CEA

CENTRO DE ECONOMÍA APLICADA

DOCUMENTOS DE TRABAJO
Serie Economía

Nº 320	**BANKING COMPETITION AND ECONOMIC STABILITY**
	R. FISCHER - N. INOSTROZA - F. J. RAMÍREZ

fcfm

Banking Competition and Economic Stability

R. Fischer

U. de Chile

N. Inostroza

Northwestern U.

F. J. Ramírez

Coopeuch [*]

September 3, 2015

Abstract

We study banking competition and stability in a 2-period economy. Firms need loans to operate, and in case of a real shock, a fraction of firms default. Banks bound by capital adequacy constraints lend less and amplify the initial shock. The magnification depends on the intensity of bank competition. The model admits *prudent* and *imprudent* equilibria, where banks collapse after shocks. We find existence conditions for a prudent equilibrium. Competition increases efficiency but leads to higher second period variance and makes imprudent equilibria more attractive. We examine the moderating effect of regulation and forbearance.

Keywords: Bank competition, stability, efficiency, forbearance.

JEL: E44, G18, L16.

1 Introduction

This paper analyzes a simple two-period model in which a banking system amplifies real economic shocks. We focus on the interaction between the amplification effect and the intensity of competition in the banking sector. A shock affects the economy through the banking channel: an initial systemic shock to productivity leads some firms to default on short term loans. Since depositors have first priority, banks are weakened after repaying short term deposits. This initial reduction in the capital base leads to a reduction in lending in the next period, because of capital adequacy restrictions. Thus the real effects of the initial shock are amplified by the banking system. As in Kiyotaki and Moore (1997) a credit multiplier amplifies the effect of a real shock.

We link this effect to competition in the financial market, because in a more competitive market, rates are lower, leading to more borrowing and to increased leverage. As the banking market becomes more competitive, the amplification effect becomes larger, even though the economy is more efficient, so competition creates a tradeoff between efficiency and stability.

There is an extensive literature on the relationship between financial market stability and competition, much of it reviewed in Vives (2010), and which we cover in the next section. Briefly, from the point of view of theory, the predictions are ambiguous. For example, Boyd and Nicoló (2005) note that reduced competition raises interest spreads, which tempts borrowers to choose riskier projects, so the loan book of banks becomes more fragile. On the other hand, in the so called *charter value* approach, a less competitive banking system means that banks are more valuable and owners are less willing to risk them, so they transfer risks to borrowers, see Beck (2008) for references. Alternatively, with more competition, there are fewer rents from screening and relationship banking (Allen and Gale, 2004), leading to more instability. Beck (2008) shows that there is corroborating empirical evidence for these contrasting arguments.

There are two kinds of financial fragility. First, fragility leading to bank runs caused by sunspots, and a second kind in which the banking system amplifies the effect of an initial real shock, by reducing lending and magnifying the economy-wide effects of the shock. In this paper we examine the relationship between competition and this second type of fragility using the

balance-sheet channel. After an initial economic shock, banks need to contract their lending in order to improve their balance sheet, which is weakened by the default of borrowers, in what Tirole (2006) denotes a *credit crunch*. An improvement in the balance sheet is often required by regulatory authorities, which may impose more stringent capital adequacy restrictions.[1]

In our model, the shock affects all banks equally. This is complementary to models which stress the interconnectedness of the banking system (Acemoglu et al. (2013); Allen et al. (2010); George (2011), among others) as a reason for fragility. In those models the fragility is given by the interconnectedness and the initial exposure of banks, so a shock can affect banks asymmetrically. In this paper, we are interested not in the fragility of the financial sector *per se*, but on how it affects economic activity more generally by increasing the response to real shocks. We show that bank competition, by overextending banks, increases their exposure to shocks and thus the impact they have on economic activity.[2]

Bank regulation reduces the effects of the associated moral hazard problem by imposing capital adequacy restrictions. At the beginning of our first period, banks lend to firms (think of it as lending for capital investment) using funds that are provided by short term deposits and their own capital. At the end of the period, if there is no shock, firms generate revenue to repay loans, and the bank can repay depositors with this income. If there is a shock, some firms are unable to repay their loans, so the banks end up with less capital and reserves. At the beginning of the second period the firms ask the banks for working capital loans, and the banks lend by obtaining new deposits.

To simplify the analysis of the second period, we assume relationship banking. Banks that lend to a firm in the first period have an advantage over their competition, and are able to retain the client against the competition. To simplify matters, we assume that the bank keeps all the

---

[1]Switzerland has imposed a stringent set of capital adequacy rules for Sistemically Important Financial Institutions (SiFis) that will constrain lending by banks. See "Swiss urge capital boost for banks" *Financial Times*, October 4, 2010. `http://www.ft.com/intl/cms/s/0/4a24a1c8-cf26-11df-9be2-00144feab49a.html#axzz2SWZy2mnj`.

[2]Our model differs from, for instance Hellman et al. (2000) who study the effectiveness of capital adequacy ratios to restrain the effects of competition on the fragility of the banking system, but where the effect of competition is to make banks choose riskier loans.

profits obtained in the second period. In the second period there are no shocks and therefore no risk of failure. Banks always want to lend to firms, but they are restricted by capital adequacy restrictions. Thus the second period has no strategic behavior nor risk. All the competitive action occurs in the first period, which we solve by backwards induction.

In the first period banks are imperfect competitors, and they maximize profits over the two periods, considering the probability of a shock. We compare Collusive, Cournot and Bertrand equilibria.[3] Banks start out with initial capital and request short term funds from depositors. Since deposits are protected by deposit insurance, depositors are willing to lend at the risk-free rate.

There are many potential entrepreneurs, who own no assets except for the idea of a project. All projects are equally profitable and equally risky. Agents are differentiated by the value of their outside option, which follows a distribution with a continuous density. In the first period, agents whose expected return from the project exceeds their outside option approach banks for loans to carry out their projects. Banks fund entrepreneurs with short period loans which must be returned at the end of each period.[4]

If there is no shock, all entrepreneurs pay back their first period loans. The income stream received by banks is large enough to allow them to pay back their deposits, and to have enough capital to fully satisfy the second period loan demand, given the capital adequacy constraint. Agents that received a first period loan will also obtain the working capital loan for the second period. However, in the case of a productivity shock, things are different. The shock wipes out the first period returns for a fraction of firms.[5] Those firms are unable to repay the bank and since

---

[3]We model competition via conjectural variations parameter. It is a convenient mechanism of mapping our three competitive alternatives, by using different values of a single parameter, see Dixit (1986). In subsection 7.3 we show that the results can be replicated under Cournot competition, where increased competition requires additional banks. For more discussion, see also section 7 on leverage.

[4]In principle, the model could allow long term deposits and long term lending (though it would lead to cumbersome modifications). In that case, the fraction of bank's assets that are covered by these long term contracts would no be subject to the problems raised in this paper. However, since banks in the model would still provide short term finance using short term funds, the problem described in the paper does not go away. In fact, in our interpretation, second period lending is for working capital. Working capital is usually short term and variations in the amount of working capital available to firms explain much of the variability in their output.

[5]They remain viable for the second period if the bank can fund them.

banks must repay depositors, at the end of the first period they have less capital and reserves than in the case of no shock. As banks must satisfy capital adequacy restrictions, the banking system amplifies economic shocks. The reduction in own capital following a shock leads to a larger reduction in loans, a well known property of fractional reserve banking. Some firms have to cease operations because they have no working capital for the second period. The intensity of the shock is conveniently measured as the fraction of firms unable to repay their first period loan.

There is another possible outcome, which occurs when banks are overextended and repayment of deposits in case of a shock wipes out the capital and reserves of the bank (we denote this case as an *imprudent* equilibrium). In that case there is a collapse of the banking system and no second period lending.[6] Since all banks are assumed identical, symmetry implies that there is a simultaneous banking collapse. In the *prudent* equilibrium, banks are judicious and lend sums which, even in the case of a shock, will allow them to survive. In the *imprudent* equilibria, banks are improvident in the sense that they lend more than is prudent, leading to banking system collapse in case of a shock. The banking regulator can exclude the *imprudent* equilibrium through judicious use of capital adequacy restrictions. Loose capital adequacy conditions may lead to *imprudent* equilibria.

Appropriate capital adequacy regulations, by precluding the collapse of the banking system, imply that there is no need for deposit insurance, so providing that service is costless to society. However, even without a banking system collapse, increased competition leads to increased variance in economic outcomes. Basically, as competition increases, banks charge a lower interest rate and lend more. In the case of no shock, there is more economic activity. On the other hand, when there is a crisis, a larger mass of entrepreneurs fail to pay their loans, leading to a bigger reduction in bank capital. This, in turn, reduces second period lending by more. Hence, second period activity is subject to more variation as competition increases.

We also examine the effect of capital adequacy rules. In response to a shock, governments

---

[6]Because of deposit insurance, there is no possibility of a bank run a la Diamond and Dybvig (1983).

usually relax the capital adequacy rules, at least in the short run. If these rules are predetermined and public, banks incorporate this value in their first period lending decisions. An interesting results is that a lower value of the capital adequacy rules after a shock leads to a reduction in first period lending. The reason for this result is that in the case of a shock, having more second period capital is more valuable if banks can lend more. Hence there is less variance in second period output. However, if the capital adequacy values depend on the level of bank reserves after the shock, first period lending can increase, adding instability.

A somewhat surprising result of the model is that for certain parameter values, the *imprudent* equilibrium leads to higher welfare and are preferred by banks. This occurs if the probability of a shock is small, but when shocks happen, they are severe, so many firms fail to repay their loans. The cost –in terms of reduced lending and economic activity– of capital adequacy restrictions that prevent the rare occurrence of a banking collapse is too high.

The main result of this paper is that independently of the type of competition, and even considering the possible switch between types of equilibria (from *prudent* to *imprudent* equilibria), an increase in the degree of competition in the banking sector increases the variance of post shock activity and hence the variance of GDP.

Our model is not a general equilibrium model, in the sense that it relies on the existence of deposit insurance and the supply of deposits is perfectly elastic. Using deposit insurance is not uncommon in the literature, as in Allen and Gale (2004). That paper includes a cost of insurance to banks that is independent of individual riskiness, but which covers the aggregate cost of deposit insurance. In our model, when the prudential equilibrium is chosen and there is no banking collapse, the real cost of insurance is zero. The model can be adapted to accommodate a flat insurance rate, with no change in the main results, and with additional difficulties, to an increasing supply schedule for deposits.[7]

The next section is a literature review, followed by a section describing the model. Section 4 derives the equilibria in the model by backwards induction. Section 5 analyzes existence and

---

[7]Guaranteeing that the insurance rate is actuarially fair would complicate the analysis.

uniqueness. Next we study the comparative statics of the equilibrium and regulatory forbearance. Section 7 analyzes extensions to other means of increasing leverage, such as financial liberalization, and section 8 contains the conclusions.

2 Literature review

There is a large literature on the relationship between stability and competition in financial markets, so this review will highlight important contributions, under the proviso that it will leave many relevant papers unmentioned.

On the theoretical side, many papers use the fact that banks can select the return and riskiness of loans to study the relationship between competition and stability. In some models, competition among banks leads to riskier lending as returns fall (Hellman et al., 2000; Allen and Gale, 2004) and thus more banking competition leads to more instability.

In other papers, as rates increase, the borrowers tend to be riskier.[8] Hence, as in Boyd and Nicoló (2005), the risk taking behavior of borrowers increases as interest rates go up due to less intense competition. This view is generalized in De Nicolo, Boyd, and Jalal (2009), which includes a safe asset to make the point that there is no one-to-one relationship between stability and competition. Hakenes and Schnabel (2011) make a similar point, in a model in which banks have equity, so that a capital adequacy ratio can be used to regulate risk taking. The authors obtain the surprising result that limiting the leverage of banks increases entrepreneurial risk taking, since less competition (due to a higher capital adequacy ratio) translates into higher interest rates on loans. The difference between these models and those of the previous paragraph is that here as rates go up the interested borrowers have riskier projects whereas in the alternative banks prefer to lend to riskier projects as competition drives down rates on safe projects.

Recently, Carletti and Leonello (2012) describes a different mechanism to relate competition and stability: in a two period model, competition leads to increased stability. With competition, banks profits from lending are low, so having large reserves is not expensive leading to a stable

---

[8]Following Stiglitz and Weiss (1981).

banking system. When there is less competition they obtain a mixed equilibrium with some banks choosing a risky strategy and others choosing a safe strategy. Hence the banking system is less stable as competition decreases.

Martinez-Miera and Repullo (2010) show that the results of Boyd and Nicoló (2005) depend crucially on having perfect correlation of loan defaults. When loans are not perfectly correlated, more competition reduces the return on loans that do not default, so the total effect of competition on stability depends not only on the reduced riskiness of loans but also on the reduced margin on loans that do not default. Using an imperfectly competitive model, these authors establish that the second effect is dominant under perfect competition and that in less competitive markets there is a U-shared relationship between competition and stability.

Wagner (2010) combines the two effects by noting that even though increased competition leads to lower rates and therefore to borrowers that choose less risky projects, the banks can also influence the level of risk of their loans. When facing lower return due to competition, they will choose borrowers with riskier projects and higher returns, and this will counteract the stabilizing effect of competition described in Boyd and Nicoló (2005).

Freixas and Ma (2012) develops a more tractable model that replicates the conclusions of Martinez-Miera and Repullo (2010) but incorporates the possibility of bank runs of the Diamond and Dybvig (1983) type.[9] They have banks that use two types of funding: insured deposits or uninsured money market funds. They differentiate between portfolio, liquidity and solvency risk and show that the conditions under which competition reduces risk depends on the fraction of insured deposits in bank liabilities, the productivity of projects and the interest rate. When productivity is low and banks are funded with insured deposits, competition increases total credit risk. They argue that their more detailed model allows them to interpret the different results obtained in the empirical literature, which they review in detail.

We have mentioned before the large and growing literature –theoretical and empirical– on

---

[9]Even in the case of bank runs there are two different approaches: the multiple equilibria-sunspot view of Diamond and Dybvig (1983) (the expectation of a collapse, coupled to the maturity mismatch leads to runs). Alternatively, as in Rochet and Vives (2004), the bank fails because the fundamentals are weak and this leads to a higher probability of a run.

interconnectedness among banks and its effects on the stability of a banking system, as for instance Allen and Gale (2004) or Cifuentes et al. (2005) and more recently Allen et al. (2010). This literature is complementary to our paper in the sense that it examines the sensibility to shocks of differing banking systems with different degrees of interconnectedness, but do not examine its interaction with competition.[10] Another paper along these lines is Acemoglu et al. (2013), who examine a model in which there is a range in which increasing interconnectedness makes the banking system more resistant to a real shock, while beyond this range increasing interconnectedness increases the sensibility to real shocks.

The empirical evidence of the relation between competition and economic stability is mixed. Early studies of the effects of bank liberalization in the USA Keeley (1990), Edwards and Mishkin (1995) and others showed that liberalization lowered the charter values of banks and this increased risk taking. For Spain, Saurina-Salas et al. (2007) found that liberalization and increased competition was associated to higher risk, measured as loan losses to total loans.

In cross country studies, diverse studies show that increase competition contributes to stability. This is the case of Schaeck, Cihak, and Wolfe (2009), who use the Panzar and Rosse H-statistic to study the probability of a crisis using 41 countries. They also point out that bank concentration is associated to higher probability of crisis, so concentration and competition capture different aspects of the fragility of banking systems. Similarly, in a working paper, Anginer, Demirgüç-Kunt, and Zhu (2012) use a sample of 63 countries to look at the effects of competition (measured by the Lerner index). They incorporate the (co-)dependency among bank risks, in order to examine systemic financial fragility, rather than at the level of individual banks. They find a stabilizing effect of competition. On the other hand, Beck, Jonghe, and Schepens (2013), who incorporate the regulatory framework and financial market characteristics as an explanatory variable in the country cross sections, find a positive relation between market power and measures of financial fragility.

---

[10] A partial exception is Cohen-Cole et al. (2011), who use a Cournot model where competition reduces total profits and interconnectedness affects the banks that are closely linked to each other. A local shock reduces loans of closely linked banks but tends to increase loans of those that are not closely linked.

After reviewing the evidence, Vives (2010) concludes:

> "Theory and empirics point to the existence of a trade-off between competition and stability along some dimensions. Indeed, runs happen independently of the level of competition but more competitive pressure worsens the coordination problem of investors/depositors and increases potential instability, the probability of a crisis and the impact of bad news on fundamentals."

3   The basic model

We consider an economy with three dates ($t = 0, 1$ and 2) and two periods. There is a continuum of risk neutral entrepreneurs with zero assets, where we denote an entrepreneur by $z \in [0, 1]$. In the first date, $t = 0$, agents decide whether to undertake a risky project which lasts two periods, or to exercise an outside option. Although the risky project is the same for all entrepreneurs, these agents are differentiated by the value of their outside option, which yields a safe return u_z for entrepreneur z at the end of the second period.[11] The distribution of u_z is given by $G(\cdot)$, which has a continuous density $g(\cdot)$ and full support $[0, U]$.

The risky project requires one unit of investment capital at $t = 0$ that the agent must borrow from one of N banks. The project provides returns at $t = 1$ and at $t = 2$. The returns in $t = 1$ depend on the state of the economy, denoted by s, which s can take two values, high (h) and low (l). In state h, which occurs with probability p, the economy has a high productivity shock, i.e., all projects are successful, in which case they return y_1. In state l, the economy suffers a low productivity shock, in which case each project succeeds and returns y_1 with probability $q \in (0, 1)$ and fails (returns 0) otherwise. Figure 1 shows the events over time:

At $t = 1$ all firms (even those that were unsuccessful) can apply for a working capital loan λ from banks, in order to operate in the second period.[12] In the second period there are no shocks

---

[11] This is similar to the assumption in Boyd and Nicoló (2005) and used for the same purpose: to differentiate among entrepreneurs and thus obtain a demand curve for loans.

[12] An alternative is to allow only successful firms to be able to ask for loans, and we have examined this case. It is more complex, because failure will affect both the demand and the supply of loans, whereas in the present case, only
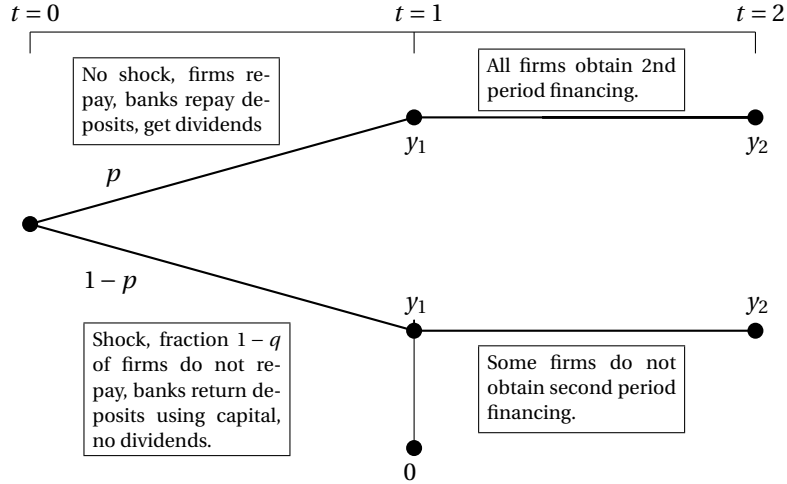
Figure 1: Scheme of returns over time.

and all firms that obtain the working capital loan receive a return of y_2 at the end of the period. The following timeline shows the relevant variables at the different points in time:

The economy has two other clases of risk neutral agents: depositors and banks. Depositors lend to banks each period and receive their money back at the end of the period. Their supply is perfectly elastic at a risk-free rate that we normalize to zero, for notational simplicity. Depositors do not ask for more than the risk-free rate because the government insures deposits at failed banks. This implies that depositors play a passive role in the model.

Define $\beta = (1 + \rho)^{-1}$ as the discount factor associated with the cost of capital ρ. We need the following assumption:

Assumption 1 $\beta p y_1 - 1 > 0$,

Assumption 1 implies that the expected net present value of the first stage of a project is positive, even in the state of nature $s = l$.

Banks are the financial intermediaries of this economy, specialized in channeling funds from investors to entrepreneurs. There are N identical banks. To fund their projects, entrepreneurs borrow from banks, and banks compete to attract entrepreneurs. At date t, each bank extends

loan supply is affected. On the other hand, we believe our formulation is reasonable if we interpret the first loan as one of initial investment plus working capital and the second one as a loan of working capital and for maintenance costs.

loans l_t that are financed by deposits d_t and inside equity e_t. Hence the budget constraint for a representative bank at date t is:

$$l_t \leq d_t + e_t \tag{1}$$

Each bank is run by a single owner-manager who provides the equity e_t; the owner's opportunity cost of capital is $\rho > 0$, so that equity financing is more expensive than deposit financing. This assumption is typically assumed in the literature.[13]

Banks compete for borrowers in the first stage and have an ongoing relationship with the borrower in the second period, so they can extract all rents from borrowers in the second stage.[14]

We assume that entrepreneurs do not use internal financing in period 2. This assumption is common in relationship-banking models.[15] Note also that if we assumed (Repullo and Suarez, 2008) that entrepreneurs' first-period profits are small relative to the amount of the working capital loan, the effects of relaxing this assumption would be negligible. Also, following Repullo and Suarez (2013), we assume that it is impossible to recapitalize a bank at date $t = 1$. Their argument, which we adopt, is that the dilution costs of an urgent equity call are high for banks with opaque assets.

Finally, there is a financial regulator that imposes capital adequacy requirements that limit the amounts that banks can lend to fixed multiple of their capital. For banks, this implies that:

$$l_0 \leq \frac{e_0}{\alpha_0}, \qquad 0 \leq \alpha_0 \leq 1 \tag{2}$$

$$l_1 \leq \frac{e_1}{\alpha_1^s}, \qquad 0 \leq \alpha_1^s \leq 1, \ s \in \{h, l\} \tag{3}$$

where $\alpha \equiv (\alpha_0, \alpha_1^h, \alpha_1^l)$ indicates the capital adequacy requirement in period 0, in period 1 at state h and in period 1 at state l respectively.

---

[13]See Berger and Ofek (1995) for a discussion of this issue; and Gorton and Winton (2003), Hellman et al. (2000) and Repullo (2004) for a similar assumption.

[14]Another simplifying assumption, that allows us to simplify the first period expressions for demand. Note that any small cost advantage to the incumben bank allows it to make a deal to share this advantage and maintain the relationship. Here we assume that all rents in the relationship are kept by the bank. For relationship lending and rents see Yafeh and Yosha (2011), Hauswald and Marquez (2006) and Degryse and Ongena (2008).

[15]See, for example, Sharpe (1990), and also von Thadden (2004).

We assume that α is fixed by the regulator and there exists commitment no to change it in the future. However, the regulator can set $\alpha_1^l \leq \alpha_1^h$, i.e., it can announce that in case of a shock, the capital adequacy requirement will be relaxed, to a predetermined level.

4   Equilibrium

Since this is a two period model, we solve it by backwards induction.

4.1   Equilibrium at $t = 1$

We assume, at seems reasonable, that only in the state of the nature l there is the possibility of a *credit crunch*[16], in the sense that some profitable projects cannot get financing –even with no uncertainty about their profitability– because banks do not have enough equity (capital plus reserves) to finance them, given the capital adequacy restriction. When the state of the nature is h, all projects succeed and managers pay back their first period loans. Thus banks have enough capital and reserves, after returning the deposits, to finance all applications for loans in period 1, and all agents know this.[17]

Suppose that the realized state of nature was s. At date $t = 1$, the following variables are taken as given by agents: (1) the equilibrium interest rate charged on first period loans, r_0; (2) the banks' capital in the state s, e_1^s; (3) the total amount of credit given by the representative bank to finance projects in the first period, l_0; (4) the number of entrepreneurs that obtained funding in the first period, $G(\bar{u})$, where $\bar{u}$ is the utility cutoff for entrepreneurs.

Entrepreneurs, even if their projects fails in the first period, can ask for a loan of amount λ from the same bank from which they asked their original loan, because of our assumption of an *ongoing relationship*.[18] Given that firms cannot apply for loans from other banks, the incumbent

[16]A *credit crunch* is defined as a situation in which there is a reduction in the general availability of credit or a sudden tightening of the conditions required to obtain a loan from the banks.

[17]We can always adjust the parameters of the model –in particular, the magnitude of the shock– to have this case in state l. That is, we do not examine the case of a capital constrained banking system.

[18]It is possible to consider as an alternative assumption that firms that fail in the first period go out of the market, and we have also examined this case (which has the added complication that in the bad state, both the demand and

bank can extract all the rents from its borrowers at this stage, and the entrepreneurs' partici-
pation constraint will be binding in equilibrium.[19] This means that banks cannot expect failed
entrepreneurs to pay a penalty fee for having defaulted on their first period loans. Hence, if
entrepreneur z gets financing, we obtain the interest rate charged to that entrepreneur:

$$y_2 - (1 + r_1)\lambda = 0 \Rightarrow 1 + r_1 = \frac{y_2}{\lambda} \tag{4}$$

As the second stage of the project is profitable for banks, banks will want to finance the
maximum number of these projects they can. The demand for second period loans is λl_0, where
we have used the fact that first period loans are of size 1. Thus, in state s, the bank solves the
following problem at $t = 1$:

$$\underset{\{l_1^s, d_1^s, Div^s\}}{\text{Max}} \beta \left[(1 + r_1) l_1^s - d_1^s \right] + Div^s$$

$$\text{s.t.} \quad e_1^s + d_1^s - l_1^s = Div^s$$

$$e_1^s - Div^s \geq \alpha_1^s l_1^s$$

$$\lambda l_0 \geq l_1^s$$

$$Div^s, l_1^s, d_1^s \geq 0$$

For each bank and in each state $s = h, l$, the decision variables are the total amount of credit
to provide, l_1^s, the total amount of deposits to raise, d_1^s and the first period dividends policy, Div^s.
The objective function is the discounted utility of the representative bank at date $t = 1$, and it
consists of two terms. The first term $\beta \left[(1 + r_1) l_1^s - d_1^s \right]$ is the net present value of the bank's net
profits of date $t = 2$, where $(1 + r_1)$ is determined as in (4). The second term Div^s is the cash left

the supply of loans depends on the fraction $1 - q$ of failing firms). However, in an interpretation of the original
investment as including initial investment plus working capital, and a second period in which only working capital
is needed, because the project is not a failure but has not met initial expectations, the interpretation we include is
more appropriate.

[19]This is not essential; the borrower could split the second period surplus with the bank, the division of the
surplus reflecting the ease of substitution with other banks. However, including this possibility would have added a
parameter to the model without materially changing our results.

over, which is used to pay dividends to shareholders at $t = 1$.

The first (equality) restriction is the time $t = 1$ budgetary restriction of the bank. The second restriction is the capital adequacy restriction, which applies after dividends are paid and determines the loanable funds. The third restriction requires that total loan supply must be smaller than loan demand in each state (otherwise the cost of loans is zero). The following proposition characterizes the equilibrium at date $t = 1$:

Proposition 1 *At date* $t = 1$*, each bank makes loans of* $l_1^s = \min\left\{\frac{e_1^s}{\alpha_1^s}, \lambda l_0\right\}$*, takes deposits of* $d_1^s = (1 - \alpha_1^s)l_1^s$ *and pays dividends* $Div^s = e_1^s - \alpha_1^s l_1^s$*,* $s = h, l$*.*

Proof: See Appendix. ∎

This result links the stock of capital of banks at $t = 1$, e_1^s, to the supply of credit that each bank provides for the second period, l_1^s. In particular, note that if the capital e_1^s is sufficiently low there will be a *credit crunch*, as banks will not be capable of meeting the effective demand for loans, λl_0. This happens because the capital adequacy restriction limits the quantity of credit that banks can supply, and this may result in an unmet demand for credit given by $\text{Max}\left\{\lambda l_0 - \frac{e_1^s}{\alpha_1^s}, 0\right\}$. Notice that there is a credit crunch only in state l if $e_1^l < \lambda \alpha_1^l l_0 \leq e_1^h$, i.e., if banks do not have enough internal capital in that state.

For later analysis, note that if there were an unexpected lowering of the capital adequacy ratios at $t = 1$ in the bad state of the world, it would be possible to eliminate the credit crunch. However, as we will see in section 6.1 below, forbearance is guaranteed to be effective in reducing the variance of second period results only if the change is unexpected, and cannot be an equilibrium.[20]

---

[20]Temporary forbearance of capital adequacy requirements is common when banks are distressed. For instance, in Mitra, Selowsky and Zalduendo, "Turmoil at Twenty: Recession, Recovery and Reform in Central and Eastern Europe and the Former Soviet Union", World Bank 2010. we find:

> "Some previous episodes of systemic banking distress, such as Argentina 2001, Bulgaria 1996, Ecuador 1999, Indonesia 1997, Korea 1997, Malaysia 1997, Mexico 1994, the Russian Federation 1998, and Thailand 1997 have also seen regulatory forbearance. Specifically, to help banks recognize losses and allow corporate and household restructuring to go forward, the *government might exercise forbearance* either on loss recognition, which gives banks more time to reduce their capital to reflect losses, or *on capital adequacy, which requires full provisioning but allows banks to operate for some time with less capital*

Consider now the value of e_1^s. In the state of nature $s = h$, we know that all projects succeed, so each entrepreneur has the resources to pay his debt at the end of the first period. Therefore, the capital of the representative bank at date $t = 1$ is (before paying dividends):

$$e_1^h = (1 + r_0)l_0 - d_0 \tag{5}$$

where $d_0 = l_0 - e_0$ are the deposits that the bank must repay at the end of the first period, just before $t = 1$. As we have assumed that in the state h the representative bank has enough capital to finance all the entrepreneurs who ask for a loan, i.e., there is no *credit crunch*, then it must hold that $\alpha_1^h l_1^h = \lambda \alpha_1^h l_0 \le e_1^h$.

Using the results of Proposition 1, we obtain the net present value of the bank at date $t = 1$ when the state of the nature is $s = h$:

$$\begin{aligned}
\Pi_1^h &= \beta \left[(1 + r_1)l_1^h - d_1^h \right] + \left[e_1^h + d_1^h - l_1^h \right] \\
&= \left[r_0 + e_0/l_0 + \beta(y_2 - \lambda) - \lambda \alpha_1^h (1 - \beta) \right] l_0
\end{aligned} \tag{6}$$

where we have used the interest rate r_1 obtained in equation (4), (5) and the results of proposition 1.

Now we study the case when the state of the world is $s = l$. Recall that in this case, from the point of view of $t = 0$, each project succeeds with probability q and fails with probability $1 - q$. By the Law of Large Numbers, exactly a fraction $1 - q$ of entrepreneurs fail, so in this economy there is no aggregate uncertainty. Therefore, ql_0 entrepreneurs succeed and pay their debts. On the other hand, we have assumed that all agents, including those who fail in the first period and are unable to repay their loans, ask for a working capital loan to continue their projects in the second period. Now, the capital of the representative bank at date $t = 1$ is:

$$e_1^l = \text{Max}\{q(1 + r_0)l_0 - (l_0 - e_0), 0\} \tag{7}$$

than prudential regulations require."

15

where the Max operator arises due to limited liability of banks. Recall that if this capital is zero then the bank fails at date $t = 1$, and its depositors are paid with the residual value of the bank $(q(1+r_0)l_0)$ plus the compensation made by the government from deposit insurance. This happens when the probability of success q satisfies:

$$q < \widehat{q} \equiv \frac{1 - (e_0/l_0)}{1 + r_0} \tag{8}$$

If the fraction of firms that manage to repay after the shock are $q \geq \widehat{q}$, then banks do not fail in the event of a crash, though their second period capital shrinks. As in the previous case, banks always want to finance as many projects as possible. As we have mentioned before, to make things interesting, we assume that in state l banks cannot finance all the entrepreneurs who ask for a second period loan, so that[21]

$$l_1^l = \frac{e_1}{\alpha_1^l} < \lambda l_0 \tag{9}$$

From the discussion above, only a fraction $\theta \in [0, 1)$ of the demand for credit λl_0 is going to be satisfied:

$$\theta = \frac{l_1^l}{\lambda l_0} = \frac{q(1 + r_0) - (1 - (e_0/l_0))}{\lambda \alpha_1^l} \tag{10}$$

The variable θ measures the ratio of the second period economy under a shock to the size of the economy without the shock, i.e., when it is close to one, the economy is able to resist the shock without many ill effects. Similarly, $1 - \theta$ is the fraction of entrepreneurs rationed by banks at date $t = 1$, and can be interpreted as the magnitude of the *credit crunch*; and $1 - q$ can be interpreted as the *magnitude of the shock*, as it represents the fraction of entrepreneurs that cannot repay their loans.

A final observation: as a consequence of proposition 1, in the case of a shock banks do not pay dividends in the first period because reinvesting all repayments into loan renewals is more profitable.

---

[21]Otherwise the case with a shock can be treated as if it were the case without a shock and nothing happens after the shock.

The discounted utility of the representative bank at $t = 1$, after a shock can be written as:

$$\Pi_1^l = \beta \left[(1 + r_1)l_1^l - d_1^l \right] + \overbrace{Div^l}^{=0}$$

$$= \frac{\beta}{\alpha_1^l} \left[\frac{y_2}{\lambda} - (1 - \alpha_1^l) \right] \text{Max} \{ q(1 + r_0)l_0 - (l_0 - e_0), 0 \} \tag{11}$$

which should be compared to the profits at $t = 1$ in the case of no shock, given by equation (6). The next step is to proceed to the analysis of the profit maximization problem at date $t = 0$.

4.2 Equilibrium at t = 0

In the first period, given an aggregate demand for loans $L(r_0)$, banks choose the profit-maximizing volumes of deposits (d_0), and loans (l_0). This automatically defines the equilibrium interest rate r_0 charged to entrepreneurs.

4.2.1 The demand for credit

Given the first period interest rate r_0 charged by banks, we define $\bar{u}(r_0) \equiv$ $[p + (1 - p)q] [y_1 - (1 + r_0)]$ as the expected net future value (at the end of the second period) that the entrepreneur will obtain if he undertakes the two-stage risky project. Observe that the entrepreneur gets no rents from operating the firm in the second period because the banks extract all profits. An entrepreneur z will be willing to embark in this venture rather than stay with the safe option only if $\bar{u}(r_0) \geq u_z$. These participation constraints implicitly define an aggregate loan demand that is decreasing in the interest rate at $t = 0$, given by:

$$L(r_0) = \int_0^{\bar{u}(r_0)} g(u)du = G \left([p + (1 - p)q][y_1 - (1 + r_0)] \right) \tag{12}$$

with $\frac{\partial L(r_0)}{\partial r_0} = g(\bar{u}(r_0)) \frac{\partial \bar{u}(r_0)}{\partial r_0} < 0$. As usual, it will be more convenient to work with the inverse demand function, $r_0(L)$. We can rearrange the last equation to obtain:

$$1 + r_0(L) = y_1 - \frac{G^{-1}(L)}{p + (1-p)q} \tag{13}$$

where $\sum_{j=1}^{j=N} L_j \equiv L$. This expression defines explicitly a downward sloping inverse demand of loans. We make the following standard assumption:

Assumption 2 *The distribution function of outside options $G(z)$ is twice-continuously differentiable, positive, and concave for all $L \in (0, 1)$*

4.2.2 The banks' optimization problem

At the beginning of the first period, each bank chooses the volume of its deposits (d_0), and loans (l_0). Given the balance sheet identity, $l_0 = d_0 + e_0$, only one of these variables can be chosen independently. Recalling that α_0 is the capital adequacy constraint at $t = 0$, the representative bank at $t = 0$ solves:

$$\underset{\{l_0\}}{\text{Max}} \, \Pi_0 \equiv \beta \left[p\Pi_1^h + (1-p)\Pi_1^l \right] - e_0$$

$$\text{s.t.} \quad l_0 \leq (e_0/\alpha_0) \tag{14}$$

where the objective function is the expected net present value of profits of the bank while the restriction corresponds to the capital adequacy condition at $t = 0$.

Recall from the comments on equation (8) that if $q < \widehat{q}$, banks go bankrupt in the low state ($e_1^l = 0$). In that case, $\Pi_1^l = 0$ and there are positive profits only in the good state ($\Pi_1^h > 0$). Noting from the definition of $\hat{q}$ that a reduction in l_0 leads to a reduction in $\hat{q}$, banks, by lending less could have remained solvent and thus would maximize over both the good and bad states of the world.[22] This corresponds to what we denote by *prudent* behavior, leading to a symmetric *prudent equilibrium*. Conversely, behavior leading to bankrupt bank is *imprudent* behavior, and

---

[22]Observe that

$$sgn \left(\frac{d\hat{q}}{dl_0} \right) = sgn \left(\frac{e_0}{l_0^2}(1+r_0) - \frac{dr_0}{dl_0}\left(1 - \frac{e_0}{l_0}\right) \right) > 0.$$

leads to an *imprudent equilibrium.* This is the type of equilibrium behavior in which banks could be accused of "privatization of profits and socialization of losses".

Hence, there are two different expressions for the profit function, depending on whether $q > \widehat{q}$, and banks survive the shock, and the case in which the inequality is reversed and banks fail. Thus we define two functions associated to profits in the two states:

$$\Omega^h(L) = \beta p \left(\left(y_1 - \frac{G^{-1}(L)}{p + (1-p)q} \right) - 1 + \beta(y_2 - \lambda) - \lambda \alpha_1^h(1 - \beta) \right)$$

$$\Omega^l(L) = \frac{\beta^2(1-p)}{\alpha_1^l} \left(\frac{y_2}{\lambda} - (1 - \alpha_1^l) \right) \left(q \left(y_1 - \frac{G^{-1}(L)}{p + (1-p)q} \right) - 1 \right) \tag{15}$$

Profits at $t = 0$ depend on whether the bank fails in period 1 in the case of a shock, i.e., if $e_1^l = 0$. Observe that even when banks follow an imprudent lending policy, the proportion of firms q that fail under a shock is relevant. The reason is that a fraction $1 - q$ of entrepreneurs make profits in the first period under a shock, and this possibility has an effect on the demand for loans. We have that total expected profits for a bank at $t = 0$ are:

$$\Pi_0(l_0) = \begin{cases} \Omega^h(L)l_0 + (\beta p - 1)e_0 & \text{if } e_1^l = 0 \\ (\Omega^h(L) + \Omega^l(L))l_0 + \left((\beta p - 1) + \frac{\beta^2(1-p)}{\alpha_1^l} \left(\frac{y_2}{\lambda} - (1 - \alpha_1^l) \right) \right) e_0 & \text{if } e_1^l > 0 \end{cases} \tag{16}$$

There are two points to make about this expression for bank profits. First, banks maximize profits subject to the capital adequacy restriction $l_0 \leq e_0/\alpha_0$. If an *imprudent* equilibrium is chosen this condition is binding, because $\beta p - 1 < 0$, and therefore $\Omega^h(L)$ has to be strictly positive or the imprudent equilibrium would have negative profits. Since the *imprudent* equilibrium is linear in l_0, the capital constrain must be binding. Second, observe that the two profit functions are different and cannot be transformed into one another via a continuously differentiable transformation, because of the non-negativity constraint on profits if the bank collapses after a shock.

Notation: We define the following notation which will be useful in the following:

$$\Psi \equiv y_1 - \frac{G^{-1}(L^\star)}{p + (1-p)q} - \frac{[1 + (N-1)v]l_0^\star}{(p + (1-p)q)G'(G^{-1}(L^\star))}, \quad N: \text{number of banks.} \tag{17}$$

$$H \equiv \frac{\beta^2(1-p)}{\alpha_1^l}\left(\frac{y_2}{\lambda} - (1 - \alpha_1^l)\right) \tag{18}$$

$$\phi \equiv 1 - \beta(y_2 - \lambda) + \lambda\alpha_1^h(1 - \beta) \tag{19}$$

where the term $(N-1)v$ is the aggregate conjectured response of the other banks to a marginal increase in l_0 by a bank. Here v is the conjectural variation parameter, corresponding to the beliefs of bank i of its rivals' reaction to its own loan supply choices. We assume that v is identical for all banks. When $v = -1/(N-1), 0, 1$ we reproduce the Bertrand, Cournot and collusive equilibria (increased v implies less competition). In subsection 7.3 we show that we can reproduce the results when considering Cournot equilibria and increased competition is caused by increasing the number N of banks.

Observe that H is the contribution to profits of one additional unit of capital at time 1 in the bad state of the world, weighed by the probability of a shock and discounted to time $t = 0$. We will use the following important assumption:

Assumption 3 $\beta p + H > 1$

The assumption means that the average value of an additional unit of bank capital at $t = 1$, discounted to $t = 0$, is bigger than one, i.e., it is profitable on average to have more period 1 capital (See appendix).

In order for imprudent equilibria to have a chance of being chosen, we must ensure that the *prudent* equilibria is interior to the capital adequacy constraint $l_0 = (e_0/\alpha_0)$ (otherwise there can be no *imprudent* equilibria , since they involve more lending than *prudent* equilibria). We need the following assumption:

Assumption 4

$$\beta p \phi + H > 0$$

This is a sufficient condition for the *prudent* equilibria to be interior to the capital adequacy constraint (proof in the appendix). We can rewrite this condition as

$$p\beta\phi + H = p\beta \left(1 - \beta(y_2 - \lambda) + \lambda\alpha_1^h(1 - \beta)\right) + H > 0. \tag{20}$$

which we use later. This condition ensures that projects are not so profitable that $1 + r_0 \leq 0$, i.e., that in a *prudent* equilibrium banks are unwilling to give away money in the first period even under Bertrand competition.

5   Existence and uniqueness of equilibrium

As there are two potential profit functions, corresponding to *prudent* and *imprudent* behavior of banks, there are potentially two families of equilibria. We use the Pareto optimality criterion, which is standard in Industrial Organization, to choose among symmetric equilibria with the same starting capital e_0.[23] We show that there is a neighborhood of $p = 0$ in which the prudent equilibrium is chosen for all intensities of competition.

Our procedure is as follows: first, we show that there is a unique symmetric equilibrium to both *prudent* and *imprudent* behaviors by banks. Next we show that as competition decreases, the gap between the profits at the *prudent* equilibrium and the *imprudent* equilibrium increases. Then we show that under Bertrand competition, when $p = 0$ (i.e., the shock is a certainty) the *prudent* equilibrium has strictly positive profits while the *imprudent* equilibrium has negative profits. Hence the result continues to hold in some neighborhood of $p = 0$ for competition that is less intense. Thus we have shown that there is a set of positive measure in which the only equilibrium is the *prudent* equilibrium, and that in general, the Pareto criterion assures us that only one equilibrium will be chosen by banks.

Lemma 1 *There is a unique equilibrium of each type (*prudent, imprudent*) to the game among*

---

[23]Global games (Morris and Shin, 2003) are not used in these settings due to technical difficulties.

banks for any intensity of competition. In the non-Bertrand case we have

$$\frac{\partial^2 \Pi_i}{\partial l_i^2} < 0 \quad and \quad \frac{\frac{\partial^2 \Pi_i}{\partial l_i^2}}{\frac{\partial^2 \Pi_i}{\partial l_i \partial l_j}} > 1$$

Proof: We examine the case of *prudent* equilibria here and the appendix contains the very similar analysis of *imprudent* equilibria. We begin by noting that the Bertrand case must be treated separately, because in that case L is presumed constant by banks. Hence, banks face a linear maximization problem, leading to *bang-bang* solutions in which either all banks do not lend (if $(\Omega^h(L) + \Omega^l(L)) < 0$) or they lend up to the capital adequacy constraint if the sign is positive. Interior solutions are possible only if $(\Omega^h(L) + \Omega^l(L)) = 0$. There is only one interior symmetrical equilibrium, since we require $Nl_0 = L$, where L satisfies

$$0 = \Omega^h(L) + \Omega^l(L)) = (\beta p + Hq)(1 + r_0) - (\beta p \phi + H)$$

Since $1 + r_0$ is strictly decreasing in L, there is a single solution L and therefore a single symmetrical level of first period lending l_0 under Bertrand competition.

In non-Bertrand cases, the profit functions satisfy the standard conditions for existence and uniqueness of equilibria. We consider the case of *prudent* equilibria:

$$\frac{\partial \Pi_i}{\partial l_i} = (\beta p + Hq)\Psi - (\beta p \phi + H) \tag{21}$$

and thus

$$\frac{\partial^2 \Pi_i}{\partial l_i^2} = (\beta p + Hq)\frac{\partial \Psi}{\partial l_i} = -(\beta p + Hq)\left(\frac{2}{P_e G'(G^{-1}(L))} - \frac{l_i G''(G^{-1}(L))}{P_e G'^3(G^{-1}(L))}\right) < 0$$

and:

$$\frac{\partial^2 \Pi_i}{\partial l_i \partial l_j} = (\beta p + Hq)\frac{\partial \Psi}{\partial l_j} = -(\beta p + Hq)\left(\frac{1}{P_e G'(G^{-1}(L))} - \frac{l_i G''(G^{-1}(L))}{P_e G'^3(G^{-1}(L))}\right)$$

from which we derive:

$$\frac{\frac{\partial^2 \Pi_i}{\partial l_i^2}}{\frac{\partial^2 \Pi_i}{\partial l_i \partial l_j}} = \frac{\left(\frac{2}{P_e G'(G^{-1}(L))} - \frac{l_i G''(G^{-1}(L))}{P_e G'^3(G^{-1}(L))} \right)}{\left(\frac{1}{P_e G'(G^{-1}(L))} - \frac{l_i G''(G^{-1}(L))}{P_e G'^3(G^{-1}(L))} \right)} > 1$$
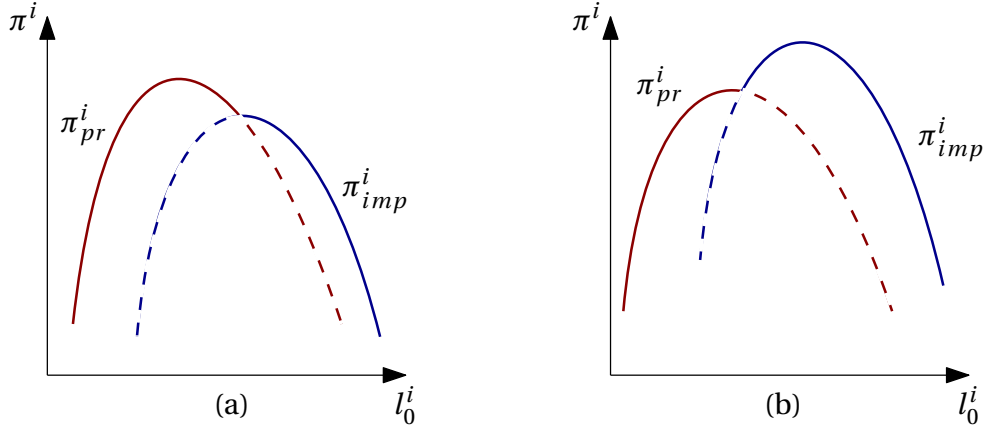
Figure 2: Equilibrium configurations

Consider diagram 2, which shows the possible configurations of the profit function for a single bank, given that the choices of the other banks are in a symmetric equilibrium. In general there will be two equilibria: a *prudent* equilibrium and an *imprudent* equilibrium. Each curve describes all possible deviations of the bank given what its rivals are choosing as their first period lending l_0 in the corresponding symmetric equilibrium. In figure (a) There is no incentive for the bank to jump to the lending associated to the imprudent equilibrium (given that the other banks are playing the prudent equilibria), since the point at which the curves cross is where $e_1^l = 0$ and its profits are lower by switching. In figure (b) the imprudent equilibrium is selected by the Pareto criterion. Note that prudent profits are not defined beyond the crossing, since $e_1^l < 0$ at those points and the prudent profit function is not defined there.

If the world were to resemble (b), then the role of regulation is to restrict l_0 so that the imprudent equilibrium cannot be attained. To see this last point, consider the case of figure 3. In the figure, the vertical line corresponds to the lending limit defined by the first period capital adequacy condition (14) and limits first period lending of any bank to that level, so that even

though the *imprudent* equilibrium is preferred by banks, it cannot be chosen and banks prefer
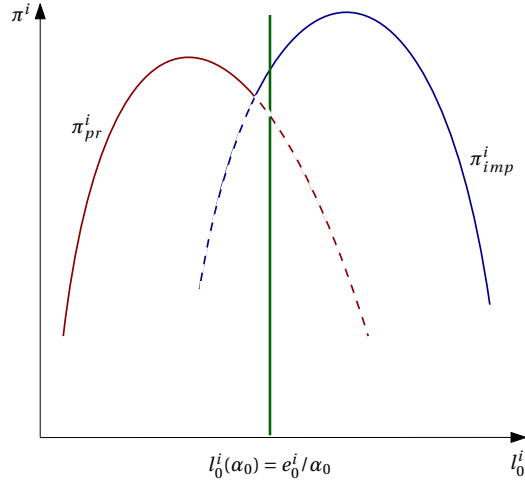the *prudent* equilibrium to their other (symmetric) options.[24]

Figure 3: A prudential limit on overlending

The next step in the proof is to show that as competition decreases, the difference between
the profits at the *prudent* and the *imprudent* equilibria increase.

Proposition 2 *As competition decreases, the* prudent *equilibrium becomes more attractive com-
pared to the* imprudent *equilibrium (* $\frac{\partial (\Pi_\star^P - \Pi_\star^I)}{\partial v} > 0$ *).*

Proof: See appendix. The intuition for this result is that with more competition a given shock
leaves a bank with smaller values of second period capital in case of shock ($e_i^l(l_0^{Pr*}; q)$). This
means that the prudent equilibrium is less attractive, since banks can finance fewer firms in the
second period. The imprudent equilibrium, which foregoes financing firms in the second period
in case of shock, becomes relatively more attractive.

The last stage in the proof is to find conditions under which the *prudent* equilibrium is prefer-
able to the *imprudent* equilibrium in the Bertrand equilibrium. By proposition 2, this means that
for any lower degree of competition, the *prudent* equilibrium continues to be chosen. More gen-
erally, if there is any level of competition for which under specified conditions the *prudent* equi-

---

[24]If the crossing between the two curves occurs to the left of the maximum of the curve corresponding to the
prudent equilibrium, one cannot use the FOC to characterize the equilibrium. In this case a *prudent* equilibrium
exists only if the capital adequacy restriction lies to the left of the crossing, where $(\partial \Pi_{pr}^i / \partial l_0^i) > 0$.

librium is preferred to the *imprudent* equilibrium, then this continues to hold true for any lower degree of competition.

The last result we need is to show that there is a region in parameter space where *prudent* equilibria exist and are preferred to *imprudent* equilibria.

Proposition 3 *There is a neighborhood of $p = 0$ in which* prudent *equilibria are preferred for all intensities of competition.*

Proof: We show that there is a neighborhood $p = 0$ where *prudent* equilibria are preferred over *imprudent* equilibria in the Bertrand case, and therefore by proposition 2, will also be preferred in less competitive banking systems.

Recall that under Bertrand competition, a prudent interior equilibrium can exist only if $\Omega^h(L) + \Omega^l(L) = 0$. In that case, the bank's profits are positive

$$\Pi_B = (\beta p + H - 1)e_0 > 0.$$

The other possibility is that $\Omega^h(L) + \Omega^l(L) > 0$ (if this term is negative there is no lending). In that case, $l_0 = e_0/\alpha_0$ in the *prudent* equilibrium. This is the same amount of lending as in the *imprudent* equilibrium. Thus the outcomes will be the same in the bad state, which is inconsistent with at least one of the two equilibria. That is, second period capital cannot be strictly positive (the requirement for a *prudent* equilibrium) and zero (corresponding to an *imprudent* equilibrium at the same time. Thus prudent equilibria can only be interior equilibria, or there are no *imprudent* equilibria.

Note that if $p = 0$, the shock always hits. The *imprudent* equilibrium always leads to bankruptcy of the bank and zero profits. Thus when $p = 0$, only interior *prudent* equilibria are viable and we showed above that profits in the Bertrand *prudent* equilibrium are strictly positive. By continuity of the profit functions, there is a neighborhood of $p = 0$ in which *prudent* equilibria are also chosen. Note also that by proposition 4 , lower intensity of competition leads to less lending, and therefore lower probabilities of collapse. ∎

It is interesting to note that there is also a range for which the *imprudent* equilibria are preferred for any intensity of competition. Consider the case where $p \approx 1$ and $q \approx 0$, so there is a small probability of a shock, but when it occurs avoiding a banking collapse does not provide an advantage because most firms fail; $e_1^l > 0$ but small in the *prudent* equilibrium. In this case, it is easy to show that an *imprudent* equilibrium is preferred (in the limit, banks do not lend in the *prudent* equilibrium). Essentially, making an effort to avoid an improbable collapse, and not gaining much by it, leads to an inferior equilibrium, given the cost of constraining lending in the highly probable state with no shock. This result becomes clearer when we examine the expressions for Expected GDP that are developed below.

6 Comparative statics

Having shown the existence ranges of Pareto selected *prudent* equilibria for certain parameter configurations, we can proceed to examine the comparative statics in this financial system. This corresponds to the case in which the economy is subject to relatively small shocks that do not endanger the banking system, or alternatively, that the banks are very well capitalized; or finally, that the value of the parameter α_0 restricts lending as in figure 3.[25] In order to do comparative statics we recall Assumption 3, which is used to show the following important result:

Proposition 4 *Increased competition in banking (lower v) increases first period lending (and reduces the first period interest rate) in both prudent and imprudent equilibria. Moreover $e_1^l \downarrow$.*

Proof: Note that

$$
\frac{\partial^2 \Pi_0}{\partial l_0 \partial v} = - \left(\frac{\beta p + \frac{\beta^2 (1-p)}{\alpha_1^l} \left(\frac{y_2}{\lambda} - (1 - \alpha_1^l) \right) q}{(p + (1-p)q)} \right) \frac{d}{dl_0} \left(\frac{l_0 \frac{\partial L}{\partial v}}{G'(G^{-1}(L))} \right)
$$

---

[25] In this last case we need, in addition, that $q < \hat{q}$ so the prudential equilibrium is viable, i.e., that the crossing of π_{pr}^i and π_{imp}^i occurs to the right of the highest point in π_{pr}^i. Otherwise the banking system is inherently unstable and we cannot perform comparative statics.

The denominator of the first expression in the RHS is positive by Assumption 3, and

$$\frac{d}{dl_0}\left(\frac{l_0\frac{\partial L}{\partial v}}{G'(G^{-1}(L))}\right) = \frac{\frac{\partial L}{\partial v} + l_0\frac{\partial^2 L}{\partial l_0 \partial v}}{G'(G^{-1}(L))} - \left(l_0\frac{\partial L}{\partial v}\frac{\partial L}{\partial l_0}\right)\left(\frac{G''(G^{-1}(L))}{G'(G^{-1}(L))^3}\right) > 0$$

where the last inequality is implied by the fact that $G(\cdot)$ is increasing and concave. Thus profits have increasing differences in $(l_0, -v)$. We can use Topkis' Lemma, which implies that first period lending l_0^* is decreasing in v. For the case of *imprudent* equilibria, see the appendix. ∎

This result implies that a more competitive banking system leads to increased economic activity in the first period, and to more efficiency in that period. There are more entrepreneurs that carry out projects given the lower interest rates. On the other hand, banks become riskier, because they are more leveraged. In case of a shock, a larger fraction of the banks' capital will be wiped out; that is, the banks' loan book is riskier. [26] Thus, it is not clear that the expected second period product is higher as competition increases.

The expected value of GDP over the two periods is:

$$Y^P = p[(y_1 - 1) + (y_2 - \lambda)]l_0^\star + (1-p)[q(y_1 - 1)l_0^\star + \frac{e_1^{l\star}}{\lambda\alpha_1^l}(y_2 - \lambda)] + \int_{\bar{U}(r_0^\star)}^{G_{max}} u\,dG(u)$$

Proposition 5 *When the risk of a shock is low, increased competition raises expected GDP and second period activity in a* prudent *equilibrium.*

Proof: See appendix.[27]

The reason for the qualifier is that increased competition raises first period activity as well as second period activity if there is no shock, but lowers it otherwise. When the probability of a shock is low ($p \approx 1$), increased competition raises expected second period activity, and thus

---

[26]Moreover individual loans are riskier for the bank: since the probability of renewing a loan is smaller when the fraction of the bank's capital that is lost increases, loans become riskier and therefore have a lower return for the bank, in addition to the fact that interest rates are lower.

[27]Note that in the event of a banking collapse, deposit insurance is a transfer (from taxpayers to depositors) so it does not alter GDP. Income is affected by the failure of projects after the shock, and that is reflected in the expression for GDP, both in the case of a banking collapse in which there is no output from any investments, and in the case without a collapse, in which output after a shock decreases.

more competition unambiguously raises expected GDP.[28] When the probability of a shock is high ($p \approx 0$), competition reduces expected second period activity.

Note that in the case $p = 1$ and $q = 0$, there is no leverage in the *prudent* equilibrium (i.e., $e_0 = l_0$) and this provides strictly less GDP than the *imprudent* equilibrium, where banks do use leverage. Therefore, there is a neighborhood in which expected output is higher under *imprudent* equilibria. As mentioned in the previous section, the effort to avoid an outcome which is unlikely (a banking collapse following the very rare shock), specially when the shock is very severe –and therefore few firms can pay back their loans– imposes too severe a constraint on lending, and it is preferable to risk the low probability shock. Thus, there is a range in parameter space where *imprudent* equilibria are preferred by banks and lead to higher welfare than the *prudent* equilibria.

The next result shows the risks associated to increased financial competition: it shrinks the range of shocks for which the prudent banking equilibrium is valid. Recall that in (8) the value $\hat{q}$ is the fraction of firms that survives a shock that leaves the banks on the threshold of failure in the case of a prudent equilibrium. An increase in $\hat{q}$ means that a smaller shock endangers the system. We have:

Lemma 2 *Increased banking competition (lower v) decreases the range of shocks ($q \in [\hat{q}, 1]$) for which the prudent equilibria ($e_1^l(l_0^{Pr*}; q) > 0$) are well defined.*

Proof: See appendix.

Next we show that even for prudent equilibrium, so there are no banking crisis, increased competition increases risk in the economy, because the magnitude of the "sudden stop" in lending in the second period after a shock is larger. Recall that $\theta \equiv (l_1^l / l_1^h)$ (see (10)) measures how much lending there is after a shock compared to lending without a shock, and that lending is directly associated to economic activity. Moreover, a fall in θ increases the variance of second period lending, and therefore the variance of second period economic activity.[29]

---

[28]Observe that in an *imprudent* equilibrium, there is second period activity only if there is no shock, and without a shock, competition increases activity. Thus increased competition always increases expected GDP in *imprudent* equilibria.

[29]Since $\text{Var}(l_1) = p(1-p)(\lambda l_0 - l_1^l)^2 = p(1-p)(1-\theta)^2(\lambda l_0)^2$.

Proposition 6 *Consider a prudent equilibrium. Increased banking competition (lower v) leads to larger reductions in lending in the second period in the case of a shock.*

' **Proof:** See appendix.

Note that this proposition, at its heart, has the notion that the equity of banks after a shock is smaller as competition increases. The next result is expected: a reduction in the size of the shocks ($q \uparrow$), i.e., a reduction in risk, leads to higher first period lending.

Proposition 7 *Consider a prudent equilibrium. Assume that under Bertrand competition, first period lending is interior to the capital adequacy constraint. Then less risk (higher q) leads to more lending in the first period (higher l_0^*).*

Proof: See appendix.

6.1 Regulatory forbearance

We have seen in section 4 that unexpected regulatory forbearance on the capital adequacy constraints can eliminate the effects of shock on second period activity (in the case of *prudent* equilibria). In many cases, banks anticipate that the regulator will exercise forbearance after the shock, and this should alter their behavior. Note that this is equivalent to studying the effect of reducing the value of the future capital adequacy parameter α_1^l in the bad state on the banker's problem at $t = 0$.[30] The following results show the effects of regulatory forbearance:

Proposition 8 (Regulatory forbearance) *In a prudent equilibrium, if the financial regulator practices forbearance in the bad state ($\alpha_1^l < \alpha_1^h$), the variance of the second period outcome decreases because l_0 decreases, i.e., $\partial l_0 / \partial \alpha_1^l > 0$.*

Proof: First, note that expected profits for banks in a prudent equilibrium at $t = 0$ can be written as (see 16):

---

[30]Note that when there is no shock, the capital adequacy condition does not bind so relaxing only affects second period behavior when there is a shock.

$$\max_{l_0} \left(\Omega^h(L) + \Omega^l(L) \right) l_0 + \text{constants}$$

After some rearranging, the FOC become:

$$\beta p(\Psi - \phi) + H(q\Psi - 1) = 0$$

the first and second terms correspond, respectively, to the marginal profits in the good and the bad state. Moreover, $\Psi - \phi > 0$, reflecting that *ex post*, the bank would have preferred to lend more in the first period since in the good state, it has more capital than it needs to finance second period projects. Similarly, $q\Psi - 1 < 0$, reflecting the fact that *ex post*, the bank would have preferred to have lent less in the first period, so that she would have had more capital in the second period, allowing it to have more second period lending.

Now note that this division of the FOC isolates the α_1^j, $j = l, h$ parameters, with α_1^l appearing only in H, that is in the condition applicable in case of shock. It is easy to show that $\partial H / \partial \alpha_1^l < 0$. Hence, forbearance (a reduction in α_1^l relative to α_1^h) decreases lending by making it more valuable to have capital in case of a shock. ∎

Observation: Note that a change in α_1^l has no effect on an *imprudent* equilibrium, because in the equilibria, in case of shock $e_1^l = 0$.

To get additional intuition about this result, note that $\text{Var}(l_1) = p(1-p)(\lambda l_0 - l_1^l)^2$. From the definition of e_1^l in (7) we have that $e_1^l = q(1 + r_0)l_0 - (l_0 - e_0)$, and $l_1^l = e_1^l / \alpha_1^l$. Therefore:

$$\frac{dl_1^l}{d\alpha_1^l} = \underbrace{- \frac{e_1^l}{\alpha_1^{l2}}}_{\text{Static Effect} < 0} + \underbrace{\frac{1}{\alpha_1^l} \frac{de_1^l}{dl_0} \frac{dl_0}{d\alpha_1^l}}_{\text{Strategic Effect}} \tag{22}$$

Thus the anticipated effect of regulatory forbearance on second period activity after a shock is composed of two terms. The static effect corresponds to the direct effect of the increased forbearance, i.e., reducing the impact of the shock on second period lending, given the value of second period banking capital. Its sign is always negative or zero. The second term in the RHS

corresponds to the changes induced by the knowledge that, in case of a shock, the regulator will exercise forbearance.

We know that the sign of $(de_1^l/dl_0) = q\Psi - 1 < 0$ in equilibrium. The previous result shows that $\frac{dl_0}{d\alpha_1^l} > 0$, so the strategic term in (22) is also negative. Counterintuitively, the effect of knowing that the government will relax the capital adequacy conditions after a shock reduces first period lending, and thus leads to reduced variance in second period outcomes due to both a strategic and a direct effect. The intuition is that if a shock occurs, equity is more valuable if there is forbearance so banks lend less. We have shown:

Corollary 1 *The effect of expected forbearance is to increase second period lending in the bad state and thus to decrease the effect of the shock and the variance in economic outcomes.*

It is interesting to examine briefly the case in which the regulator behaves inconsistently. Assume for instance that the regulator announces that it will accomodate the capital adequacy parameter act so as to eliminate the effects of a shock, in the case of *prudent* equilibria. Thus second period lending in the bad state satisfies the two conditions:

$$l_1^h = l_1^l = \frac{e_1^l}{\alpha_1^l}$$

In the case of Bertrand competition there is no *prudent* equilibrium, because banks always gain by increasing their loans slightly (they increase their profits in both states of the world), so long as $e_1^l > 0$. Thus only *imprudent* equilibria survive under Bertrand competition when governments use this inconsistent policy.

In the general case, for smaller intensities of competition, the inconsistent behavior of the regulator will lead to increased lending (in a *prudent* equilibrium), with respect to the case of a consistent regulator, which sets the capital adequacy conditions independently of the *ex post* situation. For a proof, see the appendix.

6.2 Comparative statics between types of equilibria

We have been working under the assumption that the equilibrium do not involve a collapse of the banking system (i.e., we are not in an imprudent equilibrium). However, under certain conditions, a collapse of the banking system in the bad state of the world may be convenient for banks, because they do so much better in the good state of the world. That is, under certain conditions it may be convenient for bank owners to "bet the bank" on the non-occurrence of the bad state of the world. We have shown before that it is possible to use the capital adequacy conditions to exclude this possibility, forcing them to be more conservative. However, the regulator may not always apply these conditions, or the regulator may be incapable of supervising the bank's compliance with the rule. For this reason, we explore the case in which *imprudent* equilibria are allowed by the regulator. In particular, we are interested on the possibility that competition may lead banks to become *imprudent*. We now proceed to the main result of the paper.

Proposition 9 *Increased banking competition always leads to increased variance in second period economic outcomes. This occurs within and among types of equilibria.*

Proof: Consider first the case of a prudent equilibrium. Proposition 6 shows that if we are in the range in which increased competition leads to a prudent equilibrium, so there is no switch to an *imprudent* equilibrium, second period economic results have increased variance.

Now consider *imprudent* equilibria. Since competition implies higher lending in these equilibria, economic activity without a shock is higher. On the other hand, when there is a shock, lending and the associated economic activity is always zero. Hence the variance of second period economic activity increases.

Finally, note that by proposition 2, when competition increases, the equilibria can go from *prudent* to *imprudent*, and never in the other direction. We can decompose the effects of increased competition and a switch in type of equilibria as an increase in competition among *prudent* equilibria, and a switch between types of equilibria, keeping constant the intensity of competition. The first effect increases the variance of second period economic activity. Furthermore, the jump

from a *prudent* to an *imprudent* equilibrium, keeping constant the intensity of competition increases the variance of economic results, since loans are larger under the imprudent equilibrium and thus second period economic activity is higher when there is no shock, and the effects of the shock are also more severe.

∎

7   Extensions: Leverage and Cournot Competition

The results on stability depend on competition through its effects on the leverage (or gearing) chosen by banks. At a fundamental level, it is the increase in leverage that causes instability. Thus, even though this paper only compares among Collusive, Cournot and Bertrand equilibria, one could use other models to alter the intensity of competition. In section 7.3 we consider the case of Cournot competition and we consider the added competition due to an increase in the number of banks.

Noting that leverage is key to understanding the amplification effect raises the question of whether there are other policies that increase leverage and therefore tend to increase instability. Consider the following two examples.

7.1   Financial liberalization

Up to now we have assumed that the cost of funds for banks is zero, but we now examine the case of a positive cost of funds in a closed financial market, This would not affect our qualitative results, but would reduce lending and leverage with respect to our baseline model. Now assume that this closed economy liberalizes its financial markets and savings can flow in or out of the economy, but there is no change in the structure of the banking market –foreign banks cannot operate in the country–. We also assume that the probability of a shock and its depth remain the same as before liberalization.

Then, if the domestic rate cost of funds prior to liberalization was higher than the interna-

tional rate, the reduction in the cost of funds after liberalization will increase lending and bank leverage. Moreover, since the pass-through of a cost reduction to rates on loans is higher in a more competitive market, the positive impact of financial liberalization on leverage will be higher in more competitive banking markets. Hence the impact of liberalization on second period instability is reinforced when the banking sector is more competitive. This reasoning may explain the observation that in several developing countries, initial financial liberalization was accompanied by financial instability.

7.2 Improved creditor protection

Consider improved creditor protection as described in Balmaceda and Fischer (2010). Assume that in the case of a shock, firm that do not obtain a second period loan still have a residual value for the bank. This liquidation value of the project is received in period 2, but banks cannot include it as part of first period capital.[31] Thus these liquidation values cannot be used to increase the supply of second period loans. However, since projects are more profitable overall, leverage increases. Moreover, if this residual value increases with the quality of creditor protection, leverage will increase. In this case, we get the somewhat surprising result that second period instability increases with increased creditor protection.

7.3 Increasing competition in a Cournot equilibrium

We have used conjectural variations to study the effects of increased competition in passing from a collusive equilibrium to a Cournot equilibrium and then Bertrand. However, we can also study the effects of increased competition by fixing the type of competition to be Cournot, and increasing the number of banks N.

We omit the bank index, to simplify the notation and because the equilibrium is symmetric. First, we show that for each individual bank, loans decrease as the number of banks increases. Next we show that the aggregate level of loans increases with the number of banks. Note that

---

[31]Perhaps because these liquidation values have a probability of being zero.

this means that the interest rate charged in the first period decreases ($r_0 \downarrow$). Finally, we show that profits of each bank decrease. These results are used below to show that we can reproduce the main results of the paper when increased competition occurs through an increase in the number of banks in a Cournot equilibrium.[32]

Lemma 3 *In the Cournot equilibrium, as the number of banks increases, individual bank loans decrease:* $\frac{dl_0^*}{dN} < 0.$

Proof: See appendix. ∎

Lemma 4 *As the number of banks increases, aggregate loans increase:*

$$\frac{d(Nl_0(N))}{dN} > 0.$$

Proof: See appendix. ∎

Finally we have:

Proposition 10 *As the number of banks increases, profits per bank are smaller:* $\frac{d\pi}{dN} < 0.$

Proof: See appendix.[33] ∎

Note that since both l_0 and r_0 fall with increased N, $e_1^l = [q(1+r_0)-1]l_0 + e_0$ also falls with N. Hence we can use the same proof as in the appendix to show that as the number of firms increases, the imprudent equilibrium becomes more attractive, i.e., proposition 2. Note also that increasing the number of banks amplifies the instability of the imprudent equilibria, proposition 6. For the case of prudent equilibria, we can prove proposition 4 by an analogous proof, using the signs of the derivatives in the first two lemmas above. Similarly, proposition 5 also holds when increasing

---

[32]Added competition under Cournot requires more aggregate initial bank capital (Ne_0), in contrast to the case of increased competition in the previous sections. Even then, we recover the main results. The effects would have been stronger if we had reduced the capital of each bank to keep total initial capital constant.

[33]We could also have used the general results of Amir and Lambson (2000), who examine the results of entry in general Cournot markets.

35

the number of firms, using an analogous proof. This means that the main result, proposition 9 can be proved using the same argument as in the case of conjectural variations.

Hence all of our results on the effects of increased banking competition continue to hold in the case of Cournot equilibrium when we increase the number of banks.

8 Conclusions

This paper has examined the effects of increased banking competition in two period model where a first period shock to economic activity leads to defaults on loans. These defaults lower the capital and reserves of banks, reducing their lending in the second period. Thus the first period shock is amplified by the banking system. We study the effects of varying degrees of competition in this setting.

The model allows us to understand several phenomena in the interaction between banking competition, economic activity and regulation. We have shown that there are two types of equilibria, which we denote by *prudent* and *imprudent* equilibria. Equilibria of the first type amplify the initial shock but do not cause the collapse of the banking system and the breakdown of lending activity, as occurs in the second type of equilibria. Both types of equilibria can be Pareto Optimal under different circumstances, such as the prevalence of shocks and their magnitude.

We have a series of results related to the effects of increasing competition among banks. First, as competition increases, the *imprudent* equilibria become relatively more attractive to banks. Moreover, even when we consider only *prudent* equilibria, increased competition means that the amplification of the initial shock is larger, because banks tend to lend more and therefore a shock leads a larger reduction in capital and reserves after paying back depositors. This leads to less lending in the second period, because banks are restricted by capital adequacy parameters. We also show that when the risk of a shock is low, increased competition raises GDP (in expectation), as well as expected second period activity. Our main result is to show that within and between types of equilibria, increased competition always leads to increased variance in second period

economic activity. This is true when increasing competition compares collusion, Cournot and Bertrand competition; as well as when increased competition is due to having more banks under Cournot competition.

The paper also examines the role of the banking regulator. In the model, capital adequacy rules can be used to exclude *imprudent* equilibria. This is consistent with the observation that required capital ratios have risen in response to the 2008 financial crisis. We also show that predefined regulatory forbearance in the aftermath of a shock can be used to reduce or even to eliminate the amplification effect. However, we also show that when banks predict that regulatory forbearance is adjusted in response to the magnitude of the shock, the effects of forbearance on economic activity are ambiguous. It is possible for anticipated forbearance to increase the variance of second period activity by encouraging first period lending, resulting in a larger amplification of the initial shock.

In the extensions we show that other ways of increasing leverage may also lead to increased instability. In particular, banking competition may add to the the instability caused by financial liberalization of domestic financial markets.

A worthwhile extension of this approach would be to have entrepreneurs differentiated by their capital endowments, and have credit rationing driven by informational asymmetries or legal deficiencies. Such a model would allow us to study the interaction between the legal protection for lenders and the effects of banking competition, or the interaction between the distribution of wealth, competition and stability.

Another extension is to endogenize banking capital. One possibility is to fix a minimum capital stock to have a bank and then allow free entry (or, have an exogenous fixed cost of setting a bank). In that case regulatory policy, by setting the minimum capital to establish a bank would determine the intensity of competition in the case of Cournot competition. Alternatively, we could fix the number of banks and allow capital to flow into them until the expected return on investment in banks is the same as the return in other sectors (zero in our case).

Finally, we can consider alternative policies in the event of a productivity shock leading to

a collapse. A possibility is the government recapitalizing banks as was done to some banks in the 2008 financial crisis. If the banks become the property of government (and managed by government-named controllers), from the point of view of the original controllers, at $t = 0$, the strategic problem is the same as the one we have analyzed before. Here there is no additional incentive to be imprudent, but if the initial controllers retain ownership rights, they have additional incentives to be imprudent.

References

Daron Acemoglu, Asuman Ozdaglar, and Alireza Tahbaz-Salehi. Systemic risk and stability in financial networks. Working Paper 18727, National Bureau of Economic Research, January 2013.

Franklin Allen and Douglas Gale. Competition and financial stability. *Journal of Money, Credit and Banking*, 36(3):453–479, June 2004.

Franklin Allen, Ana Babus, and Elena Carletti. Financial connections and systemic risk. Working paper 16177, NBER, Cambridge, MA, July 2010.

Rabah Amir and Val E. Lambson. On the effects of entry in Cournot markets. *Review of Economic Studies*, 67:235–254, 2000.

Deniz Anginer, Asli Demirgüç-Kunt, and Min Zhu. How does bank competition affect systemic stability? Working Paper 5981, The World Bank Development Research Group, February 2012.

Felipe Balmaceda and Ronald Fischer. Economic performance, creditor protection and labour inflexibility. *Oxford Economic Papers*, 62(3):553–577, 2010.

Thorsten Beck. Bank competition and financial stability: Friends or foes? Policy Reseacrh Working Paper 4656, The World Bank, Washuington, DC, June 2008.

Thorsten Beck, Olivier De Jonghe, and Glenn Schepens. Bank competition and stability: Cross-country heterogeneity. *Journal of Financial Intermediation*, 22(2):218 – 244, October 2013.

Philip G. Berger and Eli Ofek. Diversification's effect on firm value. *Journal of Financial Economics*, 37(1):39 – 65, 1995. ISSN 0304-405X.

John H. Boyd and Gianni De Nicoló. The theory of bank risk taking and competition revisited. *The Journal of Finance*, 60(3):1329–1343, June 2005.

Elena Carletti and Agnese Leonello. Credit market competition and liquidity crises. EUI Working Paper 14, European University Institute Department of Economics, April 2012.

Rodrigo Cifuentes, Hyun Hong Shin, and Gianluigi Ferrucci. Liquidity risk and contagion. *Journal of the European Economic Association*, 3(2-3):556–566, April-May 2005.

Ethan Cohen-Cole, Eleonora Patacchini, and Yves Zenou. Systemic risk and network formation in the interbank market. Discussion Paper 8332, CEPR, April 2011.

Gianni De Nicolo, John H. Boyd, and Abu M. Jalal. Bank competition, risk and asset allocations. Working paper 09-143, International Monetary Fund, July 2009.

Hans Degryse and Steven Ongena. *Handbook of Financial Intermediation and Banking*, chapter Competition and Regulation in the Banking Sector: A Review of the Empirical Evidence on the Sources of Bank Rents, pages 483–554. Elsevier, 2008.

Douglas W. Diamond and Philip H. Dybvig. Bank runs, deposit insurance, and liquidity. *Journal of Political Economy*, 91(3):pp. 401–419, 1983.

Avinash Dixit. Comparative statics for oligopoly. *International Economic Review*, 27 27(1):107–122, February 1986.

Franklin R. Edwards and Frederic S. Mishkin. The decline of traditional banking: Implications for financial stabilityand regulatory policy. NBER Working Papers 4993, National Bureau of Economic Research, Cambridge, MA., August 1995.

Xavier Freixas and Kevin Ma. Reconciling views on banking competition and stability: The role of leverage. October 2012.

Co-Pierre George. The effect of the interbank network structure on contagion and common shocks. Discussion Paper Series 2, 12, Deutshe Bundesbank, Frankfurt am Main, 2011.

Gary Gorton and Andrew Winton. Financial intermediation. In G.M. Constantinides, M. Harris, and R. M. Stulz, editors, *Handbook of the Economics of Finance*, volume 1 of *Handbook of the Economics of Finance*, chapter 8, pages 431–552. Elsevier, March 2003.

Hendryk Hakenes and Isabel Schnabel. Capital regulation, bank competition, and financial stability. *Economic Letters*, 113:256–258, 2011.

Robert Hauswald and Robert Marquez. Competition and strategic information acquisition in credit markets. *Review of Financial Studies*, 19(3):967–1000, Fall 2006.

Thomas F. Hellman, Kevin C. Murdock, and Joseph E. Stiglitz. Liberalization, moral hazard in banking, and prudential regulation: Are capital requirements enough? *American Economic Review*, 90(1):147–165, March 2000.

Michael C. Keeley. Deposit insurance, risk, and market power in banking. *American Economic Review*, 80(5):1183–1200, 1990.

Nobuhiro Kiyotaki and John Moore. Credit cycles. *Journal of Political Economy*, 105(2):211–248, April 1997.

David Martinez-Miera and Rafael Repullo. Does competition reduce the risk of bank failure? *Journal of Money*, 23(10):3638–3664, October 2010.

Stephen Morris and Hyun Song Shin. Global games: Theory and applications. In L. Hansen M. Dewatripont and S. Turnovsky, editors, *Advances in Economics and Econometrics*, Proceedings of the Eighth World Congress of the Econometric Society, chapter 3. Cambridge University Press, 2003.

Rafael Repullo. Capital requirements, market power, and risk-taking in banking. *Journal of Financial Intermediation*, 13(2):156–182, April 2004.

Rafael Repullo and Javier Suarez. The procyclical effects of basel ii. Discussion paper 6862, CEPR, June 2008.

Rafael Repullo and Javier Suarez. The procyclical effects of bank capital regulation. *Review of Financial Studies*, 2013.

Jean-Charles Rochet and Xavier Vives. Coordination failures and the lender of last resorter. *Journal of the European Economic Association*, 2(6):1116–1147, December 2004.

Jesus Saurina-Salas, Gabriel Jimenez, and Jose A. Lopez. How does competition impact bank risk taking? Working Paper 23, Federal Reserve Bank of San Francisco, San Francisco, February 2007.

Klaus Schaeck, Martin Cihak, and Simon Wolfe. Are competitive banking systems more stable? *Journal of Money, Credit and Banking*, 41(4):711–734, June 2009.

Steven A Sharpe. Asymmetric information, bank lending, and implicit contracts: A stylized model of customer relationships. *Journal of Finance*, 45(4):1069–87, September 1990.

Joseph E. Stiglitz and Andrew Weiss. Credit rationing in markets with imperfect information. *American Economic Review*, 71(3):393–410, 1981.

Jean Tirole. *The Theory of Corporate Finance*. Princeton University Press, 2006.

Xavier Vives. Competition and stability in banking. Working Paper 852, IESE Business School–University of Navarra, April 2010.

Ernst-Ludwig von Thadden. Asymmetric information, bank lending and implicit contracts: the winner's curse. *Finance Research Letters*, 1(1):11 – 23, 2004. ISSN 1544-6123.

Wolf Wagner. Loan market competition and bank risk-taking. *Journal of Financial Services Research*, 37:71–81, 2010.

Yishay Yafeh and Oved Yosha. Industrial organization of financial systems and strategic use of relationship banking. *European Finance Review*, 5:63–78, 2011.

A Appendix: For Online Publication

Proposition 1 *At date $t = 1$, each bank makes loans of $l_1^s = \min\left\{\frac{e_1^s}{\alpha_1}, \lambda l_0\right\}$ and takes deposits of $d_1^s = (1 - \alpha_1^s)l_1^s$ and pays dividends of $Div^s = e_1^s - \alpha_1 l_1^s$.*

Proof: Rearranging terms and using the equality condition $e_1^s + d_1^s - l_1^s = Div^s$ plus $1 + r_1 = \frac{y_2}{\lambda}$, the problem of the bank is:

$$\underset{l_1^s, Div^s}{Max} \; \beta\left(\frac{y_2}{\lambda} - 1\right)l_1^s + (1 - \beta)Div^s + \beta e_1^s$$

$$\text{s.t.} \quad e_1^s \geq \alpha_1 l_1^s + Div^s$$

$$\lambda l_0 \geq l_1^s$$

Now, note that by Assumption 2 the first term in the objective function is positive. Moreover, Assumption 2 ensures that $\beta\left(\frac{y_2}{\lambda} - 1\right) > (1 - \beta)$, so the objective function increases more with l_1^s than it increases with Div^s. Hence, Div^s is positive only if the second restriction is binding. Therefore, it is direct that in equilibrium:

$$l_1^s = \min\left\{\frac{e_1^s}{\alpha_1}, \lambda l_0\right\}$$

∎

Lemma 1 (*Imprudent* equilibria) *There is a unique equilibrium of each type (prudent, imprudent) to the game among banks, i.e.,*

$$\frac{\partial^2 \Pi_i}{\partial l_i^2} < 0$$

and

$$\frac{\frac{\partial^2 \Pi_i}{\partial l_i^2}}{\frac{\partial^2 \Pi_i}{\partial l_i \partial l_j}} > 1$$

Proof: Case of imprudent equilibrium:

43

$$\frac{\partial \Pi_i}{\partial l_i} = \beta p(\Psi - \phi)$$

and thus

$$\frac{\partial^2 \Pi_i}{\partial l_i^2} = \beta p \frac{\partial \Psi}{\partial l_i} = -\beta p \left(\frac{2}{P_e G'(G^{-1}(L))} - \frac{l_i G''(G^{-1}(L))}{P_e G'^3(G^{-1}(L))} \right) < 0$$

and

$$\frac{\partial^2 \Pi_i}{\partial l_i \partial l_j} = \beta p \frac{\partial \Psi}{\partial l_j} = -\beta p \left(\frac{1}{P_e G'(G^{-1}(L))} - \frac{l_i G''(G^{-1}(L))}{P_e G'^3(G^{-1}(L))} \right) < 0$$

From which we derive:

$$\frac{\frac{\partial^2 \Pi_i}{\partial l_i^2}}{\frac{\partial^2 \Pi_i}{\partial l_i \partial l_j}} = \frac{\left(\frac{2}{P_e G'(G^{-1}(L))} - \frac{l_i G''(G^{-1}(L))}{P_e G'^3(G^{-1}(L))} \right)}{\left(\frac{1}{P_e G'(G^{-1}(L))} - \frac{l_i G''(G^{-1}(L))}{P_e G'^3(G^{-1}(L))} \right)} > 1$$

∎

Proposition 2 *As competition decreases, the* prudent *equilibrium becomes more attractive compared to the* imprudent *equilibrium (* $\frac{\partial(\Pi_\star^P - \Pi_\star^I)}{\partial v} > 0$ *).*

Proof:: Assume first that the solution to both problems is interior. Then define the following optimization program:

$$W(x) \equiv \underset{l_0 \geq 0}{\text{Max}} \; x\Pi^P + (1-x)\Pi^I$$

which is the convex combination of the banks' problem in the two types of equilibrium. Rewriting, and using expressions 15–19 we obtain

$$W(x) \equiv \underset{l_0 \geq 0}{\text{Max}} \; x \left[\beta p((1+r_0) - \phi)l_0 + H(q(1+r_0) - 1)l_0 + (\beta p + H - 1)e_0 \right]$$

$$+ (1-x)[\beta p((1+r_0) - \phi)l_0 + (\beta p - 1)e_0]$$

Using the Envelope Theorem and the definition of e_1^l from equation 7:

$$W'(x) = H[(q(1 + r_0^\star(x)) - 1)l_0^\star(x) + e_0] \equiv He_1^{l\star}(x)$$

44

Thus:

$$\Pi^P_\star - \Pi^I_\star = W(1) - W(0) = \int_0^1 H e_1^{l\star}(x) dx$$

By taking derivatives with respect to v we obtain:

$$\frac{\partial \Delta W}{\partial v} = H \int_0^1 \frac{\partial e_1^{l\star}(x)}{\partial v} dx > 0$$

We prove that $\frac{\partial e_1^{l\star}(x)}{\partial v} > 0$ ($\forall x$). In effect:

$$\frac{\partial e_1^{l\star}(x)}{\partial v} = (q\Psi(x) - 1)\frac{\partial l_0^\star}{\partial v}$$

The derivative on the RHS is negative by proposition 4 below. For the other term in the RHS, note that from the FOC of the first period bank's problem,

$$[\beta p + Hqx]\Psi(x) = \beta p\phi + Hx$$

and thus

$$\Psi(x) = \frac{\beta p\phi + Hx}{\beta p + Hqx}$$

Finally, it is easy to check that:[34]

$$1 - q\Psi(x) = \frac{\beta p(1 - q\phi)}{\beta p + Hqx} > 0$$

finally, consider the case in which the capital adequacy conditions constrain lending in the *imprudent* equilibrium. Figure 4 shows the possible configurations and it is clear that the result continues to hold in this case.

∎

Proposition 4 *We show that as competition increases in banking (lower v), first period lending*

---

[34]This result is true when the FOC hold with equality. At $v = 0$ (Bertrand) there is the possibility of corner solutions. where the result does not necessarily hold, because solutions are of the bang-bang type.
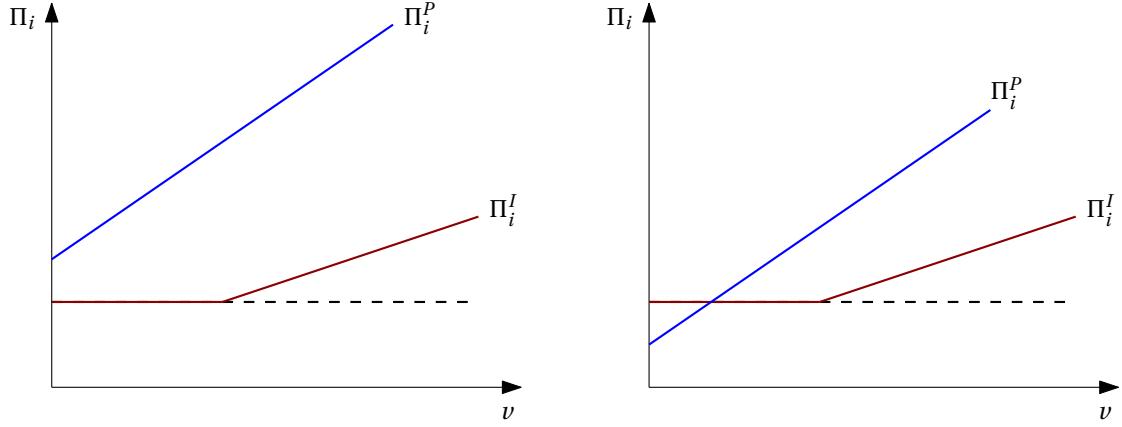
Figure 4: Profit differences increase when the imprudent equiilibrium is credit constrained for a range of competition parameters

increases in the case of an imprudent equilibrium.

Proof: Note that:

$$\frac{\partial^2 \Pi_0^I}{\partial l_0 \partial v} = -\frac{\beta p}{(p + (1-p)q)} \frac{d}{dl_0}\left(\frac{l_0 \frac{dL}{dv}}{G'(G^{-1}(L))}\right)$$

Where,

$$\frac{d}{dl_0}\left(\frac{l_0 \frac{dL}{dv}}{G'(G^{-1}(L))}\right) = \left(\frac{\frac{dL}{dv} + l_0 \frac{d^2 L}{dl_0 dv}}{G'(G^{-1}(L))}\right) - \left(\frac{l_0 \frac{dL}{dv} G''(G^{-1}(L))}{G'(G^{-1}(L))}\right)$$

Given that $G(\cdot)$ is assumed to be increasing and concave we conclude that the RHS of the expression above is strictly positive and therefore we get the result. ■

Proposition 5 *When the risk of a shock is low, increased competition raises expected GDP and second period activity in a* prudent *equilibrium.*

Proof: The expected value of GDP over the two periods is:

$$Y^P = p[(y_1 - 1) + (y_2 - \lambda)]l_0^\star + (1-p)[q(y_1 - 1)l_0^\star + \frac{e_1^{l\star}}{\lambda \alpha_1^l}(y_2 - \lambda)] + \int_{\bar{U}(r_0^\star)}^{G_{max}} u \, dG(u)$$

The effect of an increase in the degree of competition among banks can be written as:

$$\frac{dY^P}{dv} = \{[p+(1-p)q](y_1-1)+p(y_2-\lambda)\}\frac{dl_0^\star}{dv} + \frac{(1-p)}{\lambda\alpha_1^l}(y_2-\lambda)\frac{de_1^{l\star}}{dv} - U(r_0^\star)G'(U(r_0^\star))\left(\frac{-\frac{dl_0^\star}{dv}}{P_e G'(G^{-1})(L^\star)}\right)$$

Where the first and third terms are strictly negative, while the second term is positive, so the sign of the expression is ambiguous. The effect of competition on second period activity can be written as:

$$(y_2-\lambda)\left[p\frac{dl_0^*}{dv} + \frac{(1-p)}{\lambda\alpha_1}\frac{de_1^*}{dv}\right]$$

Evaluating at the polar cases $p = 0, 1$ we see that when the risk of a shock is low ($p \approx 1$) increased competition is beneficial and raises second period activity by proposition 4 (and therefore GDP is unambiguously higher). On the other hand, when the risk of a shock is large ($p \approx 0$), increased competition decreases second period GDP. ∎

Lemma 2 *Increased banking competition (lower v) decreases the range of shocks ($q \in [\hat{q}, 1]$) for which the prudent equilibria ($e_1^l(l_0^{Pr*}; q) > 0$) are well defined.*

Proof: From the definition in (8),

$$\hat{q} \equiv \frac{1 - \frac{e_0}{l_0^\star}}{1 + r_0(L(l_0^\star))} \tag{23}$$

By implicit derivation, we obtain:

$$\frac{d\hat{q}}{dv} = \left(\frac{e_0}{l_0^{\star 2}(1+r_0)}\right)\frac{dl_0^\star}{dv} - \left(\frac{1 - \frac{e_0}{l_0^\star}}{(1+r_0)^2}\right)r_0'(L^\star)N\frac{dl_0^\star}{dv} < 0$$

Because Proposition 4 shows that $\frac{dl_0^\star}{dv} > 0$.

∎

Proposition 6 *Consider a prudent equilibrium. Increased banking competition (lower v) leads to larger reductions in lending in the second period in the case of a shock. This implies increased variance in second period economic results.*

' **Proof:** Using the capital market equilibrium condition (13) we have:

$$\theta = \frac{q\left(y_1 - \frac{G^{-1}(L(l_0^\star))}{p+(1-p)q}\right) - 1 + \frac{e_0}{l_0^\star}}{\lambda\alpha_1}$$

By implicit derivation we have:

$$\frac{d\theta}{dv} = \frac{\frac{dl_0^\star}{dv}}{\lambda\alpha_1}\left(\frac{qN}{G'(G^{-1}(L)(p+(1-p)q))} + \frac{e_0}{l_0^{\star 2}}\right) > 0$$

For the last part of the proposition, recall that the variance of second period loans can be written as $\text{Var}(l_0^*) = p(1-p)(1-\theta)l_0^*$. This proves the result. ∎

Proposition 7 *Consider a prudent equilibrium. Assume that under Bertrand competition, first period lending is interior to the capital adequacy constraint. Then less risk (higher q) leads to more lending in the first period (higher l_0^*).*

Proof: The First Order Conditions of the first period maximization problem (21) imply:

$$(\beta p + Hq)\left(y_1 - \frac{G^{-1}(L^\star)}{p+(1-p)q} - \frac{[1+(N-1)v]l_0^\star}{(p+(1-p)q)G'(G^{-1}(L^\star))}\right) =$$

$$H + \beta p \underbrace{(1 + \lambda\alpha_1(1-\beta) - \beta(y_2 - \lambda))}_{\phi>0} > 0,$$

where the sign is derived from Assumption 3. Hence, the term in the large parenthesis is positive,

$$\Psi \equiv y_1 - \frac{G^{-1}(L^\star)}{p+(1-p)q} - \frac{[1+(N-1)v]l_0^\star}{(p+(1-p)q)G'(G^{-1}(L^\star))} > 0$$

Implicit differentiation of the First Order Conditions leads to (recall that the second term does

48

not involve q)

$$\frac{dl_0^\star}{dq} =$$

$$\frac{H\left(y_1 - \frac{G^{-1}(L^\star)}{p+(1-p)q} - \frac{[1+(N-1)v]l_0^\star}{(p+(1-p)q)G'(G^{-1}(L^\star))}\right) + (\beta p + Hq)\frac{(1-p)}{p+(1-p)q}\left(\frac{G^{-1}(L^\star)}{p+(1-p)q} + \frac{[1+(N-1)v]l_0^\star}{(p+(1-p)q)G'(G^{-1}(L^\star))}\right)}{(\beta p + Hq)\left(2\frac{[1+(N-1)v]}{(p+(1-p)q)G'(G^{-1}(L^\star))} - \frac{G''(G^{-1}(L^\star))[1+(N-1)v]^2l_0^\star}{(p+(1-p)q)G'^3(G^{-1}(L^\star))}\right)}$$

In this expression, the denominator is positive, so the sign of (dl_0^*/dq) is given by the sign of the numerator. Reorganizing terms, the numerator becomes:

$$\Psi\left\{\beta p(1-p)\left(\frac{\beta}{\alpha_1}\left(\frac{y_2}{\lambda} - (1-\alpha_1)\right) - 1\right)\right\} + y_1(\beta p + Hq)\frac{(1-p)}{p+(1-p)q}$$

Since we have shown that $\Psi > 0$ and all remaining terms in the numerator are positive.

<div style="text-align:right">■</div>

Proposition 8 (Regulatory forbearance) *In a prudent equilibrium, if the financial regulator practices forbearance in the bad state ($\alpha_1^l < \alpha_1^h$), the variance of the second period outcome decreases because l_0 decreases, i.e., $\partial l_0/\partial\alpha_1^l < 0$*

Proof: Recall that $\text{Var}(l_1) = p(1-p)(\lambda l_0 - l_1^l)^2$. From the definition of e_1^l in (7) we have that $e_1^l = q(1+r_0)l_0 - (l_0 - e_0)$, and $l_1^l = e_1^l/\alpha_1^l$. Therefore:

$$\frac{dl_1^l}{d\alpha_1^l} = \underbrace{-\frac{e_1^l}{\alpha_1^{l2}}}_{\text{Static Effect}<0} + \underbrace{\frac{1}{\alpha_1^l}\frac{de_1^l}{dl_0}\frac{dl_0}{d\alpha_1^l}}_{\text{Strategic Effect}} \tag{24}$$

The static effect corresponds to an unexpected change in the capital adequacy parameter, and its sign is always negative or zero. The second term in the RHS corresponds to the changes

induced by the knowledge that, in case of a shock, the regulator will exercise forbearance. Now

$$\frac{d\text{Var}(l_1)}{d\alpha_1^l} = p(1-p)2(\lambda l_0 - l_1^l)\left(\lambda\frac{dl_0}{d\alpha_1^l} - \frac{dl_1^l}{d\alpha_1^l}\right)$$

$$= p(1-p)2(\lambda l_0 - l_1^l)\left\{\underbrace{\left(\lambda - \frac{1}{\alpha^l}\frac{de_1^l}{dl_0}\right)}_{>0}\frac{dl_0}{d\alpha_1^l} + \frac{e_1}{\alpha_1^{l2}}\right\}$$

(25)

because $(de_1^l/dl_0) = q\Psi - 1 < 0$. We need to determine the sign of $(dl_0/d\alpha_1^l)$. In a *prudent* equilibrium

$$\frac{dl_0}{d\alpha_1^l} = \frac{H'(\alpha_1^l)(1-q\Psi)}{-(\beta p + Hq)\left(2\frac{[1+(N-1)v]}{(p+(1-p)q)G'(G^{-1}(L^\star))} - \frac{G''(G^{-1}(L^\star))[1+(N-1)v]^2 l_0^\star}{(p+(1-p)q)G'^3(G^{-1}(L^\star))}\right)}$$

where the denominator corresponds to the second order condition and is therefore negative. Thus the sign of the derivative $(dl_0/d\alpha_1^l)$ corresponds to the sign of

$$-H'(\alpha_1)(1-q\Psi) > 0$$

Since $(1-q\Psi) > 0$ (at the optimum) and $H'(\alpha_1) < 0$.

∎

The following proofs correspond to section 7.3, on Cournot competition. They are essential to prove all of the previous propositions for the case of increasing the number of banks in Cournot equilibrium.

Lemma 3 *In the Cournot equilibrium, as the number of banks increases, individual bank first period loans decrease:* $\frac{dl_0}{dN} < 0$.

Proof: The FOC associated to banks' optimization problem can be expressed as (here $v = 0$):

$$\Psi = y_1 - \frac{G^{-1}(Nl_0)}{p+(1-p)q} - \frac{l_0}{(p+(1-p)q)G'(G^{-1}(Nl_0))} = \frac{\beta p\phi + H}{\beta p + Hq}$$

50

Implicitly differentiating with respect to N, we get:

$$\frac{dl_0}{dN} = \frac{\frac{l_0^2 G''(G^{-1}(Nl_0))}{G'^3(G^{-1}(Nl_0))} - \frac{l_0}{G'(G^{-1}(Nl_0))}}{\frac{N+1}{G'(G^{-1}(Nl_0))} - \frac{Nl_0 G''(G^{-1}(Nl_0))}{G'^3(G^{-1}(Nl_0))}}$$

The result follows from the fact that $G(\cdot)$ is increasing and concave. ∎

Lemma 4 *As the number of banks increases, the aggregate amount of first period loans increases*

$$\frac{d(Nl_0(N))}{dN} > 0$$

Proof: Differentiating the FOC with respect to N, we get:

$$\frac{dL}{dN} = \frac{\frac{dl_0}{dN}}{l_0 \frac{G''(G^{-1}(L))}{G'^2(G^{-1}(L))} - 1} > 0$$

Where the last inequality is a consequence of Claim 1 and the concavity of $G(\cdot)$. ∎

Lemma 5 $\Omega^H(L) + \Omega^L(L) \geq 0$

Proof: Note that:

$$\Omega^H(L) + \Omega^L(L) = (\beta p + Hq)(1 + r_0) - (\beta p \phi + H) = \frac{(\beta p + Hq)l_0}{(p + (1-p)q)G'(G^{-1}(L))} > 0$$

The second equality is obtained from the FOC. ∎

Proposition 9 *As the number of banks increases, their profits decrease* $\frac{d\pi}{dN} < 0$

Proof: Given that $\pi = (\Omega^H(L(N)) + \Omega^L(L(N)))l_0(N) + $ constant, we have that:

$$\begin{aligned}
\frac{d\pi}{dN} &= l_0(N)\frac{d}{dN}(\Omega^H(L(N)) + \Omega^L(L(N))) + (\Omega^H + \Omega^L)\frac{dl_0}{dN} \\
&= -l_0(N)\left(\frac{\beta p + Hq}{G'(G^{-1}(L))}\right)\frac{dL}{dN} + (\Omega^H + \Omega^L)\frac{dl_0}{dN} \\
&< 0
\end{aligned}$$

∎

The following three results prove claims made in the text.

Result 1 *Assumption 3, i.e., $\beta p + H > 1$ implies that the average value of an additional unit of bank capital at $t = 1$, discounted to $t = 0$, is bigger than one, i.e., it is profitable on average to have more period 1 capital.*

Proof: To see this, observe first that

$$\frac{\beta}{\alpha_1}\left[\frac{y_2}{\lambda} - (1 - \alpha_1)\right] > 1 \tag{26}$$

implies that in the bad state of the world the bank prefers to invest its remaining capital rather than not lend it. On the other hand, additional capital in the good state of the world is useless, since it is plentiful, and the excess may as well be paid out in dividends. Now note the assumption 3 can be written as:

$$\beta p + H = \beta p \cdot 1 + \beta(1 - p)\left[\frac{\beta}{\alpha_1}\left(\frac{y_2}{\lambda} - (1 - \alpha_1)\right)\right] > 1.$$

Since $\beta p < 1$, assumption 3 implies that equation (26) holds. ∎

Result 2 *A sufficient condition for the* prudent *equiilibria to be interior to the capital adequacy constraint is that*

$$\beta p \phi + H > 0$$

Proof: To see this, observe that

$$\frac{\partial \Pi^p}{dl_0} = 0 \Rightarrow \Psi = \frac{\beta p \phi + H}{\beta p + Hq}$$

in the Bertrand case, $\Psi = 1 + r_0$ so that if $\beta p \phi + H \leq 0 \Rightarrow 1 + r_0 \leq 0$ and in a *prudent* equilibrium lenders will always lend to the capital adequacy constraint. Under less competitive environments it will also be the case that the capital adequacy condition is not binding. ∎

For the next result, we define a consistent regulator as one that defines *ex ante* the values of the capital adequacy parameter in the case of shock and no shock. An inconsistent regulator sets the values of α_1^l *ex post* after observing the value of equity after the shock, and so as to neutralize the effects of the shock on lending.

Result 3 *When the regulator is inconsistent, first period loans in the prudent equilibrium (l_0^{Inc}) are higher than in the case of consistent behavior.*

Define the *Inconsistent Policy* as one where the regulator intervenes by adjusting the capital adequacy parameter α_1^l so that there is no credit rationing and thus the shock has no effect on the economy:

$$\alpha_1^l : \quad \frac{e_1^l(L)}{\alpha_1^l} = \lambda l_0$$

Banks anticipate the future behavior of the regulator in case of a a shock. Note that now the capital adequacy conditions in the bad state depend on first period loans by all firms: $\alpha_1^l(l_{i0}, l_{-i0})$. Thus only Ω^- changes (where the subscript *Inc* denotes that it corresponds to the inconsistent policy):

$$\Omega^-(L) = \beta(q(1 + r_0(L)) - (1 - y_2 + \lambda))l_0$$

In deciding their initial period loans, banks take into account the future behavior of the regulator. Each bank solves:

$$\max_{l_0} \left(\Omega^h(L) + \Omega_{Inc}^-(L) \right) l_0 + \text{constants}$$

with FOC:

$$\beta p(\Psi - \phi) + \beta^2(1 - p)(q\Psi - (1 - y_2 + \lambda)) = 0$$

To show that $l_0^{Inc} > l_0^{Cons}$ (where *Cons* indicates values associated to the consistent equilibrium,

where α_1^l is predetermined), we evaluate l_0^{Cons} at the above FOC. that is

$$
\begin{aligned}
\frac{d\Pi^{Inc}}{dl_0}(l_0^{Cons}) &= \beta p(\Psi - \phi) + \beta^2(1-p)(q\Psi - (1 - y_2 + \lambda)) \\
&= -H(q\Psi(l_0^{Cons}) - 1) + \beta^2(1-p)(q(\Psi(l_0^{Cons}) - (1 - y_2 + \lambda)) \\
&> 0
\end{aligned}
$$

The second equality corresponds to the FOC in the consistent case. To show that the inequality

holds we show that:

$$
H(q\Psi(l_0^{Cons})-1) = \frac{\beta^2(1-p)}{\alpha_1^L}\left(\frac{y_2}{\lambda} - (1 - \alpha_1^L)\right)(q\Psi(l_0^{Cons})-1) < \beta^2(1-p)(q\Psi(l_0^{Cons})-(1-y_2+\lambda))
$$

Consider first that in the Consistent case, $\alpha_1^l = \alpha_0$. Simplifying, we have:

$$
\frac{1}{\alpha_0}\left(\frac{y_2}{\lambda} - 1\right)(q\Psi(l_0^{Cons}) - 1) < y_2 - \lambda
$$

As $y_2 > \lambda$ and $q\Psi(l_0^{Cons}) - 1 < 0$ we have the required inequality. Hence,

$$
\frac{d\Pi^{Inc}}{dl_0}(l_0^{Cons}) > 0
$$

and therefore $l_0^{Inc} > l_0^{Cons}$. Observe that the proof works for any level of α_1^l used by the consistent

regulator (and not only when $\alpha_1^l = \alpha_0$). ∎

Centro de Economía Aplicada
Departamento de Ingeniería Industrial
Universidad de Chile

2015

2014

306. Assessing the extent of democratic failures. A 99%-Condorcet's Jury Theorem.
Matteo Triossi

2013

305. The African Financial Development and Financial Inclusion Gaps
Franklin Allen, Elena Carletti, Robert Cull, Jun "Qj" Qian, Lemma Senbet y Patricio Valenzuela

304. Revealing Bargaining Power through Actual Wholesale Prices
Carlos Noton y Andrés Elberg

303. Structural Estimation of Price Adjustment Costs in the European Car Market
Carlos Noton

302. Remedies for Sick Insurance
Daniel McFadden, Carlos Noton y Pau Olivella

301. Minimum Coverage Regulation in Insurance Markets
Daniel McFadden, Carlos Noton y Pau Olivella

300. Rollover risk and corporate bond spreads
Patricio Valenzuela

299. Sovereign Ceilings "Lite"? The Impact of Sovereign Ratings on Corporate Ratings
Eduardo Borensztein, Kevin Cowan y Patricio Valenzuela

298. Improving Access to Banking: Evidence from Kenya
F. Allen, E. Carletti, R. Cull, J."Qj" Qian, L. Senbet y P. Valenzuela

297. Financial Openness, Market Structure and Private Credit: An Empirical Investigation
Ronald Fischer y Patricio Valenzuela

296. Banking Competition and Economic Stability
Ronald Fischer, Nicolás Inostroza y Felipe J. Ramírez

295. Trust in Cohesive Communities
Felipe Balmaceda y Juan F. Escobar

294. A Spatial Model of Voting with Endogenous Proposals: Theory and Evidence from Chilean Senate
Matteo Triossi, Patricio Valdivieso y Benjamín Villena-Roldán

2012

293. Participation in Organizations, Trust, and Social Capital Formation: Evidence from Chile
Patricio Valdivieso - Benjamín Villena-Roldán

292. Neutral Mergers Between Bilateral Markets
Antonio Romero-Medina y Matteo Triossi

291. On the Optimality of One-size-fits-all Contracts: The Limited Liability Case
Felipe Balmaceda

2011

2010

Eduardo Engel, Ronald Fischer y Alexander Galetovic

275. The Cost of Moral Hazard and Limited Liability in the Principal-Agent Problem
F. Balmaceda, S.R. Balseiro, J.R. Correa y N.E. Stier-Moses

274. Structural Unemployment and the Regulation of Product Market
Alexandre Janiak

273. Non-revelation Mechanisms in Many-to-One Markets
Antonio Romero-Medina y Matteo Triossi

272. Labor force heterogeneity: implications for the relation between aggregate volatility and government size
Alexandre Janiak y Paulo Santos Monteiro

271. Aggregate Implications of Employer Search and Recruiting Selection
Benjamín Villena Roldán

270. Wage dispersion and Recruiting Selection
Benjamín Villena Roldán

269. Parental decisions in a choice based school system: Analyzing the transition between primary and secondary school
Mattia Makovec, Alejandra Mizala y Andrés Barrera

268. Public-Private Wage Gap In Latin America (1999-2007): A Matching Approach
(Por aparecer en Labour Economics, (doi:10.1016/j.labeco.2011.08.004))
Alejandra Mizala, Pilar Romaguera y Sebastián Gallegos

267. Costly information acquisition. Better to toss a coin?
Matteo Triossi

266. Firm-Provided Training and Labor Market Institutions
Felipe Balmaceda

2009

265. Soft budgets and Renegotiations in Public-Private Partnerships
Eduardo Engel, Ronald Fischer y Alexander Galetovic

264. Information Asymmetries and an Endogenous Productivity Reversion Mechanism
Nicolás Figueroa y Oksana Leukhina

263. The Effectiveness of Private Voucher Education: Evidence from Structural School Switches
(Publicado en Educational Evaluation and Policy Analysis Vol. 33 Nº 2 2011. pp. 119-137)
Bernardo Lara, Alejandra Mizala y Andrea Repetto

262. Renegociación de concesiones en Chile
(Publicado como "Renegociación de Concesiones en Chile". Estudios Públicos, 113, Verano, 151–205. 2009)
Eduardo Engel, Ronald Fischer, Alexander Galetovic y Manuel Hermosilla

* Para ver listado de números anteriores ir a http://www.cea-uchile.cl/.