{h=1}^n x_{ih} \phi_h(x_i) \mathbb{E}(\mu_h)(1 - \ell_h) \right)$$

where

$$\phi_j(x_i) = \frac{p\sigma_{\mu_j}^2 / (p\sigma_{\mu_j}^2 + \mathbb{E}(\mu_j)^2)}{\sum_{h=1}^n x_{ih} \left( p_{ih} \sigma_{\mu_h}^2 / (p_{ih} \sigma_{\mu_h}^2 + \mathbb{E}(\mu_h)^2) \right)}$$

and a time wage

$$\gamma_i(x_i) = \sum_{j=1}^n x_{ij} \mathbb{E}(\mu_j) \phi_j(x_i) (1 - \ell_j).$$

It is instructive to see that if the firm sets  $\gamma_i = 0$ , then it follows from the first-order condition for  $\beta_{ij}$  that the optimal pay-for-performance sensitivity in each task assigned to worker  $i$  would be independent of that for the other tasks assigned to him. This means that tasks are treated as fully independent and therefore the firm is indifferent between all possible job designs. However, as shown above, from the firm's point of view this is suboptimal, since the firm wants to pay for time in order to reduce expected time costs due

to cost convexity and to keep workers' incentives to allocate time to tasks. Because the time-based pay is based on an aggregated (at the job level) time measure, the optimal contract results in the existence of a link between task-specific pay-for-performance sensitivity in task  $j$  and the beta coefficient in task  $j'$  (i.e.,  $\ell_{j'}$ ) for all  $x_{ij'} = 1$ .

Observe that the equilibrium time-based compensation falls with  $\ell_j$ ,  $\forall j = 1, \dots, n$  and the equilibrium task-specific pay-for-performance sensitivity rises with it. Thus, there is a negative relationship between time- and performance-based incentives that is of the same nature as the one arising when compensation is based on an aggregated performance measure.

It follows from the first-order condition for  $\beta_{ij}$  that the optimal pay-for-performance sensitivity in task  $j$  falls with  $\gamma_i$  and thus if  $\gamma_i$  rises when another task is bundled into the job, the task-specific pay-for-performance sensitivity in each task falls when a new task is bundled into the job. Because  $\gamma(x_i)$  is a weighted mean of  $1 - \ell_j$  and this term rises when a task with lower quality of information (i.e., lower  $\ell_j$ ) is added to the job, the optimal pay-for-performance sensitivity  $\beta_j(x_i)$  falls with it. Thus, when firms can compensate workers according to task-specific performance measures, incentive problems remain a natural source of dis-economies of scope in firms.

The expected total surplus is

$$\begin{aligned} \Pi(x, p) \equiv & \frac{1}{4} \sum_{i=1}^n (\mathbb{E}\mu(x_i))^2 \sum_{j=1}^n x_{ij} + \sum_{i=1}^N \sum_{j=1}^n x_{ij} \beta_j(x_i)^2 (p\sigma_{\mu_j}^2 + \mathbb{E}(\mu_j)^2) - \\ & \sum_{i=1}^N \left( \sum_{j=1}^n \frac{x_{ij}}{\sum_{j=1}^n x_{ij}} \beta_j(x_i) \mathbb{E}(\mu_j) \right)^2 \sum_{j=1}^n x_{ij}. \end{aligned}$$

**Proposition 11.** *Suppose the firm has access to a task-specific performance measure for each task. Suppose workers are compensated on total time and on an aggregated performance measure. Then (i) there exists a limited liability threshold, denoted by  $\hat{L} > 0$ , such that the second-best optimal job design entails only full specialized jobs for all  $L \leq \hat{L}$  and it entails multi-tasking jobs otherwise; and (ii) the second-best total surplus is lower than the first-best total surplus as long as  $\rho_j < 1$  for some  $j = 1, \dots, n$ .*

*Proof.* The proof follows immediately from the definition of maximum. Let

$$t_j(C_i, x_i) = x_{ij}(\beta_{ij}\tilde{\mu}_j + \gamma_{ij}).$$

be worker  $i$ 's time allocation when the firm can freely choose the contract  $(\alpha_i, \beta_{ij}, \gamma_{ij})$ . When the time measure is aggregated at the job level, the firm chooses  $(\alpha_i, \beta_{ij}, \gamma_{ij})$  and job design  $x \in X$  to solve the



following problem

$$\max_{x \in X, \{C_i \in \mathbb{R}^3\}_{i=1}^n} \mathbb{E} \sum_{i=1}^n \left\{ \sum_{j=1}^n \omega_j t_j(C_i, x_i) - \alpha_i - \sum_{j=1}^n \beta_{ij} \mu_j t_j(C_i, x_i) - \sum_{j=1}^n \gamma_{ij} t_j(C_i, x_i) \right\} \quad (\text{D6})$$

subject to

$$\gamma_{ij} = \gamma_i \quad \forall x_{ij} = 1 \text{ and } \forall i = 1, \dots, N.$$

The expected surplus from the full specialization job design is the same as that arises from the solution to the following problem

$$\max_{x \in X, \{C_i \in \mathbb{R}^3\}_{i=1}^n} \mathbb{E} \sum_{i=1}^n \left\{ \sum_{j=1}^n \omega_j t_j(C_i, x_i) - \alpha_i - \sum_{j=1}^n \beta_{ij} \mu_j t_j(C_i, x_i) - \sum_{j=1}^n \gamma_{ij} t_j(C_i, x_i) \right\} \quad (\text{D7})$$

Because the maximization problem in equation (D6) is equal to that in equation (D7) but with the restriction that the time-based compensation cannot be customized to the the level of task only to the job level, the expected total surplus arising from program (D7) is at least as large as that of (D6).  $\square$

**Centro de Economía Aplicada  
Departamento de Ingeniería Industrial  
Universidad de Chile**

**2011**

- 279. Job Design and Incentives  
Felipe Balmaceda**
  
- 278. Unemployment, Participation and Worker Flows Over the Life Cycle  
Sekyu Choi - Alexandre Janiak - Benjamín Villena-Roldán**
  
- 277. Public-Private Partnerships and Infrastructure Provision in the United States  
Eduardo Engel, Ronald Fischer, Alexander Galetovic**

**2010**

- 276. The economics of infrastructure finance: Public-private partnerships versus public provision  
Eduardo Engel, Ronald Fischer y Alexander Galetovic**
  
- 275. The Cost of Moral Hazard and Limited Liability in the Principal-Agent Problem  
F. Balmaceda, S.R. Balseiro, J.R. Correa, N.E. Stier-Moses**
  
- 274. Structural Unemployment and the Regulation of Product Market  
Alexandre Janiak**
  
- 273. Non-revelation Mechanisms in Many-to-One Markets  
Antonio Romero-Medina y Matteo Triossi**
  
- 272. Labor force heterogeneity: implications for the relation between aggregate volatility and government size  
Alexandre Janiak y Paulo Santos Monteiro**
  
- 271. Aggregate Implications of Employer Search and Recruiting Selection  
Benjamín Villena Roldán**
  
- 270. Wage dispersion and Recruiting Selection  
Benjamín Villena Roldán**

269. **Parental decisions in a choice based school system: Analyzing the transition between primary and secondary school**  
Mattia Makovec , Alejandra Mizala y Andrés Barrera
268. **Public-Private Wage Gap In Latin America (1999-2007): A Matching Approach**  
Alejandra Mizala, Pilar Romaguera y Sebastián Gallegos
267. **Costly information acquisition. Better to toss a coin?**  
Matteo Triossi
266. **Firm-Provided Training and Labor Market Institutions**  
Felipe Balmaceda

## **2009**

265. **Soft budgets and Renegotiations in Public-Private Partnerships**  
Eduardo Engel, Ronald Fischer y Alexander Galetovic
264. **Information Asymmetries and an Endogenous Productivity Reversion Mechanism**  
Nicolás Figueroa y Oksana Leukhina
263. **The Effectiveness of Private Voucher Education: Evidence from Structural School Switches**  
Bernardo Lara, Alejandra Mizala y Andrea Repetto
262. **Renegociación de concesiones en Chile**  
Eduardo Engel, Ronald Fischer, Alexander Galetovic y Manuel Hermosilla
261. **Inflation and welfare in long-run equilibrium with firm dynamics**  
Alexandre Janiak y Paulo Santos Monteiro
260. **Conflict Resolution in the Electricity Sector - The Experts Panel of Chile**  
R. Fischer, R. Palma-Behnke y J. Guevara-Cedeño
259. **Economic Performance, creditor protection and labor inflexibility**  
Felipe Balmaceda y Ronald Fischer
258. **Effective Schools for Low Income Children: a Study of Chile's Sociedad de Instrucción Primaria**  
Francisco Henríquez, Alejandra Mizala y Andrea Repetto
257. **Public-Private Partnerships: when and how**  
Eduardo Engel, Ronald Fischer y Alexander Galetovic

## 2008

256. Pricing with markups in industries with increasing marginal costs  
José R. Correa, Nicolás Figueroa y Nicolás E. Stier-Moses
255. Implementation with renegotiation when preferences and feasible sets are state dependent  
Luis Corchón y Matteo Triossi
254. Evaluación de estrategias de desarrollo para alcanzar los objetivos del Milenio en América Latina. El caso de Chile  
Raúl O’Ryan, Carlos J. de Miguel y Camilo Lagos
253. Welfare in models of trade with heterogeneous firms  
Alexandre Janiak
252. Firm-Provided Training and Labor Market Policies  
Felipe Balmaceda
251. Emerging Markets Variance Shocks: Local or International in Origin?  
Viviana Fernández y Brian M. Lucey
250. Economic performance, creditor protection and labor inflexibility  
Ronald Fischer
249. Loyalty inducing programs and competition with homogeneous goods  
N. Figueroa, R. Fischer y S. Infante
248. Local social capital and geographical mobility. A theory  
Quentin David, Alexandre Janiak y Etienne Wasmer
247. On the planner’s loss due to lack of information in bayesian mechanism design  
José R. Correa y Nicolás Figueroa
246. Política comercial estratégica en el mercado aéreo chileno  
Ronald Fischer
245. A large firm model of the labor market with entry, exit and search frictions  
Alexandre Janiak
244. Optimal resource extraction contracts under threat of expropriation  
Eduardo Engel y Ronald Fischer

## 2007

243. The behavior of stock returns in the Asia-Pacific mining industry following the Iraq war  
Viviana Fernandez
242. Multi-period hedge ratios for a multi-asset portfolio when accounting for returns comovement  
Viviana Fernández
241. Competition with asymmetric switching costs  
S. Infante, N. Figueroa y R. Fischer
240. A Note on the Comparative Statics of Optimal Procurement Auctions  
Gonzalo Cisternas y Nicolás Figueroa
239. Parental choice and school markets: The impact of information approximating school effectiveness  
Alejandra Mizala y Miguel Urquiola
238. Marginal Cost Pricing in Hydro-Thermal Power Industries: Is a Capacity Charge Always Needed?  
M. Soledad Arellano and Pablo Serra
237. What to put on the table  
Nicolas Figueroa y Vasiliki Skreta
236. Estimating Discount Functions with Consumption Choices over the Lifecycle  
David Laibson, Andrea Repetto y Jeremy Tobacman
235. La economía política de la reforma educacional en Chile  
Alejandra Mizala
234. The Basic Public Finance of Public-Private Partnerships  
Eduardo Engel, Ronald Fischer y Alexander Galetovic
233. Sustitución entre Telefonía Fija y Móvil en Chile  
M. Soledad Arellano y José Miguel Benavente
232. Note on Optimal Auctions  
Nicolás Figueroa y Vasiliki Skreta